

Discussion Paper No.245

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Volunteer's Dilemma and Step-level Public Goods  
Games with Binary Decision

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February 2015



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# Quantal Response Equilibria in a Generalized Volunteer's Dilemma and Step-level Public Goods Games with Binary Decision<sup>§</sup>

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February 3, 2015

## Abstract

The present paper fully characterizes equilibria of a generalized Volunteer's Dilemma game, which is an integration of the volunteer's dilemma game and the step-level public goods game with binary decision. We also examined the explanatory power of a widely accepted model with bounded rationality, the quantal response equilibrium (QRE). It is shown that the performance of the QRE model is better in explaining laboratory data.

**Keywords:** Volunteer's dilemma, public goods, binary decision, quantal response equilibrium

**JEL classification number:** C72, D72, D74, H41

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<sup>§</sup> We thank Andreas Diekmann and Axel Franzen for sharing raw data of their experiments. We also thank Sacha Bourgeois-Gironde for drawing our attention to the Volunteer's Dilemma game during our discussion.

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## 1. Introduction

The Volunteer's Dilemma game (VOD) was first formulated by Diekmann (1993) to elucidate "social dilemmas" or "social traps" broader than those covered by the prisoner's dilemma. A typical social situation is the helping behavior of people witnessing an accident or crime, as best exemplified by the murder case of Kitty Genovese examined by Darley and Latané (1968). It is said that her life could have been saved if only one bystander had paid a small amount of cost (e.g., making an emergency call to the police).

An interesting issue concerning VOD is the effect of the group size on the tendency to cooperate or contribute, the so called "bystander effect." A large amount of evidence has been accumulated by political scientists and psychologists to investigate factors affecting this effect (Latané and Nida, 1981).

Biologists have also found that similar situations arise in groups of animals. It is known that one member of the group occasionally looks up and checks for a predator for protecting the group as a whole. Archetti and Scheuring (2010) extend VOD to the situation where more than one volunteer is necessary (we call it a generalized VOD) and derive an approximation formula for the equilibrium probability of contribution. Technically speaking, a generalized VOD is a special case of the step-level public goods game with binary decision (Croson and Marks, 2000). However, to the best of our knowledge, the formal characterization of the mixed strategy equilibria in this class of games has not yet appeared.

Against these backdrops, this paper is first seen as an attempt to derive the necessary and sufficient condition for the existence of the mixed strategy equilibrium of generalized VOD. Furthermore we will analyze a generalized VOD with a widely accepted behavioral model with bounded rationality, quantal response equilibrium (QRE). Goeree et al. (2005a) report their QRE analysis of this game, but their analysis does not seem to cover all cases. We will provide almost complete characterization of the QRE of this class of games. Level-k analysis of a generalized VOD is brand new. Finally, we will give the econometric estimation for the two models using laboratory data in previous research. Our results show that QRE is better than level-k model in the explanation of the data.

## 2. Model

### 2.1 General environment and pure strategy equilibria

Each of  $n$  players simultaneously decides whether to contribute (denoted by  $C$ ), or not to contribute (denoted by  $N$ ). If more than or equal to  $m$  players contribute, public goods is provided and a fixed amount of public goods benefit,  $V > 0$ , goes to all the players regardless of their decisions. Otherwise, every player receives a fixed amount of payoff,  $L \geq 0$ . Cost of contribution is  $K \geq 0$ . Assume that  $V - K > L$ .

There are two kinds of pure strategy Nash equilibria, cooperative and non-cooperative. If contribution by  $m$  players is necessary for the provision of the public goods, exactly  $m$  players' choosing  $C$  constitutes a pure strategy Nash equilibrium. For  $m > 1$ , there exists a non-cooperative equilibrium in which no player contributes. For  $m = 1$ , no such equilibrium exists.

