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Inflation-deflation expectations and economic stability in a Kaleckian system

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Inflation-deflation expectations and economic stability in a Kaleckian system

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Abstract

In this paper, we analyze the impact of inflation-deflation expectations and the effect of the monetary authority's inflation-targeting policy in a Kaleckian system, on the basis of macro-foundation approaches. For this purpose, we build a dynamical system composed of four variables (the rate of utilization, the wage share, the nominal rate of interest and the expected rate of inflation) and examine the properties, especially stability, of this dynamical system. We then find that the existence of (adaptive) inflation-deflation expectations always destabilizes our Kaleckian system (irrespective of the revision speed of expectations) while that the monetary authority's intensive inflation-targeting policy can make the system stable but the effect of this policy depends heavily upon the public credibility of it. We also perform numerical simulations to check that our analysis is valid.

Keywords: Kaleckian economics; Economic stability; Inflation-deflation expectations; Inflation-targeting policy
JEL Classification: E11; E12; E32

1 Introduction

Today, it is widely thought that macroeconomics should be based upon microeconomics, i.e., that macroeconomics needs a rigorous micro-foundation. This way of thinking is seen in a lot of contemporary representative textbooks (e.g., Woodford 2003; Romer 2012). At first sight, micro-foundations may seem highly reasonable processes to build macroeconomics because the macro economy consists of a myriad of micro agents, but they may provide a wrong description of the true macro economy. In micro-foundations, usually, it is supposed that only one type of agent exists and that s/he makes rational choices. As a consequence, there are no failures in coordination or malfunctions in markets, and involuntary unemployment or deficiency of aggregate demand cannot exist. In this way, a lot of realistic aspects of the macro economy are lost through micro-foundations.

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An alternative approach of macroeconomics was proposed by Carl Chiarella with Peter Flaschel under the name of "macro-foundations" (e.g., Chiarella and Flaschel 2000). In macro-foundations, the macro economy is treated as an aggregate system and is modeled as such. In other words, the traditional Keynesian approaches are adopted in both static and dynamic ways. In particular, Chiarella and Flaschel emphasized the importance of dynamics driven by the Keynesian macro-foundations such as business cycles and growth cycles. Various mathematical methods of dynamic analysis were supplied by them. The macro-foundation approaches were also succeeded in other works by Chiarella, such as Asada et al. (2003), Chiarella et al. (2003), Chiarella et al. (2005), Asada et al. (2006), Asada et al. (2010) and Chiarella et al. (2013). We believe that the macro-foundations proposed by Chiarella and others offer the only way to discuss economic implications of the dynamics of the macro economy.

Post Keynesian economists have taken up inter-class distribution of aggregate income as the main subject matter following the tradition of Kaldor (1955-1956) and Pasinetti (1962), and the early post Keynesian theory of economic growth dealt mainly with full-employment situations. Since Asimakopulos (1975), Rowthorn (1977, 1981) and Dutt (1984) laid the cornerstone of the so-called Kaleckian theory by blending the principle of effective demand of Kalecki and Keynes[1] with long-term elements of the post Keynesian theory of economic growth, however, this theory has gradually been assimilated into the post Keynesian school and it is now one of the basic frameworks in this school. The characteristics of a Kaleckian theory are that it can address the problem of income distribution as a result of inter-class conflict and that it can also allow for variations of employment and capital utilization due to changes in effective demand even in the long run, that is, the Kaleckian theory supplies the systematic analytical framework to analyze simultaneously the long run effects both of the principle of effective demand and of inter-class income distribution.

In the Kaleckian theory, the price of each good or service (or the general price level) is assumed to be set by the so-called mark-up principle. In other words, the price level is equal to the unit labor cost multiplied by the constant ratio of mark-up in the Kaleckian framework. To reflect the Marxian conflict between capitalists and workers, Rowthorn (1977) extended the Kaleckian mark-up pricing rule by formalizing the (dynamic) Kaleckian price and wage adjustment processes, in which the price level and the nominal wage are adjusted to fill the gaps, respectively, between the actual and target shares of profit and between the actual and target shares of wage, and this dynamic formalization of the Kaleckian pricing rules was also utilized in post Keynesian works by, for instance, Marglin (1984), Taylor (1985), Dutt (1992), Cassetti (2003), Sasaki (2010, 2013) or Ohno (2014). Their studies cast a brilliant light on the effects of the inter-class conflict suggested by Kalecki (1971) in the Kaleckian theory and

[1] As is well known, Michal Kalecki, as well as John M. Keynes, was one of the first founders of the principle of effective demand. Kalecki (1935) established a unique theory of business cycles by making use of some Keynesian concepts such as the "multiplier." Moreover, Kalecki (1935) conducted a thorough mathematical analysis by employing the theory of mixed difference-differential equations (delay differential equations). In this sense, it can be said that Kalecki's (1935) analysis was superior to Keynes' (1936) one. According to Kalecki's (1936) review of Keynes (1936), which was translated by Targetti and Kinda-Hass (1982) in English, however, Kalecki (1936) agreed with the Keynesian mechanism of determination of aggregate income or employment for a given volume of investment, which corresponds to the concept of "multiplier," but disagreed with the mechanism of determination of investment developed by Keynes (1936, chap. 11). Kalecki (1936) argued that Keynes' (1936) theory of investment lacked the dynamic perspective on the effects of the current investment activities on the current expectation on the future profit. Note, however, that Kalecki was in favor of Keynes' (1936) principle of effective demand.

contributed to the development of the Kaleckian (post Keynesian) theory. However, they had a common flaw from a theoretical viewpoint; they ignored the influence of inflation-deflation expectations.[2] In the Keynesian theory, the impact of inflation-deflation expectations has already been explored through the aforementioned macro-foundation approaches in a lot of preceding works such as Tobin (1975, 1993), Chiarella and Flachel (2000), Flaschel and Franke (2000), Asada et al. (2005), Asada (2004), Chiarella et al. (2005), Palley (2008), Chiarella et al. (2013), Murakami (2014, 2016b) and Asada et al. (2016). It may be worthwhile to examine the influence of inflation-deflation expectations on the behavior of the Kaleckian theory, following the spirit of macro-foundations.

The purpose of this paper is to analyze the influence of inflation-deflation expectations and the stabilizing effect of the monetary authority's inflation-targeting policy on a Kaleckian economy. To do so, we shall incorporate the process of adaptive inflation-deflation expectation formations and of the inflation-targeting policy rule in a Kaleckian system and discuss the (de)stabilizing effects of inflation-deflation expectations and the inflation-targeting policy.

This paper is organized as follows. In Section 2, we shall formalize a model of the Kaleckian macroeconomic system by taking care of the influence of inflation-deflation expectations. In Section 3, we shall analyze the properties of our Kaleckian model and examine the stability of equilibrium. In Section 4, we shall perform some numerical simulations to check the validity of our analysis. In Section 5, we shall conclude this paper.

2 The model

In this section, we shall set up a model of the Kaleckian macroeconomic system so as to discuss the influence of inflation-deflation expectations and the effect of the monetary authority's inflation-targeting policy on our Kaleckian system.[3]

2.1 Dynamic process of the rate of utilization

We shall first describe the process of capital accumulation. We shall assume that the rate of capital accumulation (i.e., the ratio of *ex ante* (gross) capital accumulation I to capital stock K) is influenced by the rate of utilization u measured by the output-capital ratio Y/K,[4] the wage share v and the (expected) real rate of interest $\rho^e \equiv r - \pi^e$ (where r and π^e stand for the nominal rate of interest and the expected rate of inflation, respectively) in the following way:

$$\frac{I}{K} = g(u, v, \rho^e), \tag{1}$$

[2]Rowthorn (1977) cared about the influence of inflation-deflation expectations but he did not provide a formal analysis of this influence.

[3]In this paper, we shall mean by the "monetary authority" the institution which can control, directly or indirectly, the supply of money or the nominal rate of interest. In a narrow sense, the monetary authority is synonymous with the "central bank."

[4]Throughout this paper, we shall assume that potential output Y^* is proportional to capital stock K. This assumption implies that the output-capital ratio $u = Y/K$ can be viewed as the rate of utilization.

where g is the capital accumulation function which is assumed to be twice continuously differentiable[5] with

$$g_u \equiv \frac{\partial g}{\partial u} > 0, \; g_v \equiv \frac{\partial g}{\partial v} \leq 0, \; g_{\rho^e} \equiv \frac{\partial g}{\partial \rho^e} < 0. \tag{2}$$

Eq. (1) with (2) is based upon the utilization principle[6] and consistent with the profit principle,[7] because the rate of profit is given by $(1-v)u$. We can safely expect that the wage share v has a negative (or non-positive) influence on capital accumulation because a rise in v means that capitalists acquire less fraction of their revenue as income and can afford to spend less on investment.

Second, we shall formalize savings behavior. As in the usual Kaleckian analysis (e.g., Kalecki 1939, 1971; Asimakopulos 1975), we shall suppose that there are two kinds of agents: workers and capitalists. We shall also assume that workers consume all of their income while capitalists save part of their income. Then, the ratio of *ex ante* (gross) savings S to capital stock K is assumed to be represented by

$$\frac{S}{K} = s(1-v)u, \tag{3}$$

where $s \in (0,1)$ is the rate of savings of capitalists.

Finally, we shall assume as in the usual Keynesian quantity adjustment process that the output-capital ratio or the rate of utilization u is increased (resp. decreased) when the rate of capital accumulation I/K is larger (resp. less) than the ratio of savings to capital S/K.[8] By (1) and (3), the dynamic process of u can thus be represented in the following form:

$$\dot{u} = \alpha\Big(\frac{I}{K} - \frac{S}{K}\Big) = \alpha[g(u, v, \rho^e) - s(1-v)u], \tag{4}$$

where $\alpha > 0$ denotes the speed of utilization adjustment.

