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Chaotic Itinerancy in Regional Business Cycle Synchronization

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Abstract

Chaotic itinerancy is complex behavior in high-dimensional dynamical systems characterized by itinerant motion among many different ordered states through chaotic transition. In this study, we robustly observe this behavior in a model of regional business cycles, in which all regions are homogeneous and connected through producers' expectations. Although producers adjust price and output expectations quite slowly toward the average level announced by the government, regional business cycles begin to synchronize from the entrainment effect. Moreover, the economy is more likely to exhibit chaotic itinerancy when the producers emphasize the expected profit maximization and when they adjust their expectations slowly toward the average.

Key words: Regional business cycle, Nonlinear dynamics, Synchronization, Globally coupled map, Chaotic itinerancy

JEL Classifications: C61, E32

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1 Introduction

Business cycle synchronization has become a topic of growing interest from around the end of the 20th century (Antonakakis, 2012; Baxter and Kouparitsas, 2005; Yetman, 2011). The vast empirical literature elucidates that countries with intensified trade linkages have resemblant business cycles, which Selover and Jensen (1999) and Süssmuth (2003) explain by proposing a mode-locking model. However, little is known about the business cycle synchronization across subnational regions within a country, as mentioned by Kouparitsas and Nakajima (2006). Selover *et al.* (2005) also apply a mode-locking model to this topic under a scenario in which the cycles of disparate regions synchronize through interregional trade linkages. Onozaki *et al.* (2007) propose another scenario in which regional business cycles may synchronize through producers' expectations based on global information via a system of globally coupled maps (GCM), first proposed by Kaneko (1990).

Related to a GCM model, complex behavior called "chaotic itinerancy" is sometimes observed. The term describes the phenomenon of an orbit successively itinerating among many ordered states through chaotic transitions in dynamical systems. The phenomenon was independently discovered in a model of optical turbulence by Ikeda *et al.* (1989), a globally coupled chaotic system by Kaneko (1990), and nonequilibrium neural networks by Tsuda (1990). The term "chaotic itinerancy" was coined unanimously by its discoverers to denote universal dynamics in a class of high-dimensional dynamical systems (Kaneko and Tsuda, 2000). To the best of our knowledge, only Yasutomi (2003) has modeled chaotic itinerancy in an economic context. In this study, we reexamine the model of Onozaki *et al.* (2007) and robustly observe chaotic itinerancy for various constellations of parameters. This study is organized as follows: Section 2 presents a regional business cycle model and Section 3 presents the study on chaotic itinerancy by using the model. Section 4 presents the conclusion of our study.

2 Model

We begin by introducing a GCM model represented as follows:

$$x_{t+1}(i) = (1-\varepsilon)f(x_t(i)) + \frac{\varepsilon}{N}\sum_{j=1}^{N} f(x_t(j)), \qquad i = 1, \cdots, N, \tag{1}$$

where $x_t(i)$ denotes the value of the ith element at discrete time period t, and N the number of elements. A map $f(x)$ describes each element's endogenous dynamics. In this study, we use a noninvertible map as $f(x)$ that can exhibit chaotic behavior. The second term on the right-hand side of (1) represents the global interaction of each element through the "mean field," i.e., the all-to-all interaction. Therefore, two opposite effects coexist: the all-to-all interaction is inclined to synchronize all elements, and the chaotic instability in each element tends to desynchronize them. Depending on the value of $\varepsilon \in (0, 1)$, i.e., the balance between these two effects, the GCM model exhibits a rich variety of complex phenomena (Kaneko, 1990). The remainder of this section shows that the model analyzed in this study takes the same form as (1).

The economy has N regions, each with a single producer and a separate market. Each producer makes homogeneous goods and delivers them only to the market of its own region. Consumers are uniformly distributed over all regions and purchase goods from the market they belong to. Since the business cycles of different regions may synchronize through interregional trade, the model is intended to concentrate on factors other than trade. Therefore, we assume that there is no interregional trade to eliminate its effects. Government announces the average price and output of all regions in each period, and each producer acts based on this information; i.e., regions are linked via global information.