### 2.2 Mixed strategy equilibria in normal-form

The following analyses focus on symmetric equilibria. Suppose that every player contributes with probability  $p$ . Then, the expected payoff for  $C$  is

$$E(C) = q(m-1)(V-K) + (1-q(m-1))(L-K),$$

where

$$q(z) \equiv \sum_{k=z}^{n-1} \binom{n-1}{k} p^k (1-p)^{n-k-1}$$

is the probability that at least  $z$  out of  $n-1$  players choose  $C$ . Similarly, the expected payoff for  $N$  is

$$E(N) = q(m)V + (1-q(m))L.$$

In equilibria in totally mixed strategy, the expected payoffs to  $C$  and  $N$  should be equal. Thus we have

$$q(m-1) - q(m) = \frac{K}{V-L} \quad (1)$$

The left-hand side of (1) is the "marginal probability," the difference of probability between the event that at least  $m-1$  players contribute and the event that at least  $m$  players contribute.

The equation (1) can be simplified as follows.

$$f(p) \equiv \binom{n-1}{m-1} p^{m-1} (1-p)^{n-m} = \frac{K}{V-L} \quad (2)$$

Solving the equation (2) numerically gives the mixed strategy probability. For the cases with  $m = 1$  or  $m = n$ , a unique solution can be explicitly solved as below (for  $m = 1$  case, Diekmann (1993)).

**Proposition 1.** *For  $m = 1$  or  $m = n > 1$ , there exists a unique symmetric mixed strategy equilibrium. The probability of contribution in the symmetric equilibrium in mixed strategy is decreasing in  $n$  when  $m = 1$  and increasing in  $n$  when  $m = n$ .*

*Proof.* When  $m = 1$ , the equation (2) becomes

$$(1 - p)^{n-1} = \frac{K}{V - L} < 1,$$

and the solution is obviously unique. It is also obvious that  $p$  is decreasing in  $n$ . When  $m = n$ , the equation (2) becomes

$$p^{n-1} = \frac{K}{V - L}.$$

In this case too, the solution is unique and  $p$  is increasing in  $n$  as  $V - K > L$  by assumption. Q.E.D.

The reason that the equilibrium is unique when  $m = 1$  or  $m = n > 1$  is that  $f(p)$  is monotone. When  $1 < m < n$ ,  $f(p)$  is bell shaped with  $f(0) = f(1) = 0$  as the following calculation shows. Differentiating the left side of equation (2) with respect to  $p$  yields

$$f'(p) = \binom{n-1}{m-1} p^{m-2} (1-p)^{n-m-1} \{(m-1) - (n-1)p\}. \quad (3)$$

Obviously,  $f'(p) > 0$  for  $0 < p < \frac{m-1}{n-1}$ ,  $f'(p) < 0$  for  $\frac{m-1}{n-1} < p < 1$  and  $f(p)$  takes the maximum value when

$$p^* = \frac{m-1}{n-1}.$$

In such cases, we have the following proposition.

**Proposition 2.** *Suppose  $1 < m < n$ . Let  $p^* = \frac{m-1}{n-1}$ . (1) if  $f(p^*) < K/(V - L)$ , then there is no symmetric equilibrium in totally mixed strategy. (2) if  $f(p^*) = K/(V - L)$ , then  $p = p^*$  is the unique symmetric equilibrium. (3) if  $f(p^*) > K/(V - L)$ , then*

there are two symmetric equilibria in totally mixed strategy, one with  $p > \frac{m-1}{n-1}$  and the

other  $p < \frac{m-1}{n-1}$ .