2.2 Dynamic process of the wage share

To formulate the dynamic process of the wage share v, we shall specify those of the price level p and of the nominal wage w.

To begin, we shall suppose that capitalists alter their prices p in response to the rate of utilization u and to the wage share v. In our analysis, p is assumed to be influenced by u because a rise in u is, for capitalists, a sign of demand increase, and they have an incentive to raise p. The reason why p is assumed to be affected by v is, on the other hand, that capitalists have in mind their target wage share, which is assumed to be constant, and that

[5]The assumption of twice continuous differentiability is imposed to employ the Hopf bifurcation theorem (cf. Marsden and McCracken 1976, pp. 197-198).

[6]For details on the utilization principle, see, for instance, Steindl (1952, 1979), Rowthorn (1981), Dutt (1984, 2006), Amadeo (1986), Skott (1989), Marglin and Bhaduri (1990) or Lavoie (1992).

[7]For details on the profit principle, see, for example, Kalecki (1935, 1939), Kaldor (1940), Robinson (1962), Malinvaud (1980), Rowthorn (1981), Dutt (1984), Skott (1989), Marglin and Bhaduri (1990) or Lavoie (1992). For microeconomic foundation of the profit principle, see, for instance, Murakami (2016a).

[8]By definition, our investment I is *ex ante* one (or planned one), and it is not necessarily equal to savings S.

they alter p so as to fill the so-called "aspiration gap," i.e., the gap between the current profit share $1 - v$ and their target profit share. This hypothesis is consistent with the related works such as Rowthorn (1977), Marglin (1984), Taylor (1985), Dutt (1992), Cassetti (2003), Sasaki (2010, 2013) and Ohno (2014). Moreover, we shall assume that capitalists also care about their expectations on other capitalists' price settings[9] and that changes in the price p are also affected by the expected rate of inflation π^e. Therefore, the rate of inflation $\dot{p}/p$ can be represented in the following form:[10]

$$\frac{\dot{p}}{p} = f(u, v) + \pi^e, \tag{5}$$

where f is the price inflation function which is assumed to be twice continuously differentiable with

$$f_u \equiv \frac{\partial f}{\partial u} > 0, \; f_v \equiv \frac{\partial f}{\partial v} > 0. \tag{6}$$

Next, we shall assume, as in the above literatures, that capitalists and workers change the nominal wage w, through their wage bargaining, so as to fill the gap between the current wage share v and the target wage share of workers and that their target wage share is influenced by the rate of utilization u, which is assumed to be positively correlated with the rate of employment.[11] This assumption reflects the hypothesis that the higher the rate of employment is, the stronger the bargaining power of workers (or of labor unions) is. We shall further assume that the expected rate of inflation π^e is passed on to the rate of changes in the nominal wage w.[12] This assumption is made on the basis of the reality that inflation-deflation expectations are taken into consideration in revisions of wages. The dynamic process of w can thus be expressed as follows:

$$\frac{\dot{w}}{w} = h(u, v) + \pi^e, \tag{7}$$

where h is the wage inflation function which is assumed to be twice continuously differentiable with

$$h_u \equiv \frac{\partial h}{\partial u} > 0, \; h_v \equiv \frac{\partial h}{\partial v} < 0. \tag{8}$$

Eq. (7) with (8) can easily be confirmed to be consistent with Phillips' (1958) curve. Note that, unlike in the above literatures, the inflation-deflation expectation effect on wage adjustment is reflected in (7) and that what affects the pressure of wage changes is not the actual rate of inflation (deflation) but the expected rate of inflation (deflation).

Therefore, we can, by (5) and (7), obtain the dynamic process of the wage share v as follows:

$$\dot{v} = \Big(\frac{\dot{w}}{w} - \frac{\dot{p}}{p} - a\Big)v = [h(u, v) - f(u, v) - a]v, \tag{9}$$

[9] It is assumed that *each* capitalist can set the price of his/her product but that s/he cannot have access to full information on the other capitalists' price settings.

[10] Eq. (5) is consistent with Rowthorn (1977), but he did not provide a formal analysis using it.

[11] The positive correlation between the rate of employment and that of utilization is established as Okun's (1962) law.

[12] We shall assume for simplicity that workers and capitalists hold the common expected rate of inflation.

where a stands for the constant rate of growth of labor productivity.

2.3 Dynamic process of the nominal rate of interest

As for the nominal rate of interest r, we shall suppose that r is controlled through monetary policy by the monetary authority. Specifically, we shall assume, following Chiarella et al. (2003), Asada (2013, 2014), Asada et al. (2016) and Murakami (2016b), that r is varied in the following way:

$$\dot{r} = \begin{cases} \gamma\left(\frac{\dot{p}}{p} - \overline{\pi}\right) & \text{if } r > 0 \\ \max\left\{0, \gamma\left(\frac{\dot{p}}{p} - \overline{\pi}\right)\right\} & \text{if } r = 0 \end{cases} = \begin{cases} \gamma[f(u,v) + \pi^e - \overline{\pi}] & \text{if } r > 0 \\ \max\{0, \gamma[f(u,v) + \pi^e - \overline{\pi}]\} & \text{if } r = 0. \end{cases} \tag{10}$$

where $\overline{\pi}$ denotes the target rate of inflation set by the monetary authority; $\gamma \geq 0$ stands for the degree of willingness of the monetary authority to attain the target rate of inflation. Eq. (10) is a kind of feedback policy of Taylor (1993) type. It is assumed that the monetary authority adjusts the nominal rate of interest towards the realization of the target rate of inflation, i.e., that it conducts the so-called "inflation-targeting" policy.[13] Note that care is taken of the nonnegative constraint or zero bound of the nominal rate of interest in (10).

2.4 Dynamic process of the expected rate of inflation

To close our Kaleckian model, we shall formulate the law of motion of the expected rate of inflation π^e. In what follows, we shall suppose that the expected rate of inflation π^e is revised adaptively. Since the target rate of inflation $\overline{\pi}$ is set (and announced) by the monetary authority, however, π^e can be influenced by $\overline{\pi}$. Following Groth (1988), Asada et al. (2003), Asada (2013, 2014), Asada et al. (2016) and Murakami (2016b), we shall formalize the dynamics of π^e in the following way:

$$\dot{\pi}^e = \beta\left[\theta\overline{\pi} + (1-\theta)\frac{\dot{p}}{p} - \pi^e\right] = \beta[\theta(\overline{\pi} - \pi^e) + (1-\theta)f(u,v)], \tag{11}$$

where $\beta \geq 0$ is the speed of revisions of inflation-deflation expectations; $\theta \in [0,1]$ is the parameter that measures the credibility of the "inflation-targeting" policy conducted by the monetary authority. If $\theta = 0$, Eq. (11) reduces to the usual formula of adaptive (or backward-looking) expectation formations, while if $\theta = 1$, Eq. (11) may be viewed as the formula of forward-looking expectation formations. In this respect, Eq. (11) can be interpreted to mean that inflation-deflation expectations are formed by the combination of backward-looking (adaptive) and forward-looking expectations.

[13] For simplicity, we shall only consider the inflation-targeting policy, but it is possible to generalize our monetary policy rule (10), without so much difficulty, by allowing for the possibility that the monetary authority also pursues the attainment of the target rate of employment (or of utilization).

2.5 Full model: System (K)

Summarizing the previous subsections, we can obtain the following system of equations:

$$\dot{u} = \alpha[g(u, v, r - \pi^e) - s(1 - v)u], \tag{4}$$

$$\dot{v} = [h(u, v) - f(u, v) - a]v, \tag{9}$$

$$\dot{r} = \begin{cases} \gamma[f(u, v) + \pi^e - \overline{\pi}] & \text{if } r > 0 \\ \max\{0, \gamma[f(u, v) + \pi^e - \overline{\pi}]\} & \text{if } r = 0, \end{cases} \tag{10}$$

$$\dot{\pi}^e = \beta[\theta(\overline{\pi} - \pi^e) + (1 - \theta)f(u, v)]. \tag{11}$$

In what follows, the system of Eqs. (4), (9), (10) and (11) shall be labeled as "System (K)" (to signify "Kalecki").

3 Analysis

In this section, we shall proceed to analyze our Kaleckian model, System (K).

We shall define an equilibrium point of System (K) as a point $(u^*, v^*, r^*, \pi^{e*}) \in \mathbb{R}^3_{++} \times \mathbb{R}$ that satisfies $\dot{u} = \dot{v} = \dot{r} = \dot{\pi}^e = 0$.[14] It then follows from (4), (9)-(11) that an equilibrium point of System (K), $(u^*, v^*, r^*, \pi^{e*})$, is a solution of the following system of simultaneous equations:

$$0 = g(u, v, r - \pi^e) - s(1 - v)u, \tag{12}$$

$$0 = h(u, v) - f(u, v) - a, \tag{13}$$

$$0 = f(u, v) + \pi^e - \overline{\pi}, \tag{14}$$

$$0 = \theta(\overline{\pi} - \pi^e) + (1 - \theta)f(u, v). \tag{15}$$

In what follows, we shall assume that there exists at least one equilibrium point of System (K).[15]

Fortunately, we can prove the uniqueness of an equilibrium point of System (K) as in the following proposition.