Each producer's decisions involve two stages as follows. First, each producer dislikes discrepancies between the actual and average levels of price and output, and intends to resolve them in response to the government's announced averages. At period t, the ith producer expects that the actual price $p_t(i)$ in its market will be adjusted adaptively at period $t+1$ toward the announced average level $\bar{p}_t := (1/N)\sum_{j=1}^{N} p_t(j)$ such that

$$p^e_{t+1}(i) = (1-\varepsilon)p_t(i) + \frac{\varepsilon}{N}\sum_{j=1}^{N} p_t(j), \tag{2}$$

where superscript e denotes expectations and $\varepsilon \in (0,1)$ is an expectation adjustment coefficient common among producers. Simultaneously, the ith producer establishes a provisional target of the output $\hat{x}_t(i)$ as a weighted average of the actual output $x_t(i)$ and average output $\bar{x}_t := (1/N)\sum_{j=1}^{N} x_t(j)$ with a weight ε, which equals the coefficient of the price expectation adjustment. Thus, $\hat{x}_t(i) = (1-\varepsilon)x_t(i) + \varepsilon\bar{x}_t(i)$.

Second, each producer calculates the output level maximizing the expected profit $\tilde{x}_{t+1}(i)$ based on its adaptively expected price (2) and the quadratic cost function $C(x) = x^2/2$ such that $\tilde{x}_{t+1}(i) = p^e_{t+1}(i)$. Ultimately, it sets a final output plan as a weighted average of the provisional target $\hat{x}_t(i)$ and the expected profit maximizing output $\tilde{x}_{t+1}(i)$ with a weight $\phi \in (0,1)$. The resulting formula for each producer's output is as follows:

$$\begin{aligned} x_{t+1}(i) &= (1-\phi)\,\hat{x}_t(i) + \phi\,\tilde{x}_{t+1}(i) \\ &= (1-\phi)(1-\varepsilon)\,x_t(i) + (1-\phi)\,\varepsilon\,\bar{x}_t + \phi\,\tilde{x}_{t+1}(i). \end{aligned}$$

Since the sum of the above three coefficients on the right-hand side is unity, we can paraphrase the producer's decision making as follows: Each producer's output is determined as a weighted average of $x_t(i)$, $\bar{x}_t(i)$, and $\tilde{x}_{t+1}(i)$.

The demand in each region is assumed to be identical and described by the same monotonic inverse demand function as follows:

$$p_t(i) = \frac{1}{(y_t(i))^\eta},$$

where $y_t(i)$ is the demand of the ith region at period t, and $\eta > 0$ is the inverse of the price elasticity of the demand. By assuming that in each period, prices are determined in each market for equilibrating supply and demand, we obtain a GCM model (1) with a noninvertible map as follows:

$$f(x_t(i)) = (1-\phi)x_t(i) + \frac{\phi}{(x_t(i))^\eta}, \tag{3}$$

the behavior of which is well studied by Onozaki *et al.* (2000) and known to exhibit chaotic behavior depending on the set of parameters. The larger ϕ and η are, the more

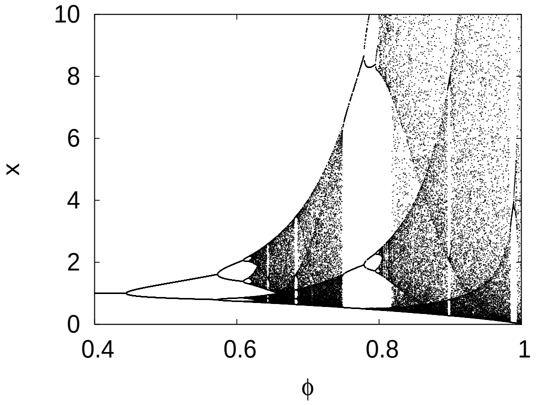

Fig. 1: Bifurcation diagram of one-dimensional map $x_{t+1} = f(x_t)$ by changing ϕ.

likely the economy behaves chaotically. After all, the regional business cycle model to be analyzed in this study consists of (1) and (3).