*Proof.* First suppose that

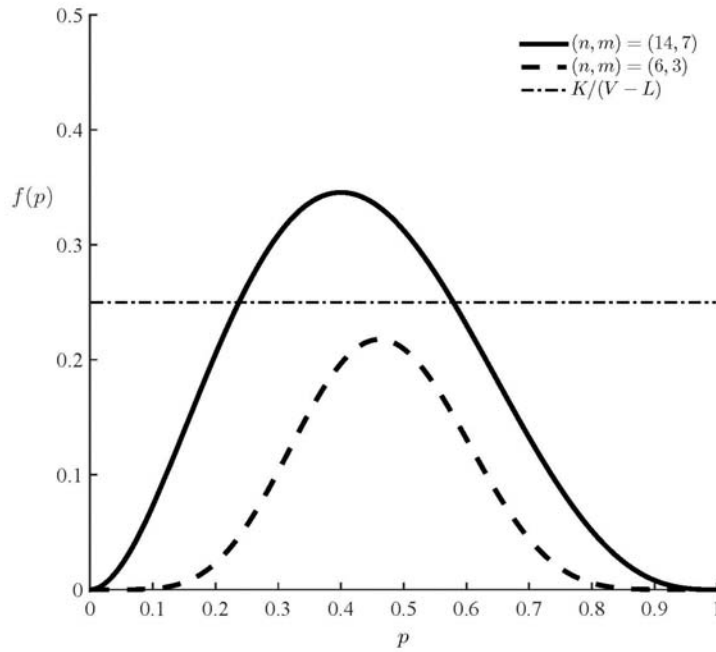
$$f(p^*) < K/(V - L). \quad (4)$$

Equation (2) has obviously no solution. If

$$f(p^*) > K/(V - L), \quad (5)$$

there are two equilibria. If  $f(p^*) = K/(V - L)$ ,  $p = p^*$  is the unique symmetric equilibrium. *Q.E.D.*

With the parameter values  $V = 1.0$ ,  $L = 0.2$ , and  $K = 0.2$  used by Goeree et al. (2005b), the numerical solutions of mixed strategies for  $(n, m) = (6, 3)$  and  $(n, m) = (14, 7)$  are presented in Figure 1. The former has two solutions but the latter has no solution.



**Figure 1.** Mixed strategy probability for  $(n, m) = (6, 3)$  and  $(n, m) = (14, 7)$ .

### 2.3 Quantal response equilibrium (QRE)

Quantal response equilibrium (QRE) is an equilibrium concept based on boundedly rational strategic behavior, assuming that players play a noisy best response (McKelvey and Palfrey, 1995). We focus on symmetric equilibria. For a parameter  $\lambda \in [0, \infty)$ , the

stochastic best response in terms of choice probability of  $C$  is given by

$$p = \frac{\exp(\lambda \cdot E(C))}{\exp(\lambda \cdot E(C)) + \exp(\lambda \cdot E(N))} = \frac{1}{1 + \exp[\lambda \cdot \{E(N) - E(C)\}]}. \quad (6)$$

QRE is a fixed point of this mapping.

The parameter  $\lambda \in [0, \infty)$  represents the degree of rationality such that  $\lambda = 0$  implies complete randomizing over pure strategies. If  $\lambda = 0$ , then  $p = 1/2$ , which is usually called the centroid of the simplex of the strategy space. McKelvey and Palfrey (1995) show that, for almost all games, the graph  $(\lambda, \text{QRE}(\lambda))$  contains a unique branch which starts at the centroid and converges to a unique Nash equilibrium as  $\lambda$  goes to infinity. The limit point of this principal branch is called limiting (logit) QRE. Thus, limiting QRE can be interpreted as an equilibrium selection criterion.

Assuming  $E(C) \neq E(N)$  allows us to rearranging Equation (6) as

$$\lambda = \frac{\log \frac{1-p}{p}}{-f(p)(V-L) + K}. \quad (7)$$

Note that the numerator  $\log(1-p)/p$  is monotone decreasing, and zero at  $p = 1/2$ . It is positive in  $(0, 1/2)$ , and negative in  $(1/2, 1)$ . With this in mind, we need to consider the behavior of the denominator to analyze equation (7). As the proof of Proposition 1 shows, the value of  $p$  satisfying  $f(p) = K/(V-L)$  exists and unique, when  $m = 1$  or  $m = n > 2$ . Let the value be denoted as  $\hat{p}$ .