Proposition 1. *There exists at most one equilibrium point of System (K).*

Proof. We can find from (14) and (15) that the equilibrium value of π^e, or π^{e*} is given by $\pi^{e*} = \overline{\pi}$ and that the equilibrium values of u and v, or u^* and v^*, respectively, must satisfy the following:

$$f(u^*, v^*) = 0. \tag{16}$$

[14] By this definition, we can exclude the pathological case of $r^* = 0$.

[15] It is technically possible to ensure the existence of this equilibrium point by imposing some "boundary conditions," but these additional conditions are irrelevant to the essence of our analysis. For some examples of boundary conditions in similar models, see, for example, Murakami (2014, 2015, 2017).

Due to (13), u^* and v^* must also fulfill the following:

$$h(u^*, v^*) = a. \tag{17}$$

Once u^* and v^* are given, the equilibrium value of r, r^*, is uniquely determined by (12) because of (2). Therefore, to prove the assertion of this proposition, it suffices to show that at most one combination of u and v, (u^*, v^*), can be a solution of the simultaneous equations (16) and (17).

Assume, for the sake of contradiction, that two distinct combinations (u^*, v^*) and (u^{**}, v^{**}) fulfill (16) and (17). By (6) and (8), we have $u^* \neq u^{**}$ and $v^* \neq v^{**}$. Then, we can set the following nonempty, compact, rectangular domain Ω:

$$\Omega \equiv \{(u, v) : u \in [\min\{u^*, u^{**}\}, \max\{u^*, u^{**}\}],\ v \in [\min\{v^*, v^{**}\}, \max\{v^*, v^{**}\}].$$

Define the vector-valued function F on Ω as follows:

$$F(u, v) \equiv \begin{pmatrix} f(u, v) \\ h(u, v) - a \end{pmatrix}.$$

The Jacobian matrix of F, denoted by J_F, is given by

$$J_F = \begin{pmatrix} f_u & f_v \\ h_u & h_v \end{pmatrix}.$$

On the minors of J_F, we know from (6) and (8) that

$$f_u > 0,\ f_v > 0,\ h_u > 0,\ h_v < 0,$$
$$\det J_F = f_u h_v - f_v h_u < 0.$$

Hence, the determinants of J_F and all the principal minors of it are nonzero. Therefore, we can conclude from Gale and Nikaido (1965, p. 91, Theorem 7 (i)) that F is univalent on Ω. But this contradicts our hypothesis that distinct points (u^*, v^*) and (u^{**}, v^{**}) satisfy $F = 0$. Thus, a solution of the simultaneous equations (16) and (17) is unique.

□

Proposition 1 implies that for given u^* and v^*, the equilibrium value of the expected real rate of interest ρ^e, ρ^{e*}, can be expressed by the function R such that

$$g(u^*, v^*, R(u^*, v^*)) = s(1 - v^*)u^*. \tag{18}$$

Since the equilibrium value of π^e, or π^{e*}, is equal to $\overline{\pi}$, we have

$$r^* = R(u^*, v^*) + \overline{\pi}.$$

Then, for the equilibrium value of the nominal rate of interest r^* to be positive,[16] we must have

$$\overline{\pi} > -R(u^*, v^*), \tag{19}$$

where R is defined by (18). Condition (19) means that the target rate of inflation $\overline{\pi}$ should be sufficiently high because of the positivity constraint of r^*. In particular, if the equilibrium value of the expected real rate of interest R is negative, $\overline{\pi}$ has to be positive. As for the equilibrium value of the rate of inflation $\dot{p}/p$, it can easily be seen that this value is equal to $\overline{\pi}$. In other words, the target rate of inflation $\overline{\pi}$ can be achieved as the long-run equilibrium value of the actual rate of inflation.

In the next subsections, we shall examine the effects of inflation-deflation expectations and active monetary policy on our Kaleckian system by comparing the cases of $\beta = \gamma = 0$, of $\beta > 0$ and $\gamma = 0$ and of $\beta > 0$ and $\gamma > 0$.

3.1 Stationary inflation-deflation expectations and passive monetary policy

To begin with, we shall consider the case in which the monetary authority does not conduct any active monetary policy and the public hold static inflation-deflation expectations. In this case, we may set the speed of inflation-deflation expectations β and the responsiveness of the monetary authority γ as $\beta = \gamma = 0$. In the absence of the inflation-targeting policy, we may assume that the target rate of inflation $\overline{\pi}$ and the credibility parameter θ are both 0. We shall further suppose, for simplicity, that the nominal rate of interest r and that of inflation π^e are fixed at the equilibrium value of the *expected real* rate of interest $r^* - \overline{\pi}$ and 0, respectively.

In this case, System (K) may be written as

$$\dot{u} = \alpha[g(u, v, r^* - \overline{\pi}) - s(1 - v)u], \tag{20}$$

$$\dot{v} = [h(u, v) - f(u, v) - a]v. \tag{21}$$

In this section, the system of (20) and (21) shall be denoted by "System (K-S)." One can easily find that one of the equilibrium points of System (K-S) is given by (u^*, v^*).[17]

To investigate the asymptotic stability of the equilibrium point of System (K-S) under consideration, we shall calculate the Jacobian matrix of System (K-S) evaluated at this equilibrium point. The resultant Jacobian matrix,

[16] Our definition of equilibrium rules out the case of $r = 0$.

[17] Although the uniqueness of an equilibrium point of System (K) is guaranteed, that of System (K-S) may not be ensured by the assumptions made so far. To guarantee the uniqueness by the same method as in the proof of Proposition 1, it suffices to assume that conditions (24) and (25) hold not only at the equilibrium under consideration but also at every point else.

denoted by J^*_{KS}, is given by

$$J^*_{KS} = \begin{pmatrix} \alpha[g^*_u - s(1-v^*)] & \alpha(g^*_v + su^*) \\ (h^*_u - f^*_u)v^* & (h^*_v - f^*_v)v^* \end{pmatrix},$$

where $*$ denotes the value evaluated at the equilibrium point. The trace and determinant of J^*_{KS} are given as follows

$$\mathrm{tr}\mathbf{J}^*_{\mathrm{KS}} = \alpha[g^*_u - s(1-v^*)] + (h^*_v - f^*_v)v^*, \tag{22}$$

$$\det J^*_{KS} = \alpha\{[g^*_u - s(1-v^*)](h^*_v - f^*_v) - (g^*_v + su^*)(h^*_u - f^*_u)\}v^*. \tag{23}$$

In what follows, we shall assume that the following condition holds:

$$g^*_u < s(1-v^*). \tag{24}$$

Condition (24) means that the marginal effect of changes in the rate of utilization u on the rate of gross capital accumulation g is less than that of changes in u on the saving-capital ratio at the equilibrium point. This implies that the so-called Keynesian stability condition (cf. Marglin and Bhaduri 1990) is satisfied.[18]

For the analysis below, we shall make the following realistic assumption.

$$[g^*_u - s(1-v^*)](h^*_v - f^*_v) > (g^*_v + su^*)(h^*_u - f^*_u). \tag{25}$$

Under the Keynesian stability condition (24), the left hand side of (25) is positive, but the sign of the right hand side is unknown from our assumptions alone. Since $g_v < 0$ and $-su < 0$ are, respectively, the marginal effects of a change in the wage share v on the rate of gross capital accumulation g and the savings-capital ratio $s(1-v)u$, the sign of $g_v + su$ is determined by which of them is greater in the absolute value.[19] Similarly, the sign of $h^*_u - f^*_u$ is determined by the difference between the marginal effect of a change in the rate of utilization u on the rate of changes in the nominal wage w and that on the price level p. Since these two effects, f_u and h_u, are not considered to differ largely from each other, the absolute value of $h^*_u - f^*_u$ may be taken as relatively small. For this reason, condition (25) can be considered to be fulfilled in reality.

Concerning the stability of the unique equilibrium of System (K-S), we have the following proposition.

[18]Recently there have been some arguments on the validity of the Keynesian stability condition. For instance, Skott (2012) argued that if the rate of capital accumulation is assumed to be influenced not only by the current rate of utilization but also by the past rates of utilization, the Keynesian stability condition may be satisfied in the short run (i.e., the marginal effect of the current rate of utilization on the rate of capital accumulation may be less than the rate of savings), but this condition is violated in the long run (i.e., the sum of the marginal effects of all the past rates of utilization on the rate of capital accumulation is larger than the rate of savings). For details in the role of the Keynesian stability condition in the Kaleckian analysis, see, for instance, Hein et al. (2011) or Franke (2016).

[19]Borrowing the terminology from Marglin and Bhaduri (1990), we may say that the cases of $g^*_v + su^* > 0$ and of $g^*_v + su^* < 0$ can be labeled as the "wage-led" case and the "profit-led" case, respectively. Making use of these terms, it is seen that condition (25) is always satisfied in the "profit-led" case.

Proposition 2. *The equilibrium point of System (K-S), (u^*, v^*), is locally asymptotically stable.*[20]

Proof. We know from (2), (6), (8), (22)-(25) that the trace and determinant of the Jacobian matrix of System (K-S) evaluated at the equilibrium point are negative and positive, respectively. It then follows from the Routh-Hurwitz criterion that the equilibrium point of System (K-S) is locally asymptotically stable. □

Under the Keynesian stability condition (24), condition (25) is the necessary and sufficient condition for the local asymptotic stability of System (K-S). Using the terminology of Marglin and Bhaduri (1990), we may say that in the profit-led case, the unique equilibrium point of System (K-S) is locally asymptotically stable but that, in the wage-led case, it may not depending upon the magnitude of $g_v^* + su^*$.