In what follows, the system dimension N is fixed as 10. Unless otherwise noted, η, ϕ, and ε are fixed as 3.5, 0.7, and 0.315, respectively [1]. We randomly select initial conditions $x_0(i)(i = 1, \cdots, 10)$ from the range $[0.5, 1.5]$. Note that one-dimensional map $x_{t+1} = f(x_t)$ can generate chaotic behavior with respect to ϕ around 0.62 through a period-doubling bifurcation, as shown in Fig. 1.

3 Chaotic Itinerancy

This section studies chaotic itinerancy in the regional business cycle model (1) and (3). Chaotic itinerancy is typical in high-dimensional chaos. In its presence, an orbit wanders among the different states of complexity (Kaneko and Tsuda, 2000). We first show the time developments of the model in Fig. 2 [2]. In coupled systems, separate oscillators sometimes synchronize, a phenomenon called "entrainment". A set of synchronizing oscillators is a cluster. Ten types of regimes (from one-cluster to 10-cluster regime) appear in the model's behavior. The one- and 10-cluster regimes are shown in Fig. 3 (left, right).

To definitely characterize chaotic itinerancy, let us define the effective dimension and its mean (Komuro, 2005). The effective dimension (ED) of a point $x \in \mathbb{R}^N$ with the precision δ, denoted by $ED(x, \delta)$, is defined as a number of clusters. Here, variables within a distance δ are considered to belong to the same cluster [3]. We rewrite the model (1) and (3) as a map $F : \mathbb{R}^N \to \mathbb{R}^N$ defined as follows:

$$(x_{t+1}(1), \cdots, x_{t+1}(N)) = F(x_t(1), \cdots, x_t(N)). \tag{4}$$

Then the mean of the effective dimension (MED) of a point $x \in \mathbb{R}^N$ with the precision δ

[1] Onozaki *et al.* (2007) show that the very long transient behaviors exist when the system dimension N is fixed as 100 and the parameter η is selected from the range $[1.0, 8.0]$.

[2] To obtain this figure, we neglect the first 10^4 iterations as transient behaviors and use 10^5 iterations.

[3] The value of δ is fixed as 10^{-4} in this paper.

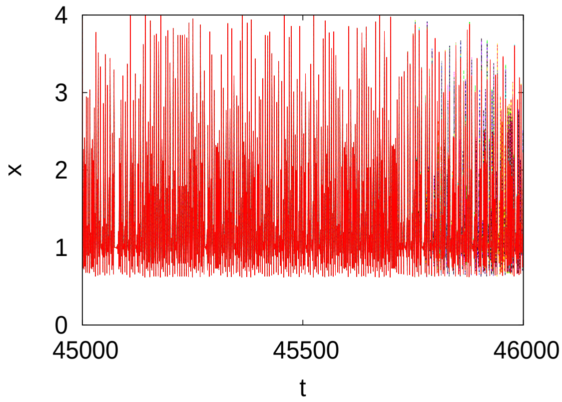

Fig. 2: Time development of $x_t(i)(i = 1, \cdots, 10)$.

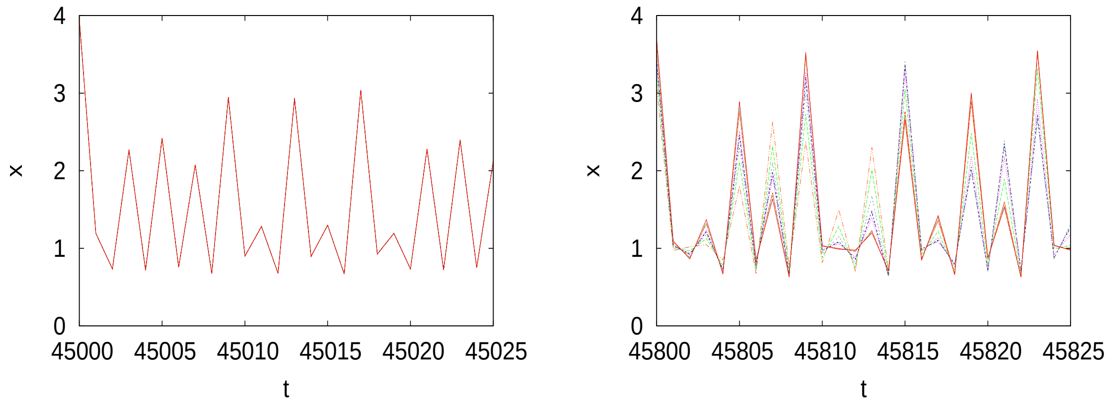

Fig. 3: Enlarged view of Fig. 2: Detailed observation of time development of $x_t(i)$ $(i = 1, \cdots, 10)$ in an orbit (one-cluster regime (left), 10-cluster regime (right)).