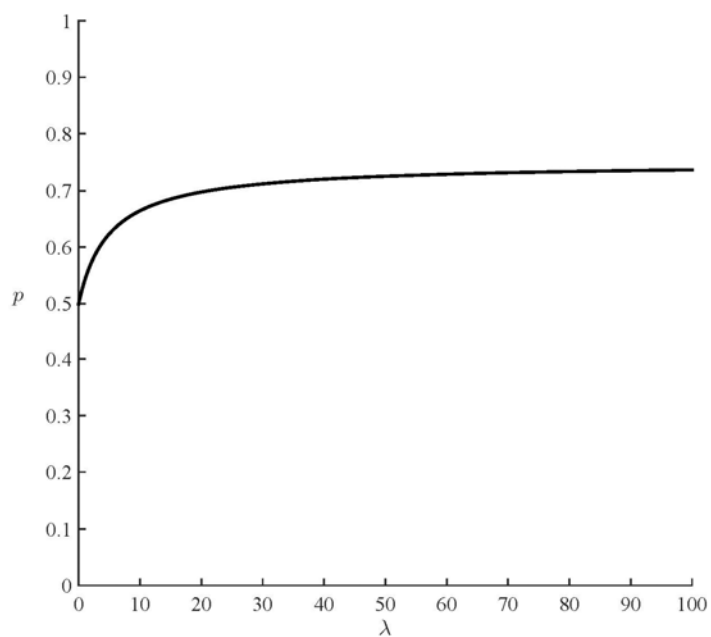
**Proposition 3.** *Suppose  $m = 1$ . Then the limiting logit QRE is  $p^* = \hat{p}$  (the unique symmetric Nash equilibrium in totally mixed strategy). For  $m = n > 1$ , the limiting logit QRE is  $p^* = 0$  if  $\hat{p} > 1/2$ ,  $p^* = 1$  if  $\hat{p} < 1/2$  and  $p^* = 1/2$  if  $\hat{p} = 1/2$ .*

*Proof.* Consider the case with  $m = 1$ . First consider the case that  $\hat{p} < 1/2$ . Then considering the branch starting at the centroid, we observe it is disconnected at  $\hat{p}$ , at which  $\lambda$  goes to infinity as  $p$  approaches  $\hat{p}$  from  $1/2$ . This establishes the statement. In the case  $\hat{p} > 1/2$ , the branch starting at the centroid is connected over the interval  $(0, \hat{p})$  and  $\lambda$  goes to infinity as  $p$  approaches  $\hat{p}$  from  $1/2$ . When  $\hat{p} = 1/2$ , then  $p$  stays at  $1/2$  along the graph of QRE.

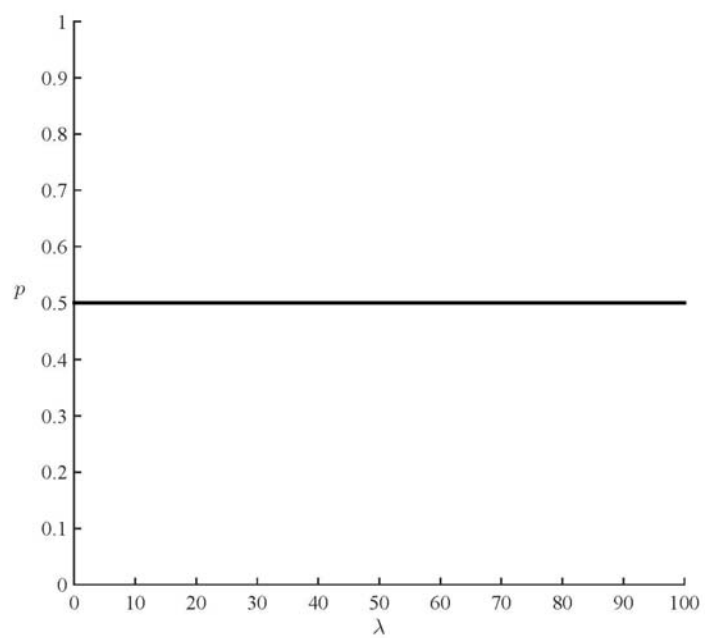
Next suppose  $m = n > 1$ . Then  $f(p) = p^{m-1}$ . Over the interval  $(0, \hat{p})$ , the denominator of equation (7) is positive. If  $\hat{p} > 1/2$ , then the branch starting at the centroid is connected in  $(0, \hat{p})$ . Furthermore,  $\lambda$  goes to infinity as  $p$  approaches zero. If  $\hat{p} < 1/2$ , the branch starting at the centroid is connected in  $(\hat{p}, 1)$ .  $\lambda$  goes to infinity as  $p$  approaches unity. If  $\hat{p} = 1/2$ ,  $p = 1/2$  is compatible with any  $\lambda \geq 0$ . Other