3.2 Adaptive inflation-deflation expectations and passive monetary policy

We shall next examine the case in which the public revise adaptively inflation-deflation expectations but no active monetary policy is conducted. In this case, we have $\beta > 0$ and $\gamma = 0$. We shall assume, as before, that $\overline{\pi}$ and θ are 0 and that r is fixed at $r^* - \overline{\pi}$.

In this case, System (K) can be written as

$$\dot{u} = \alpha[g(u, v, r^* - \pi^e) - s(1 - v)u], \tag{26}$$

$$\dot{v} = [h(u, v) - f(u, v) - a]v, \tag{27}$$

$$\dot{\pi}^e = \beta f(u, v). \tag{28}$$

In what follows, the system of (26)-(28) shall be called "System (K-A)." We may prove that the unique equilibrium point of System (K-A) is given by $(u^*, v^*, 0)$.

To investigate the local asymptotic stability, we shall derive the Jacobian matrix of System (K-A) evaluated at $(u^*, v^*, 0)$, denote by J_{KA}^*:

$$J_{KA}^* = \begin{pmatrix} \alpha[g_u^* - s(1 - v^*)] & \alpha(g_v^* + su^*) & -\alpha g_{\rho^e}^* \\ (h_u^* - f_u^*)v^* & (h_v^* - f_v^*)v^* & 0 \\ \beta f_u^* & \beta f_v^* & 0 \end{pmatrix}.$$

Therefore, the characteristic equation associated with J_K^* is given as follows:

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0,$$

[20]If conditions corresponding to (24) and (25) are fulfilled everywhere on the u-v plane, the equilibrium point of System (K-S), (u^*, v^*), may be proved to be globally asymptotically stable by Olech's theorem (1963, p. 395, Theorem 4), provided that the equilibrium point is unique. To verify this fact, however, we have to take care of the positivity constraints of u and v. To cope with this problem, see Ito (1978). Another way to establish the global asymptotic stability (in two-dimensional differential equations) is to apply the Poincarè-Bendixson theorem (cf. Coddington and Levinson 1955, chap. 16) and Bendixson's criterion (cf. Andronov, et al. 1966, p. 305) to positively invariant (compact) sets. For applications of this method in economic models, see, for example, Minagawa (2008) or Murakami (2014, 2015).

where

$$a_1 = -\alpha[g_u^* - s(1 - v^*)] - (h_v^* - f_v^*)v^* > 0, \tag{29}$$

$$a_2 = \alpha\langle\{[g_u^* - s(1 - v^*)](h_v^* - f_v^*) - (g_v^* + su^*)(h_u^* - f_u^*)\}v^* + \beta g_{\rho^e}^* f_u^*\rangle, \tag{30}$$

$$a_3 = -\alpha\beta g_{\rho^e}^*(f_u^* h_v^* - f_v^* h_u^*)v^* < 0. \tag{31}$$

The inequalities are derived from (2), (6), (8), (24) and (25).

We can immediately reach from (31) the following unpleasant conclusion on the stability of System (K-A).

Proposition 3. *The unique equilibrium point of System (K-A) is locally asymptotically unstable irrespective of the value of $\beta > 0$.*

Proof. According to the Routh-Hurwitz criterion, we can know that one of the necessary condition for the local asymptotic stability $a_3 > 0$ is violated by (31). Then, the conclusion follows. □

Proposition (3) states the "destructive" conclusion that if inflation-deflation expectations are adaptively revised in our Kaleckian system, the stability can never be obtained (except for the case in which the state of the economy is on the stable manifold or on what is known as the "stable arm" in the context of economics). This consequence is really devastating because, in usual Keynesian models, which are akin to the present Kaleckian system, fast revisions of inflation-deflation expectations certainly destabilize the system, but the stability can be gained as long as these revisions are not so fast.[21] In this sense, the destabilizing effect of inflation-deflation expectations works more strongly in the Kaleckian system than in the Keynesian system. That is, the so-called Mundell-Tobin effect is more powerful in the former than in the latter.

We shall explain the mechanism of revisions of inflation-deflation expectations destabilizing our Kaleckian system. For this purpose, assume that the state of the economy is originally at the equilibrium $(u^*, v^*, \overline{\pi})$ but that u is suddenly reduced from u^* for some reason. In this situation, the rate of utilization u rises through (4) due to (24) but the expected rate of inflation π^e decreases through (11) because of (6); this reduction of π^e causes u to go further down through (4) because of (2); if the revision speed β is fast enough, the decline in u by the decrease in π^e exceeds the rise in u by the first reduction of u. This mechanism can be summarized in the following schematic diagram (provided that β is large enough):

$$u \downarrow \begin{cases} \xrightarrow{(4)} \quad u \uparrow \\ \xrightarrow{(11)} \quad \pi^e \downarrow\downarrow \xrightarrow{(4)} \quad u \downarrow\downarrow \end{cases}, \tag{DS}$$

[21]This conclusion is common in the preceding works including Tobin (1975, 1993), Chiarella and Flaschel (2000), Flaschel and Franke (2000), Asada et al. (2003), Asada (2004, 2013, 2014), Asada et al. (2006), Asada et al. (2010), Palley (2008), Chiarella et al. (2013), Murakami (2014, 2016b) and Asada et al. (2016).

where ↑, ↓ and ↓↓ indicate a rise, decline and sharp decline in the variable on the left side, respectively. The schematic diagram (DS) describes the destabilizing force of inflation-deflation expectations.

Our analysis in this section has shown that adaptive revisions in inflation-deflation expectations deprive our Kaleckian system of its stability through the Mundell-Tobin effect. In the next subsection, we shall examine the possibility that the stability of our Kaleckian system can be restored by active monetary policy, or the inflation-targeting policy.

3.3 Adaptive inflation-deflation expectations and active monetary policy

We shall turn to the full version of System (K), in which the public change their inflation-deflation expectations and the monetary authority performs its monetary policy through adjustments in the nominal rate of interest. For reference, System (K) is reproduced below:

$$\dot{u} = \alpha[g(u, v, r - \pi^e) - s(1 - v)u], \tag{4}$$

$$\dot{v} = [h(u, v) - f(u, v) - a]v, \tag{9}$$

$$\dot{r} = \begin{cases} \gamma[f(u, v) + \pi^e - \overline{\pi}] & \text{if } r > 0 \\ \max\{0, \gamma[f(u, v) + \pi^e - \overline{\pi}]\} & \text{if } r = 0, \end{cases} \tag{10}$$

$$\dot{\pi}^e = \beta[\theta(\overline{\pi} - \pi^e) + (1 - \theta)f(u, v)]. \tag{11}$$

In what follows, we shall assume that the following condition also holds:

$$\alpha[g_u^* - s(1 - v^*)]f_u^* < (f_u^* - h_u^*)f_v^* v^*. \tag{32}$$

Under (2), (6), (8) and (24), this condition is satisfied if the speed of utilization (quantity) adjustment α is sufficiently high because the left hand side is negative. As Leijonhufvud (1968) argued, the Keynesian or Kaleckian economics is characterized by the principle of effective demand and by the hypothesis of fast quantity adjustment.[22] In this sense, we can assume that α is sufficiently large in our Keleckian system. Thus, condition (32) can be justified as an assumption in our analysis.

Now that the existence of an equilibrium point of System (K) has been assumed and the uniqueness of it has been established, we shall proceed to the analysis of the stability of this system. The Jacobian matrix of System

[22] Leijonhufvud (1968) made the following statement:

> In [neoclassical] general equilibrium flow models, *prices* are the only endogenous variables which enter as arguments into the demand and supply functions of individual households. Tastes and initial resource endowments are parametric. In "Keynesian" flow models the corresponding arguments are real income and the interest rate. Of these, real income is a measure of *quantity*, not of prices. On a highly abstract level, the fundamental distinction between general equilibrium and Keynesian models lies in the appearance of this quantity variable in the excess demand relation to the latter. The difference is due to the assumptions made about the adjustment behavior of the two systems. In the short run, the "[Neo]Classical" system adjusts to changes in money expenditures by means of price-level movements; the Keynesian adjusts primarily by way of real income movements. (p. 51)

(K) evaluated at the unique equilibrium point $(u^*, v^*, r^*, \overline{\pi})$ is given by

$$J_K^* = \begin{pmatrix} \alpha[g_u^* - s(1-v^*)] & \alpha(g_v^* + su^*) & \alpha g_{\rho^e}^* & -\alpha g_{\rho^e}^* \\ (h_u^* - f_u^*)v^* & (h_v^* - f_v^*)v^* & 0 & 0 \\ \gamma f_u^* & \gamma f_v^* & 0 & \gamma \\ \beta(1-\theta)f_u^* & \beta(1-\theta)f_v^* & 0 & -\beta\theta \end{pmatrix}.$$

Hence, the characteristic equation associated with J_K^* is as follows:

$$\lambda^4 + b_1\lambda^3 + b_2\lambda^2 + b_3\lambda + b_4 = 0,$$

where

$$b_1 = -\alpha[g_u^* - s(1-v^*)] - (h_v^* - f_v^*)v^* + \beta\theta > 0, \tag{33}$$

$$\begin{aligned} b_2 =& \alpha\langle\{[g_u^* - s(1-v^*)](h_v^* - f_v^*) - (g_v^* + su^*)(h_u^* - f_u^*)\}v^* + [\beta(1-\theta) - \gamma]g_{\rho^e}^* f_u^*\rangle \\ &- \{\alpha[g_u^* - s(1-v^*)] + (h_v^* - f_v^*)v^*\}\beta\theta, \end{aligned} \tag{34}$$

$$\begin{aligned} b_3 =& \alpha\langle\beta[\theta\{[g_u^* - s(1-v^*)](h_v^* - f_v^*) - (g_v^* + su^*)(h_u^* - f_u^*)\} - (1-\theta)g_{\rho^e}^*(f_u^* h_v^* - f_v^* h_u^*)]v^* \\ &+ [(f_u^* h_v^* - f_v^* h_u^*)v^* - \beta f_u^*]\gamma g_{\rho^e}^*\rangle, \end{aligned} \tag{35}$$

$$b_4 = \alpha\beta\gamma g_{\rho^e}^*(f_u^* h_v^* - f_v^* h_u^*)v^* > 0. \tag{36}$$

The inequalities follow from (2), (6), (8), (24) and (25).