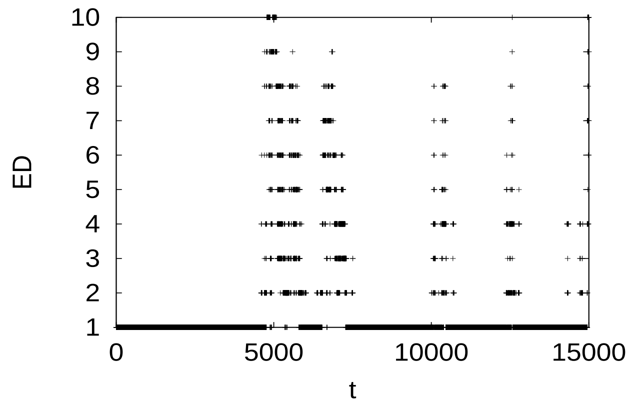

Fig. 4: Time development of effective dimension (ED) of the economy. It primarily remains within a one-cluster regime (ED = 1). Once it departs from the one-cluster regime, it wanders among various regimes of effective dimensions before returning to the former regime.

for duration τ [4] is defined as follows:

$$MED(x, \delta, \tau) = \frac{1}{\tau} \sum_{i=0}^{\tau-1} ED(F^i(x), \delta).$$

Because of entrainment effect, synchronization strengthens and the economy remains within the one-cluster regime (ED = 1) as a laminar state for some time. Then, the economy leaves the regime and wanders among various regimes as a bursting state. This process is repeated for an extended period (Fig. 4).

In Fig. 5, the distribution of duration time in a regime of ED (= 1) is shown to obey a power-law with exponents estimated to be $-3/2$. This implies that extended trapping in such a regime is less infrequent than in cases involving popular exponential distribution [5]. Conversely, the exponential distribution is confirmed for the duration time in a regime of ED $(= 2, 5, 10)$ [6]. MEDs of duration $\tau = 10^5$ are calculated after the transients of 10^5 iterations for various εs, as shown in Fig. 6. An integer MED indicates that the economy remains in the same regime over 10^5 iterations. A non-integer MED indicates that the economy wanders among various regimes of different effective dimensions. Chaotic itinerancy is considered to occur in such cases.

As shown in Fig. 7, we calculate MEDs of the economy by changing a set of parameters (ϕ, ε), and identify the parameter region (ϕ, ε) where MEDs are non-integers, indicating that the economy wanders among various regimes [7]. Figure 7 shows the robustness of chaotic itinerancy with respect to large ϕ and small ε. The larger ϕ is and the smaller ε is, i.e., when the producers emphasize the profit maximization and when they adjust their

[4] The value of τ is fixed as 10^5 in this paper.

[5] Power-law distributions are observed in a well-known critical phenomenon called "Type I intermittency" in a low-dimensional system, but they are not robust in a parameter space. In contrast, the distributions in our model are robustly found.

[6] Exponential distribution is confirmed for each regime of ED $(= 2, \cdots, 10)$.

[7] The MEDs are calculated from 10 randomly chosen initial conditions $x_0(i)$ $(i = 1, \cdots, 10)$ after neglecting transitions of 10^3 iterations. If the average is a non-integer, the corresponding set of parameters (ϕ, ε) is plotted in Fig. 6. These calculations are performed for 10^5 points in a parameter region (ϕ, ε).