branches are separated from the centroid.

*Q.E.D.*



(a)



(b)

**Figure 2. Typical QRE graphs when  $m = 1$ . (a)  $n = 2, m = 1, V = 1, L = K = 0.2$ , (b)  $n = 2, m = 1, V = 100, L = 0, K = 50$ .**



Diekman (1993), Franzen (1995) and Goeree et al. (2005b) all consider cases with  $m = 1$ , where the above proposition applies. Figure 2 (a) depicts the QRE graph in the case of Goeree et al. (2005b) where  $n = 2, m = 1, V = 1, L = 0.2, K = 0.2$ . The denominator of the right side of Equation (7) is negative iff  $p \in (0, 3/4)$  and the numerator is nonpositive iff  $p \in [1/2, 1)$ . Thus only the interval  $[1/2, 3/4)$  is compatible with  $\lambda \geq 0$ . Figure 2 (b) shows the QRE graph in the case of Diekman (1993) and Frazen (1995), where  $n = 2, m = 1, V = 100, L = 0, K = 50$ . In this case, only  $p = 1/2$  is compatible with nonnegative value of  $\lambda$ . It is also compatible with any  $\lambda \geq 0$ .

Now consider the case with  $1 < m < n$  as in Dawes et al (1986) and Rapoport and Ehed-Levy (1989). From Proposition 2, if inequality (4) holds, only pure strategy equilibria exists. If inequality (5) holds, there exist two values of  $p$  such that  $f(p) = K/(V - L)$ . Denote the smaller of these as  $\underline{p}$  and the larger one as  $\bar{p}$  in such a case. Otherwise, there exists unique mixed strategy equilibrium. For each of these cases, we have following characterizations for the limiting QRE.

**Proposition 4.** *If Inequality (4) holds, the limiting QRE is  $p^* = 0$ .*

*Proof.* Inequality (4) ensures the denominator of the right hand side of Equation (7) is positive. Then the only possible principal branch starting at the centroid for  $\lambda \geq 0$  is the graph of Equation (7) restricted on  $p \in (0, 1/2]$ . This corresponds to the branch leading to the limiting QRE. Obviously  $\lambda \rightarrow \infty$  as  $p \rightarrow 0$ . *Q.E.D.*

**Proposition 5.** *Suppose Inequality (5) holds. If  $1/2 < \underline{p}$ , the limiting QRE is  $p^* = 0$ . If  $\underline{p} < 1/2$ , the limiting QRE is  $p^* = \bar{p}$ . If  $\underline{p} = 1/2$ , then  $p^* = 1/2$ .*

*Proof.* If Inequality (5) and  $1/2 < \underline{p}$  hold, the denominator of the right hand side of Equation (7) is positive in  $(0, \underline{p})$  and the numerator is positive in  $(0, 1/2)$ . Focusing on the branch starting from the centroid, the same logic applies as the previous case.