For the unique equilibrium of System (K) to possess the local asymptotic stability, according to the Routh-Hurwitz criterion, the following condition must be satisfied:[23]

$$\begin{aligned} &G(\gamma;\theta)(\equiv[(b_1b_2 - b_3)b_3 - b_1^2 b_4]]/\alpha) \\ &= \alpha\langle\{\alpha[g_u^* - s(1-v^*)]f_u^* + (h_u^* - f_u^*)f_v^* v^* + \beta(1-\theta)f_u^*\rangle[(f_u^* h_v^* - f_v^* h_u^*)v^* - \beta f_u^*](g_{\rho^e}^*)^2\gamma^2 \\ &+ c_1^*\gamma + c_2^* > 0 \end{aligned} \tag{37}$$

We shall touch on this condition in the propositions below.

We shall now look into how the credibility of monetary policy (measured by the parameter θ) influences the stability of our Kaleckian system. To see this influence clearly, we shall separately examine the cases in which the credibility parameter θ is near 0 and in which this parameter is sufficiently close to 1.

First, we shall take a look at the case with θ sufficiently close to 0. Concerning the (local asymptotic) stability of the equilibrium of System (K) with θ sufficiently close to 0, we have the following proposition.

[23]The exact values of c_1^* and c_2^* are omitted because they are irrelevant to our analysis.

Proposition 4. *Assume that θ is sufficiently close to 0.*

(i) If β is sufficiently small, there exists a positive value γ^* such that the unique equilibrium point of System (K) is locally asymptotically unstable (resp. stable) for $\gamma < \gamma^*$ (resp. for $\gamma > \gamma^*$) and that a periodic orbit is generated by way of a Hopf bifurcation if γ is sufficiently close to γ^*.

(ii) If β is sufficiently large, the unique equilibrium point of System (K) is (locally asymptotically) unstable irrespective of the value of $\gamma > 0$.

Proof. We shall only prove this proposition in the case of $\theta = 0$. The conclusion still holds by continuity if θ is sufficiently close to 0.

(i) Assume, as the proposition requires, that β is small enough to satisfy the following:

$$\beta < -\alpha[g_u^* - s(1 - v^*)] + \frac{(f_u^* - h_u^*)f_v^* v^*}{f_u^*} \tag{38}$$

Let

$$\gamma_0 = \beta\Big[1 + \frac{\beta f_u^*}{(f_u^* h_v^* - f_v^* h_u^*)v^* - \beta f_u^*}\Big].$$

It follows from (2), (6), (8) and (35) that, for $\gamma < \gamma_0$, we have

$$b_3|_{\theta=0} = \alpha[(f_u^* h_v^* - f_v^* h_u^*)v^* - \beta f_u^*]g_{p^e}^*(\gamma - \gamma_0) < 0. \tag{39}$$

According to the Routh-Hurwitz criterion, this indicates that a necessary condition for the local asymptotic stability is violated for $\gamma < \gamma_0$.

Next, we know from (2), (6), (8), (24) and (35) that, for $\gamma > \gamma_0$, we have

$$b_3|_{\theta=0} = \alpha[(f_u^* h_v^* - f_v^* h_u^*)v^* - \beta f_u^*]g_{p^e}^*(\gamma - \gamma_0) > 0.$$

It follows from (33) and (36) that if $[(b_1 b_2 - b_3)b_3 - b_1^2 b_4]|_{\theta=0} \geq 0$, then $(b_1 b_2 - b_3)|_{\theta=0} > 0$ holds. Therefore, according to the Routh-Hurwitz criterion, the necessary and sufficient condition for the local asymptotic stability is, under $\gamma > \gamma_0$, given by[24]

$$\begin{aligned} G(\gamma;0) =& \alpha\langle\{\alpha[g_u^* - s(1 - v^*) + \beta]f_u^* + (h_u^* - f_u^*)f_v^* v^*\}[(f_u^* h_v^* - f_v^* h_u^*)v^* - \beta f_u^*](g_{p^e}^*)^2\gamma^2 + b_0^*\gamma \\ &+ \alpha\langle\alpha[g_u^* - s(1 - v^*)] + (h_v^* - f_v^*)v^*\}\{[g_u^* - s(1 - v^*)](h_v^* - f_v^*) - (g_v^* + su^*)(h_u^* - f_u^*)\} \\ &+ \beta\{\alpha[g_u^* - s(1 - v^*)]f_u^* - (f_u^* - h_u^*)f_v^*\}g_{p^e}^*\rangle(g_{p^e}^*)^2(v^*)^2 > 0. \end{aligned} \tag{40}$$

where G is defined in (37). It is immediately seen from (2), (6), (8), (24), (32) and (38) that the coefficient of γ^2

[24] The exact value of b_0^* is not provided because it is irrelevant to our analysis.

and the constant term (or $G(0;0)$) of the quadratic $G(\gamma;0)$ are both positive. Moreover, since we have $b_3|_{\theta=0} = 0$ for $\gamma = \gamma_0$, we find from (33) and (36) that

$$G(\gamma_0;0) = -b_1^2 b_4|_{\theta=0,\ \gamma=\gamma_0} < 0. \tag{41}$$

Then, $G(\gamma;0) = 0$ has two distinct positive roots $\gamma^{**} < \gamma_0$ and $\gamma^* > \gamma_0$. Hence, condition (40) is violated for $\gamma \in [\gamma_0, \gamma^*]$ but fulfilled for $\gamma > \gamma^*$.

Thus, the local asymptotic stability is lost for $\gamma < \gamma^*$ but obtained for $\gamma > \gamma*$.

Finally, we can know from the above argument that $\gamma = \gamma^*$ is not a multiple root of the quadratic $G(\gamma;0) = 0$ and that $G_\gamma(\gamma^*;0) > 0$. We can thus conclude from Asada and Yoshida (2003, p. 527, Theorem 3) that a Hopf bifurcation is generated for $\gamma = \gamma^*$ and that a periodic orbit appears for γ sufficiently close to γ^*.

(ii) Suppose that β is large enough to satisfy the following:

$$\beta > -\alpha[g_u^* - s(1 - v^*)] + \frac{(f_u^* - h_u^*) f_v^* v^*}{f_u^*} \tag{42}$$

First, it is already seen from (39) that a necessary condition for the local asymptotic stability is not satisfied for $\gamma < \gamma_0$.

Next, we find from (40) that, under (2), (6), (8), (24), (32) and (42), the the coefficient of γ^2 and the constant term (or $G(0;0)$) of the quadratic $G(\gamma;0)$ are negative and positive, respectively. Then, $G(\gamma;0) = 0$ has a positive and negative root. Let γ^{***} be the positive root. Because of (41), we know that $\gamma^{***} < \gamma_0$. Hence, we have $G(\gamma;0) < 0$ for $\gamma \geq \gamma_0$. Thus, the necessary and sufficient condition for the local asymptotic stability (for $\gamma \geq \gamma_0$) is always violated.

Therefore, the unique equilibrium is always locally asymptotically stable for $\gamma > 0$. □

Proposition 4 suggests that, in the case of θ sufficiently close to 0, the local asymptotic stability of System (K) is conditional. Specifically, if the revision speed of inflation-deflation expectations β is slow enough, the local asymptotic stability can be gained for γ sufficiently large, while, if β is relatively fast, the stability always fails to be gained. We can conclude from this result that, in our Kaleckian system (K), when the public do not put trust on the inflation targeting policy, this policy can be useful for achieving the macroeconomic stability but it depends upon the public revision speed of inflation-deflation expectations.

We can explain the stabilizing force of the monetary authority's inflation-targeting policy in the case of $\theta = 0$ by making use of a schematic digram. Assume as usual that the state of the economy was originally at the equilibrium $(u^*, v^*, r^*, \overline{\pi})$ but that the rate of utilization u suddenly falls. If the degree of intensity of the inflation-targeting policy γ is sufficiently large, the phenomena described in the following schematic diagram can be observed (provided that the revision speed of inflation-deflation expectations β is slow enough to meet (38)):

$$u\downarrow \begin{cases} \xrightarrow{(4)} \ u\uparrow \\ \left\{ \begin{array}{l} \xrightarrow{(10)} \ r\downarrow\downarrow \\ \xrightarrow{(11)} \ \pi^e\downarrow ...(\#) \end{array} \right\} \xrightarrow{(4)} \ u\uparrow ...(*) \end{cases} \tag{DK0}$$

The stabilizing effect of the inflation-target policy is illustrated in (*). Note however that in the case of $\theta = 0$, the above schematic diagram is effective only when the revision β is sufficiently small.

Next, we shall move on to the analysis of the case of θ equal or sufficiently close to 1. In this case, we can obtain the following proposition concerning the local asymptotic stability.