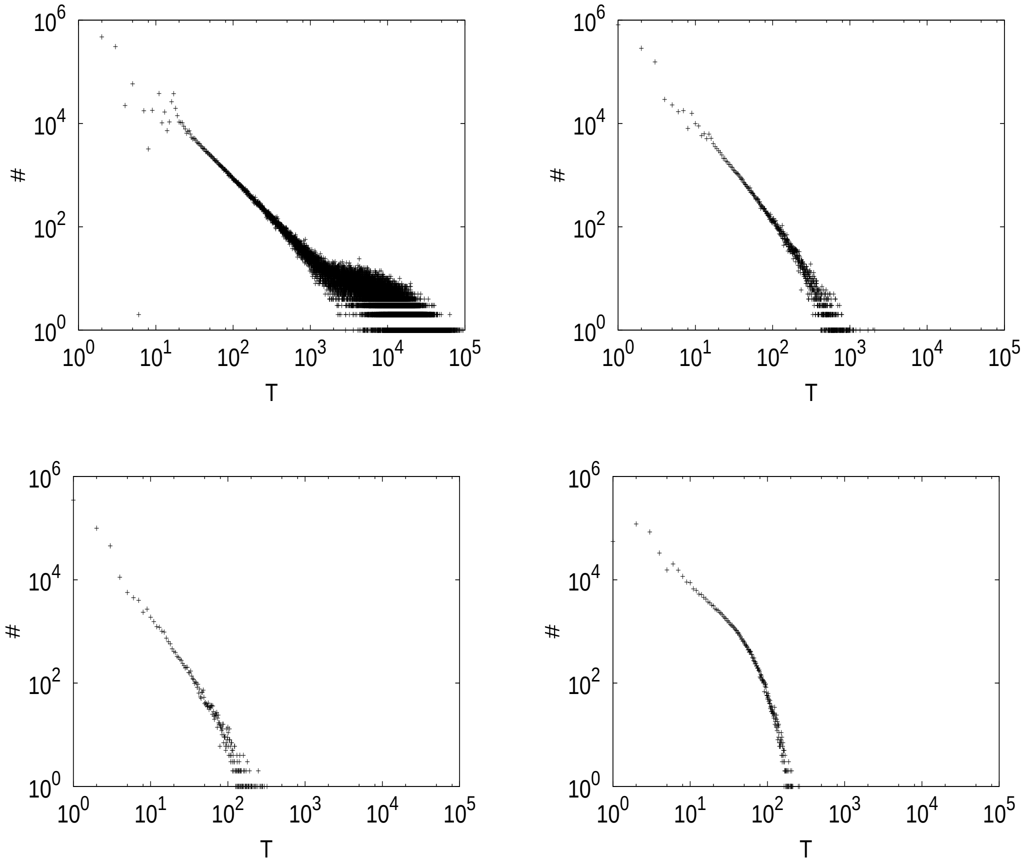

Fig. 5: Distribution of duration time T for remaining in a regime of effective dimension ED (= 1 (upper left), 2 (upper right), 5 (lower left), and 10 (lower right)). The distribution of ED (= 1) is shown to obey a power-law and that of ED ($= 2, 5, 10$) to obey exponential.