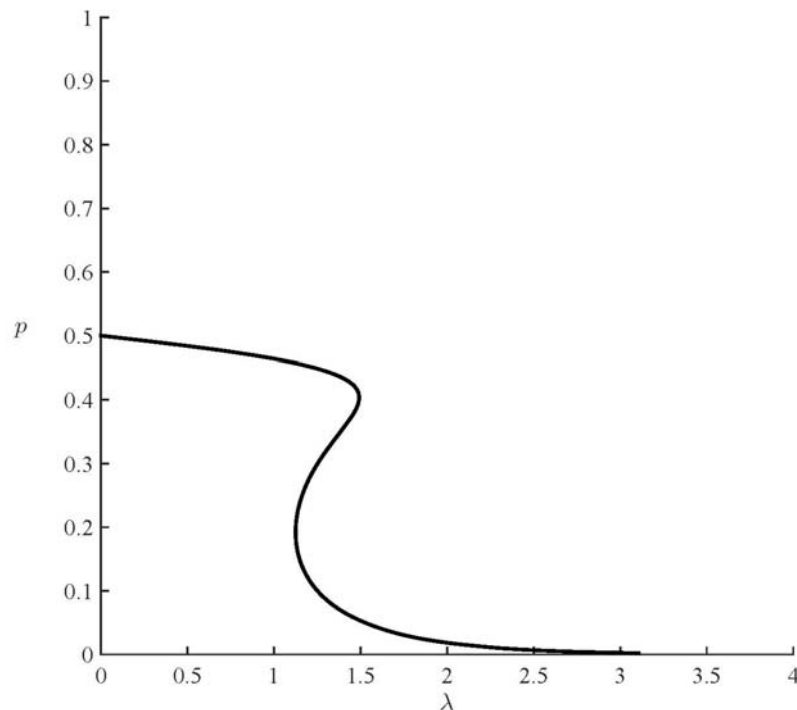
Suppose Inequality (5) holds. First consider the case where  $\underline{p} < 1/2 < \bar{p}$ . The right hand side of Equation (7) is undefined at  $\underline{p}$  and  $\bar{p}$ , between which  $1/2$  lies. This allows us to restrict attention to  $(\underline{p}, \bar{p})$  for finding the branch starting at the centroid. In this interval,  $1/2 < p < \bar{p}$  is compatible with  $\lambda \geq 0$ . Obviously  $\lambda \rightarrow \infty$  as  $p \rightarrow \bar{p}$ . Next consider the case where  $\bar{p} < 1/2$ . The branch starting from the centroid obviously lies in  $(\bar{p}, 1/2)$ . Obviously  $\lambda \rightarrow \infty$  as  $p \rightarrow \bar{p}$  along this branch. Finally if  $\underline{p} = 1/2$ , then

$p = 1/2$  is compatible with any  $\lambda \geq 0$ . Other branches are separated from the centroid.  
*Q.E.D.*

**Proposition 6.** *Suppose that there exist unique value of  $p$  such that  $f(p) = K/(V - L)$  and Let  $\tilde{p}$  be that value. If  $\tilde{p} < 1/2$ , then the limiting QRE is  $p^* = \tilde{p}$ . If  $\tilde{p} = 1/2$ , then the limiting QRE is  $p^* = 1/2$ . If  $\tilde{p} > 1/2$ , then the limiting QRE is  $p^* = 0$ .*

*Proof.* If  $\tilde{p} < 1/2$ , then the branch starting at the centroid is connected in  $(\tilde{p}, 1/2]$  and  $\lambda$  goes to infinity as  $p$  approach  $\tilde{p}$  from  $1/2$ . This proves the first claim. If  $\tilde{p} = 1/2$ , then the branch starting at  $p = 1/2$  is a straight horizontal line. Although there is a branch in  $(0, 1/2)$ , it is disconnected from the centroid. If  $\tilde{p} > 1/2$ , then along the branch starting at the centroid,  $\lambda$  goes to infinity as  $p$  approach 0. *Q.E.D.*

Figure 3 shows the QRE graph for the case in Rapoport and Ehed-Levy (1989), where  $n = 5, m = 3, V = 5, L = 0, K = 2$ . Note that the correspondence QRE( $\lambda$ ) is upper hemi-continuous and one-dimensional manifold, as shown in McKelvey and Palfrey (1995).