Proposition 5. *Assume that β is fixed at some finite positive value. Then, if θ is sufficiently close to 1, the unique equilibrium point of System (K) is locally asymptotically stable for γ sufficiently large.*

Proof. It is easily seen from (2), (6), (8), (24) and (35) that we have

$$b_3|_{\theta=1} = \alpha\langle\beta\{[g_u^* - s(1-v^*)](h_v^* - f_v^*) - (g_v^* + su^*)(h_u^* - f_u^*)\}v^* + [(f_u^* h_v^* - f_v^* h_u^*)v^* - \beta f_u^*]g_{\rho^e}^*\gamma\rangle > 0.$$

Then, we know from the same argument as made in the proof of Proposition 4 (i) that the necessary and sufficient condition for the local asymptotic stability is given by (37). We can easily see from (2), (6), (8), (24) and (32) that since β is fixed (at some finite value), the coefficient of γ^2 of the quadratic $G(\gamma;\theta)$ is positive if θ is chosen as sufficiently close to 1. Hence, we have $G(\gamma;1) \to \infty$ as $\gamma \to \infty$. Thus, if θ is sufficiently close to 1, for γ sufficiently large, condition (37) is satisfied and the local asymptotic stability obtains. □

Proposition 5 indicates that if the credibility parameter θ is equal (or sufficiently close) to 1, the local asymptotic stability of System (K) can be gained when the degree of intensity of the inflation-targeting policy γ is large enough. This consequence means that, if the monetary authority's inflation-targeting policy is credited much by the public, this policy can fully exert its power to stabilize our Kaleckian system.

As in the case of $\theta = 0$, we can expound the stabilizing force of the monetary authority's inflation-targeting policy in the case of $\theta = 1$ with a schematic digram. Assume that the state of the economy was originally at the equilibrium (u^*, v^*, r^*, π^*) but that the rate of utilization u suddenly falls. If the degree of intensity of the inflation-targeting policy γ is sufficiently large, the phenomena described in the following schematic diagram can be observed:

$$u\downarrow \begin{cases} \xrightarrow{(4)} \ u\uparrow \\ \xrightarrow{(10)} \ r\downarrow\downarrow \xrightarrow{(4)} \ u\uparrow\uparrow ...(*) \end{cases} \tag{DK1}$$

where ↑↑ stands for a "sharp rise" in the variable on the left side. The stabilizing effect of the inflation-target policy is illustrated in (*). Note however that unlike in the case of $\theta = 0$, the above schematic diagram can, in the case of $\theta = 1$, be effective even when the revision speed of inflation-deflation expectations β is sufficiently large. This difference can be seen by the disappearance of the destabilizing effect of inflation-deflation expectations emphasized by (#) in (DK0).

Comparing Propositions 4 and 5 with each other, one can immediately see that the credibility of monetary policy makes a sharp difference on the effect on stability of our Kaleckian system. In particular, the monetary authority's inflation-targeting policy is always conducive to the stability when the credibility parameter θ is sufficiently near 1, while the effect of this policy depends upon the value of the revision speed of inflation-deflation expectations in the case of θ equal or sufficiently close to 0. This difference is due to the existence of the negative feedback process of the expected rate of inflation π^e in the case of $\theta = 1$ (cf. (11)). All in all, what determines the effectiveness of the inflation-targeting policy is the degree of the public trust on the feasibility of the target rate of inflation. To achieve the goal, the monetary authority must make an endless effort to gain trust with the public.

4 Numerical Analysis

In this section, we shall perform numerical simulations to confirm that the above analytical results are true. In particular, we shall conduct check if the conclusion obtained in Section 3.3 is valid.

For this purpose, we shall specify the functional forms of the capital accumulation function g, the price inflation function f and the wage inflation function h. Specifically, we shall specify these functions in the following forms:[25]

$$g(u, v) = 0.09 + 0.15[(1 - v)u - (r - \pi^e)] + 0.03u, \tag{43}$$

$$f(u, v) = 0.04u + 0.1v - 0.08, \tag{44}$$

$$h(u, v) = 0.04 + 0.08u - 0.1v. \tag{45}$$

Eq. (43) says that the rate of capital accumulation is a linear function of the difference between the rate of profit $(1 - v)u$ and the expected real rate of interest $r - \pi^e$. This reflects the profit principle. In (44) and (45), it is assumed that the rate of price inflation and that of wage inflation are linear functions of u and v.

Referring to Chiarella et al. (2003), we shall set the saving rate of capitalists s as follows:

$$s = 0.6. \tag{46}$$

[25]The coefficients of these functions are chosen so as to be consistent with the empirical study of the U.S. economy by Chiarella et al. (2003).

We shall set the speed of utilization adjustment α and the target rate of inflation $\overline{\pi}$ as follows:

$$\alpha = 1, \tag{47}$$

$$\overline{\pi} = 0.02. \tag{48}$$

Substituting (43)-(48) in System (K), we can obtain the following system:

$$\dot{u} = 0.075 + 0.03u - 0.45(1-v)u - 0.15(r - \pi^e), \tag{49}$$

$$\dot{v} = (0.1 + 0.04u - 0.2v)v, \tag{50}$$

$$\dot{r} = \begin{cases} \gamma(0.04u + 0.1v + \pi^e - 0.1) & \text{if } r > 0 \\ \max\{0, \gamma(0.04u + 0.1v + \pi^e - 0.1)\} & \text{if } r = 0, \end{cases} \tag{51}$$

$$\dot{\pi}^e = \beta[\theta(0.02 - \pi^e) + (1-\theta)(0.04u + 0.1v - 0.08)]. \tag{52}$$

In this section, the system of (49)-(52) shall be redefined as System (K).

The unique equilibrium point of System (K) can easily be obtained as:

$$(u^*, v^*, r^*, \pi^{e*}) = (0.5, 0.6, 0.02, 0.02). \tag{53}$$

For numerical simulations, we shall fix the initial condition of System (K) as follows:

$$(u(0), v(0), r(0), \pi^e(0)) = (0.45, 0.6, 0.02, 0.02). \tag{54}$$

To begin, we shall check if the conclusion in Proposition 4 (i) holds. For this purpose, we shall fix the parameter of credibility on monetary policy θ at 0:

$$\theta = 0. \tag{55}$$

To represent the situation of Proposition 4 (i), we shall set the revision speed of inflation-deflation expectations β as follows so that condition (38) would be satisfied:

$$\beta = 0.05. \tag{56}$$

By making use of (53), we can calculate the threshold (bifurcation) value of the intensity parameter of the inflation-targeting policy γ denoted by γ^* as follows:

$$\gamma^* = 0.232.$$

To see that the value of γ makes a great difference in this case, we shall perform numerical simulations in the following three cases:

$$\gamma = 0.01, \tag{57}$$

$$\gamma = 0.23, \tag{58}$$

$$\gamma = 0.5. \tag{59}$$

In the following figure illustrated are the solution paths of System (K) with (54), (55), (56) and (57).

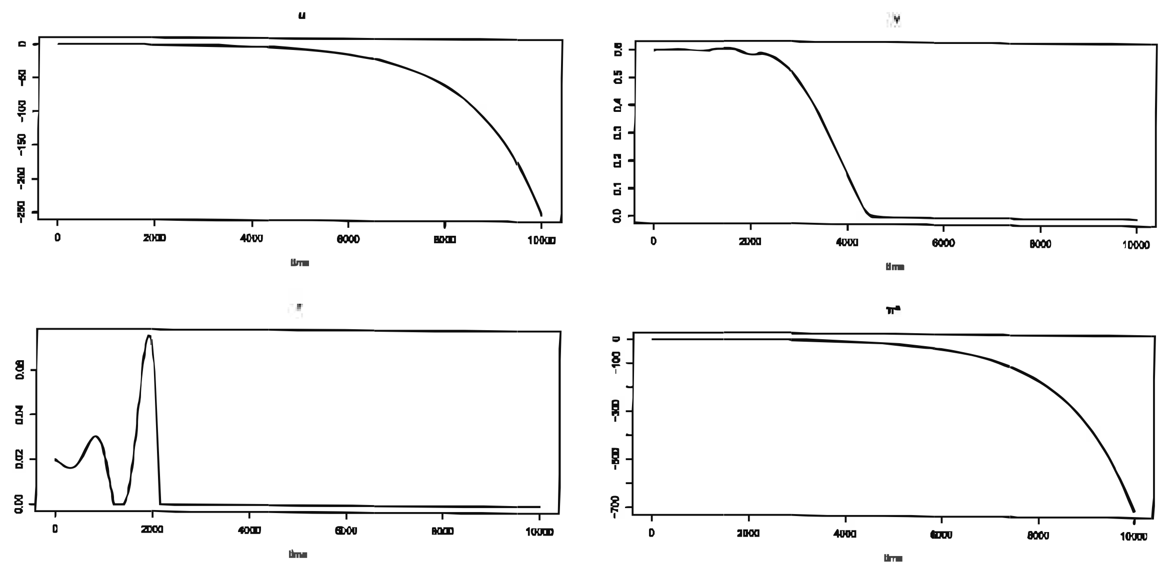

Figure 1: Solution paths of System (K) with $\theta = 0$, $\beta = 0.05$ and $\gamma = 0.01$

In the following figure, the solution paths of System (K) with (54), (55), (56) and (58) are described.