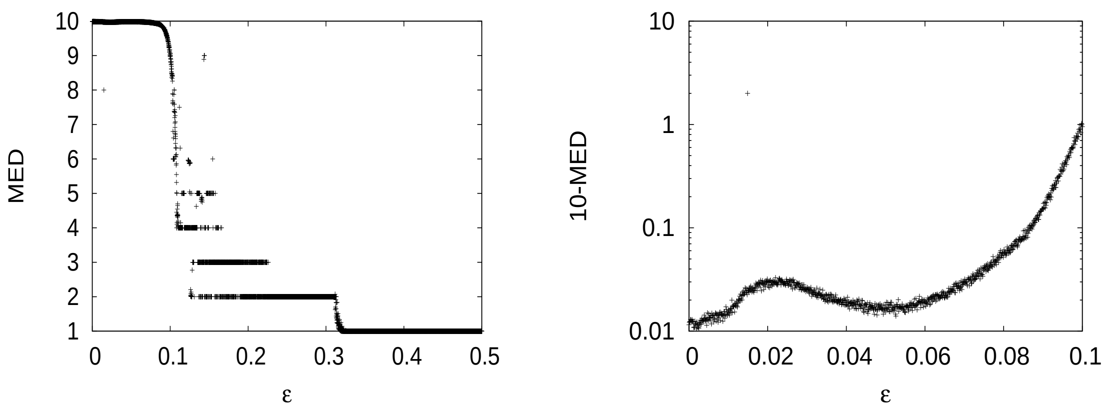

Fig. 6: MEDs of duration $\tau = 10^5$ of the economy with respect to ε. A non-integer MED indicates that the economy wanders among various regimes of different clusters (EDs) (left). The graph of $10-$MED for small values of ε (right: logarithmic scale). The value of $10-$MED for small ε is found not to be 0, implying that the orbit wanders among various regimes.

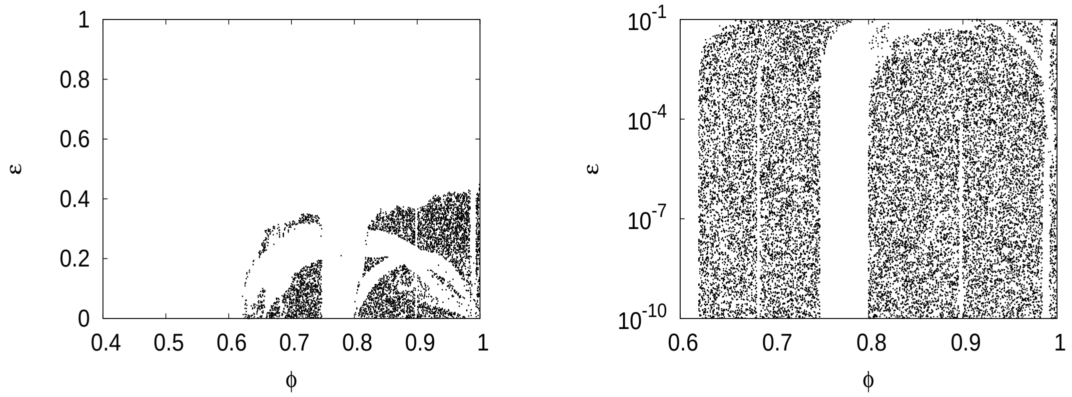

Fig. 7: Parameter regions (ϕ, ε) where the economy wanders among various regimes in $\tau = 10^5$ iterations from 10 initial conditions after the transients of 10^5 iterations (η is still fixed as 3.5.) (left). Enlarged view of the left figure (right). The economy for various parameters (ϕ, ε), including the small ε region, shows chaotic itinerancy.

expectations slowly toward the average announced by the government, the more likely the economy will exhibit chaotic itinerancy. By comparing the bifurcation diagram of a single map (Fig. 1) with Fig. 7, it is apparent that the model shows chaotic itinerancy of complex behaviors with multiple regimes around bifurcation points and chaotic regions of a single map. This phenomena is closely related to the structural stability (Robinson, 2013): The single map (3) is nonhyperbolic or less hyperbolic in these parameter regions, and thus the GCM (1) can easily destroy the respective dynamics of ED (= 10) and show chaotic itinerancy by the extremely small perturbation described by the coupling parameter ε.

4 Conclusion

In this study, we have robustly observed chaotic itinerancy in a model of regional business cycles coupled through producers' expectations derived from global information. The economy wanders among various regimes featuring different numbers of clusters for particular constellations of parameters. Only a small coupling effect is required for this phenomenon to occur. This implies that although all regions or agents are economically homogeneous, the situation should not compel attention to a "representative" region or agent; all must be considered simultaneously. Furthermore, in this study, we have shown that when the producers emphasize the profit maximization and when they adjust their expectations slowly toward the average level announced by the government, the more likely the economy will exhibit chaotic itinerancy.

The dependency of chaotic itinerancy on the system dimension remains an important issue for future work. Especially, the relation between chaotic itinerancy and on-off intermittency, sometimes observed in two-dimensional coupled systems, appears to be interesting for understanding the mechanism of chaotic itinerancy.

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