**Figure 3.** A typical QRE correspondence when  $1 < m < n$  ( $n = 5, m = 3, V = 5, L = 0, K = 2$ ).

*Remark:* Our Propositions 3-6 correspond to Propositions 3 (Volunteer's Dilemma) and 4 (step-level public goods game with binary decision) in Goeree et al. (2005a). They assert that the choice probability of contributing,  $p$ , in the limiting QRE in both cases unconditionally converges to 0. But it is not the case with  $1 < m < n$  when inequality (4) doesn't hold. So, our analysis adds some qualification to their assertion.

### 3. Estimation

We now conduct econometric comparisons to check the explanatory power of QRE. Data are taken from the experiments reported in Diekmann (1993), Franzen (1995) and Goeree et al. (2005b) for  $m = 1$  (Volunteer's Dilemma), and from those in Dawes et al. (1986) and Rapoport and Ehed-Levy (1989) for  $m > 1$  (step-level public goods game with binary decision). In Diekmann (1993), Franzen (1995) and Dawes et al. (1986), the game was played once and for all. In Goeree et al. (2005b), the game is repeated twenty times and Rapoport and Ehed-Levy (1989) twenty-five times. Tables 1 and 2 show the data, Nash equilibria in mixed strategies and the estimation results.

Tables 1 and 2 here.

In the QRE estimation, the following log-likelihood function is maximized with respect to  $\lambda$ :

$$LL = g_c \log p(\lambda) + (g - g_c) \log(1 - p(\lambda)),$$

where  $p(\lambda)$  is the choice probability of  $C$  in QRE corresponding to  $\lambda$ ,  $g_c$  is the number of instances in which  $C$  is chosen,  $g$  is the total number of choices.

Then, as for the Volunteer's Dilemma game, substantial deviation from mixed strategy equilibria is observed in the experiments of Diekmann (1993), Franzen (1995), while the experimental result is very close to mixed strategy equilibria in Goeree et al. (2005b). In  $n = 2$  in Diekmann (1993) and Franzen (1995) and  $n = 3$  in Goeree et al. (2005b), as the choice probability of contributing is constant, i.e., 0.5, for any value of  $\lambda$  (see again Figure 2 (b)), the goodness-of-fit of the QRE cannot improve no more than the mixed strategy equilibria. So, except for those cases, the QRE model gives better explanation for the data in general.

Next, we check the data in step-level public goods games with binary decision. Note that in the games examined by Dawes et al. (1986) and Rapoport and Ehed-Levy (1989), there is no Nash equilibrium in totally mixed strategy ( $0 < p < 1$ ), that is, there

are only pure strategy equilibria. The limiting QRE selects the non-cooperative equilibrium,  $p = 0$ , in these cases. However, substantial deviation from these equilibria was observed. Subjects actually played more cooperatively. We see that  $p(\lambda)$ , the choice probability of C based on the estimated  $\lambda$ , is very close to the observed data, except for one of the sessions with  $(n, m) = (7, 5)$  in Dawes et al. (1986), where the proportion of C substantially exceeds 0.5. As the upper bound of the QRE probability is 0.5 (see the proof of Proposition 4), the goodness-of-fit of the QRE cannot improve any more than that at the centroid. But as there is no mixed strategy equilibrium in this case, the QRE model gives better prediction than pure strategy equilibrium.

In sum, QRE's prediction is (weakly, at least) better than equilibrium prediction in pure and mixed strategies both in Volunteer's Dilemma and step-level public goods games.