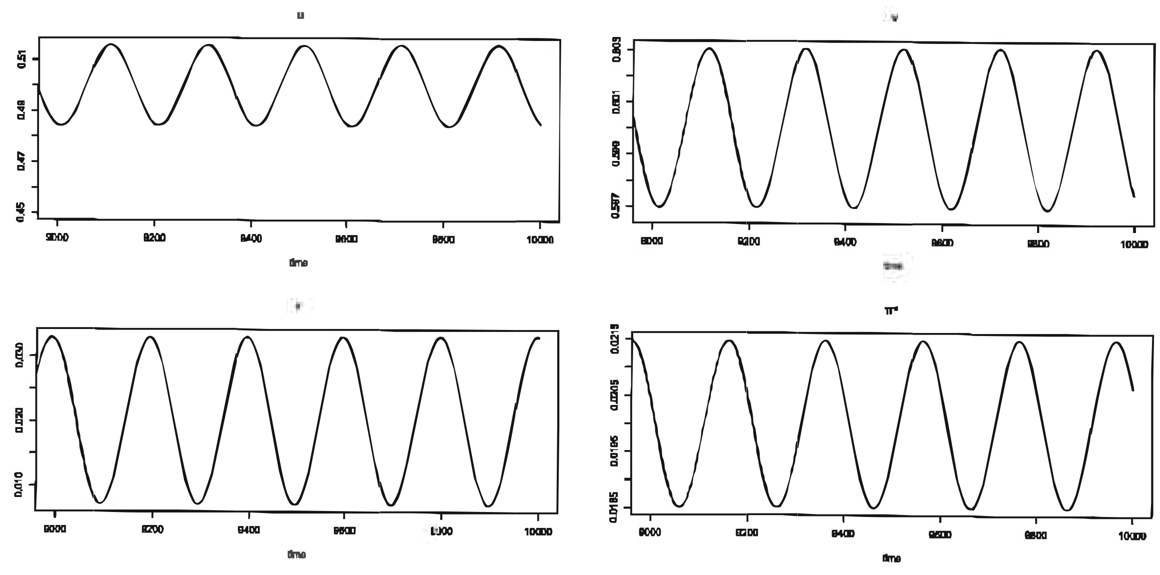

Figure 2: Solution paths of System (K) with $\theta = 0$, $\beta = 0.05$ and $\gamma = 0.23$

Finally, the following figure depicts the solution paths of System (K) with (54), (55), (56) and (59)

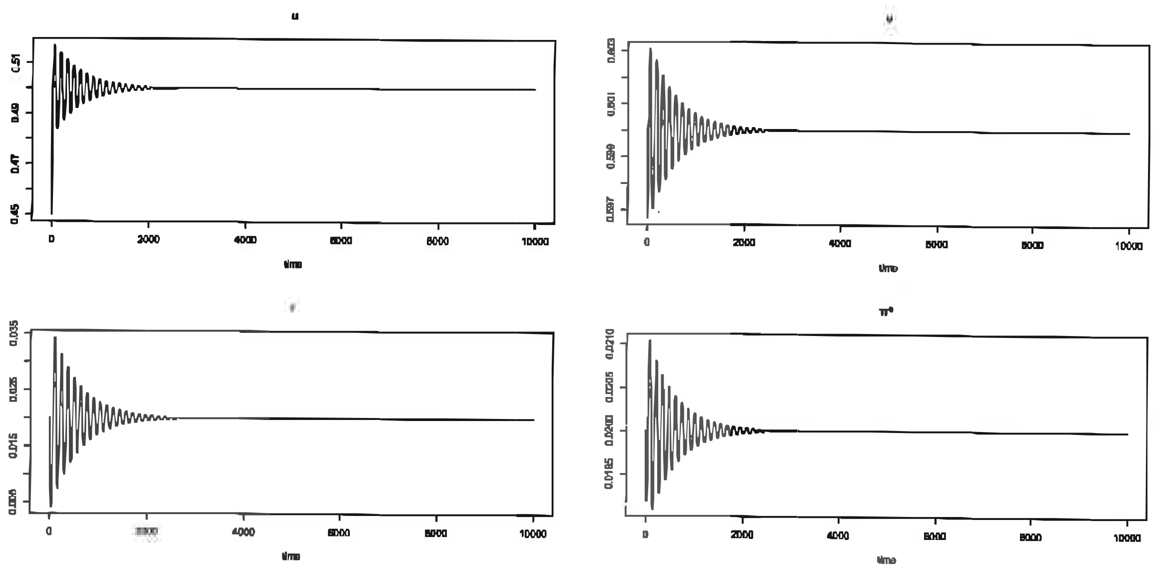

Figure 3: Solution paths of System (K) with $\theta = 0$, $\beta = 0.05$ and $\gamma = 0.5$

Figs. 1-3 describe the cases where the four variables diverge, fluctuate with constant amplitudes and converge, respectively. They show vicious circles of depression and deflation, persistent business cycles and stable equilibrium, respectively. We can say that the larger the value of γ is, the smaller the amplitudes of fluctuations of variables are. In other words, the stabilizing effect of the inflation-deflation policy whose intensity is measured by γ can be seen from figs. 1-3.

We shall proceed to confirm the validity of Proposition 4 (ii). In this case, the revision speed of inflation-deflation expectations β should be large enough to meet (42). So we shall fix β as follows:

$$\beta = 0.3. \tag{60}$$

To emphasize the destabilizing effect of inflation-deflation expectations, we shall dare to fix the value of γ as (59), for which the stability of System (K) obtains provided that β is sufficiently small. In the following figure described are the solution paths of System (K) with (54), (55), (60) and (59).

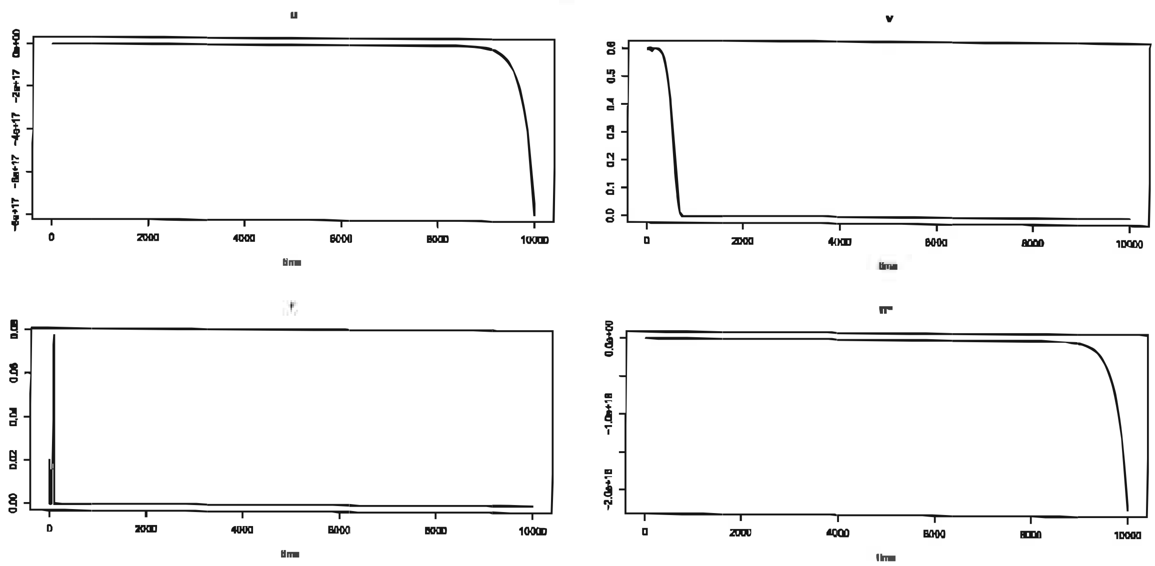

Figure 4: Solution paths of System (K) with $\theta = 0$, $\beta = 0.3$ and $\gamma = 0.5$

It is seen from fig 4 that when β is sufficiently large and θ is 0, the stability of System (K) cannot be revitalized even when γ is large enough. This indicates that fast revisions of inflation-deflation expectations have a powerful destabilizing force.

Next, we shall proceed to check if the conclusion of Proposition 5 actually holds. For this purpose, we shall fix the parameter of credibility on monetary policy θ at 1:

$$\theta = 1. \tag{61}$$

For the values of the revision speed of inflation-deflation expectations γ and the intensity parameter of monetary policy γ, we shall adopt (56) and (59) so that the values of β and γ would be sufficiently large. The following figure depicts the solution paths of System (K) with (54), (59), (60) and (61).

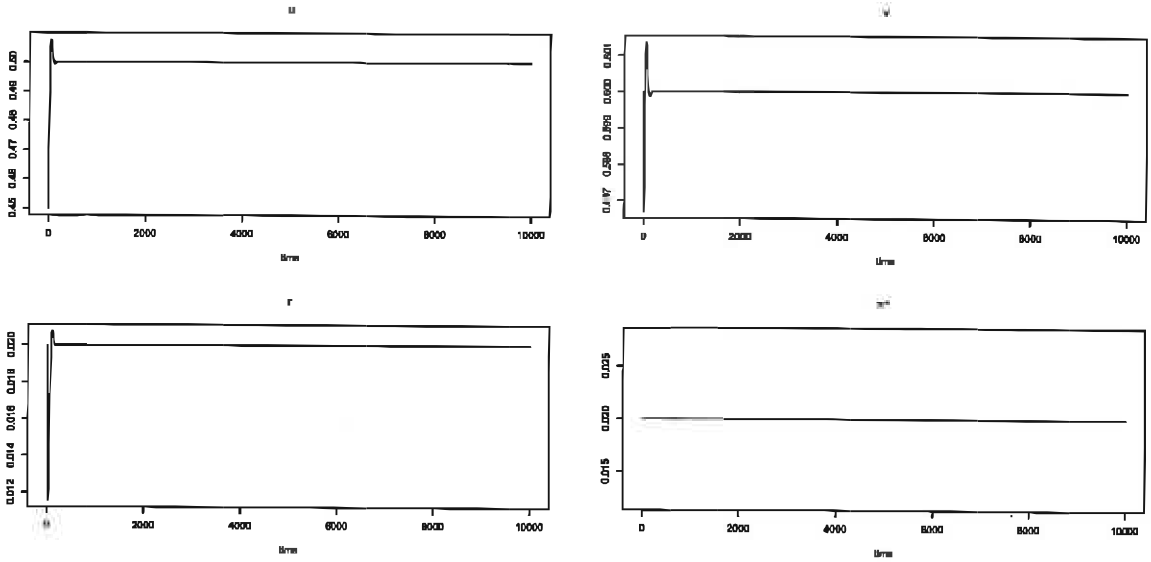

Figure 5: Solution paths of System (K) with $\theta = 1$, $\beta = 0.3$ and $\gamma = 0.5$

We can see from this figure that though the value of β is sufficiently large, the stability of System (K) can be regained in the case of $\theta = 1$ if γ is large enough.

Finally, we shall take a look at some role of the non-negativity constraint of the nominal rate of interest. In our System (K), the nominal rate of interest r is not allowed to be negative due to the non-negativity constraint in (10). But we can expect that if this constraint is removed, the dynamic properties of System (K) may change. To see if the relaxation of this constraint makes a difference, we shall consider the following system:

$$\dot{u} = 0.075 + 0.03u - 0.45(1 - v)u - 0.15(r - \pi^e), \tag{49}$$

$$\dot{v} = (0.1 + 0.04u - 0.2v)v, \tag{50}$$

$$\dot{r} = \gamma(0.04u + 0.1v + \pi^e - 0.1), \tag{62}$$

$$\dot{\pi}^e = \beta[\theta(0.1 - \pi^e) + (1 - \theta)(0.04u + 0.1v - 0.08)]. \tag{52}$$

The main and only difference from System (K) ((49)-(52)) is that r is allowed to be negative in this system. In this section, we shall call the system of equations (49), (50), (52) and (62) "System (K*)."

We shall compare the properties of Systems (K) and (K*) through numerical simulations. To do so, we shall tentatively choose the values of β, γ and θ as follows:

$$\beta = 0.05, \tag{56}$$

$$\gamma = 0.2, \tag{63}$$

$$\theta = 0. \tag{55}$$

In the following figure illustrated is the solution paths of System (K) (with the non-negativity constraint) with (54), (55), (56) and (63).

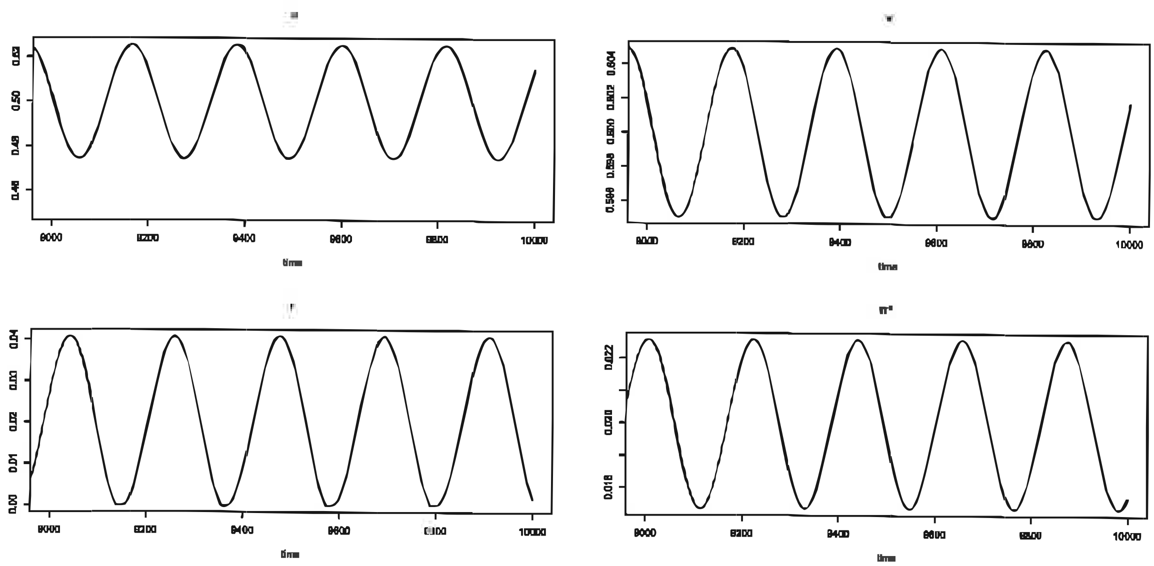

Figure 6: Solution paths of System (K) with $\theta = 0$, $\beta = 0.05$ and $\gamma = 0.2$

On the other hand, the following figure describes the solution paths of System (K*) (without the non-negativity constraint) with (54), (55), (56) and (63).

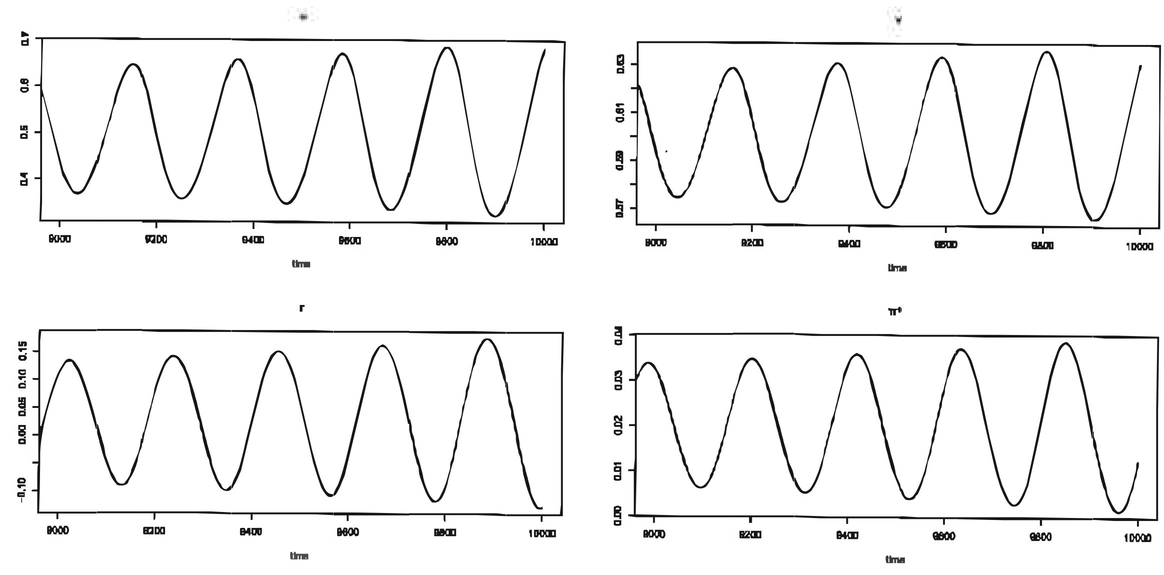

Figure 7: Solution paths of System (K*) with $\theta = 0$, $\beta = 0.05$ and $\gamma = 0.2$

By comparing figs. 6 and 7, we can easily see that the amplitudes of fluctuations of all variables are constant in System (K) (with the non-negativity constraint) while they expand in System (K*) (without the non-negativity constraint). Indeed, it is seen in fig. 6 that the non-negativity constraint binds in System (K). The difference between Systems (K) and (K*) implies that the non-negativity constraint of the nominal rate of interest works as a kind of stabilizer in this case. This consequence is reminiscent of Hicks' (1950) concept of "floor." Note, however, that this consequence depends upon our specifications on the functional forms and the parameter values.[26]

5 Concluding remarks

We shall summarize our present analysis.

In this paper, we have extended the traditional Kaleckian system by incorporating the effects of inflation-deflation expectations and of the monetary authority's inflation-targeting policy. In Section 2, we have set up our Kaleckian model, which consists of the four dynamic equations: the dynamic processes of the rate of utilization, of the wage share, the nominal rate of interest and the expected rate of inflation. In Section 3, we have analyzed the dynamic properties of our Kaleckian system. In Section 3.1, we have confirmed that our Kaleckian system possesses the (local asymptotic) stability in the case of stationary inflation-deflation expectations and in the absence of the inflation-targeting policy. We have then revealed in Section 3.2 that, in the absence of the inflation-targeting policy, the stability of our Kaleckian system is always lost if inflation-deflation expectations are revised adaptively (regardless of the value of the revision speed of these expectations). This consequence is more destructive than those obtained

[26]In other settings, the non-negativity constraint can be a "destabilizer." For such a case, see, for instance, Asada et al. (2016).

in the related works. In Section 3.3, we have verified that the inflation-targeting policy can stabilize our Kaleckian system but that the stabilizing force of this policy depends upon the credibility of it. In particular, we have found that when the inflation-targeting policy is not trusted enough, this policy can serve as a stabilizer only if the revision speed of inflation-deflation expectations is sufficiently small, but that when this policy is credited enough by the public, it can always possess the stabilizing effect regardless of the revision speed. This consequence is consistent with the preceding works. In short, the destabilizing force of inflation-deflation expectations is more powerful in the Kaleckian system than in the Keynesian one but this force can be weakened or combated by the monetary authority's intensive inflation-targeting policy. In Section 4, we have confirmed that some of our analytical results are valid.

Throughout this paper, we have evaluated the impact of inflation-deflation expectations and the stabilizing force of the inflation-targeting policy in a Kaleckian system. These two factors have been unexplored in the preceding works on the Kaleckian analysis. We hope that our analysis is helpful for the discussion on proper economic policies in the macroeconomic system.

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