## 4. Conclusion

We have characterized equilibria in a generalized Volunteer Dilemma game, and have derived predictions with a boundedly rational model, quantal response equilibrium (QRE). It is found that the QRE model better performed in econometric analysis of the laboratory data. In this paper, we only focus on symmetric game where cost for contribution is the same among players. Analyzing the game with asymmetric cost will be our next research agendum. Finally, our approach is also applicable to a certain kind of voting games. Such an application is also of great interest.

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	Diekmann (1993)		Franzen (1995) <sup>1)</sup>					Goeree et al. (2005b) <sup>2)</sup>						
$n$	2	5	2	3	5	7	9	21	51	2	3	6	9	12
$(V, L, K)$	(100, 0, 50)	(100, 0, 50)	(100, 0, 50)	(100, 0, 50)	(100, 0, 50)	(100, 0, 50)	(100, 0, 50)	(100, 0, 50)	(100, 0, 50)	(1, 0.2, 0.2)	(1, 0.2, 0.2)	(1, 0.2, 0.2)	(1, 0.2, 0.2)	(1, 0.2, 0.2)
# of $C$	20	7	15	18	9	6	12	7	5	333	281	298	137	182
# of $N$	13	18	8	13	12	18	22	16	20	347	439	662	583	778
Proportion of $C$	0.610	0.280	0.652	0.580	0.429	0.250	0.353	0.304	0.200	0.490	0.390	0.310	0.190	0.190
Equilibrium Prob.( $C$ )	0.5	0.159	0.5	0.293	0.159	0.109	0.083	0.034	0.014	0.750	0.500	0.242	0.159	0.118
limiting QRE	0.500	0.160	0.500	0.293	0.160	0.110	0.083	0.034	0.014	0.750	0.500	0.242	0.159	0.119
Prob. ( $C$ ) in QRE	0.500	0.280	0.500	0.500	0.4286	0.250	0.353	0.3043	0.200	0.500	0.500	0.310	0.190	0.190
$\lambda$	$[0, \infty)$	0.041	$[0, \infty)$	0.000	0.007	0.034	0.013	0.017	0.028	0.000	0.990	10.606	27.762	12.029
$-LL(\lambda)$	22.87	14.824	15.942	21.488	14.341	13.500	22.074	14.134	12.510	471.340	499.066	594.656	350.370	466.193

1) In Franzen (1995)'s experiment, there was a case where  $n = 101$ . As the computation does not converge in that case, we omit it here.

2) As the game was repeated 20 times in Goeree et al. (2005b)'s experiment, here we show their aggregate data.

**Table 1. Experimental data and the estimation results for  $m = 1$  (Volunteer's Dilemma game).**

	<b>Dawes et al. (1986)</b>			<b>Rapoport and Ehed-Levy (1989) <sup>1)</sup></b>
<b><math>(n, m)</math></b>	(7, 3)	(7, 5)	(7, 5) <sup>2)</sup>	(5, 3)
<b><math>(V, L, K)</math></b>	(10, 0, 5)	(10, 0, 5)	(10, 0, 5)	(5, 0, 2)
<b># of <math>C</math></b>	36	45	9	548
<b># of <math>N</math></b>	34	25	26	952
<b>Proportion of <math>C</math></b>	0.514	0.643	0.257	0.365
<b>Prob.(<math>C</math>) in mixed strategy <sup>3)</sup></b>	N/A	N/A	N/A	N/A
<b>limiting QRE</b>	0	0	0	0
<b><math>p(\lambda)</math> in QRE</b>	0.5	0.5	0.257	0.365
<b><math>\lambda</math></b>	0.000	0.000	0.229	1.427
<b><math>-LL(\lambda)</math></b>	48.520	48.520	19.952	984.638

1) As the game was repeated 25 times in Rapoport and Ehed-Levy (1989)'s experiment, here we show their aggregate data.

2) This is the follow-up experiment.

3) In the games examined by Dawes et al. (1986) and Rapoport and Ehed-Levy (1989), there is no completely mixed strategy ( $0 < p < 1$ ).

**Table 2. Experimental data and the estimation results for  $m > 1$  (step-level public goods game with binary decision).**