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Existence of Nash equilibria in sporting contests

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Abstract

This article considers a contest model of an n-team professional sports league. The market areas in which teams are located may differ from one another and each team may have different preferences for winning. In a general asymmetric sporting contest, we demonstrate that under standard assumptions, there exists a unique non-trivial Nash equilibrium in which at least two teams must be active in equilibrium. In addition, we prove that at the non-trivial equilibrium, each team's winning percentage and playing talent are determined by its composite strength—market size and win preference.

JEL Classification: L83, C72, L13

Keywords: Sporting contests; Nash equilibrium; Existence; Win preference

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1 Introduction

The main purpose of this paper is to demonstrate the existence of pure-strategy Nash equilibria in an "n" team sporting contest. Since the seminal papers of Szymanski (2003, 2004) and Szymanski and Késenne (2004), the Nash equilibrium model has been used in the analysis of professional team sports. However, many papers have been restricted to a two-team league model (Chang and Sanders, 2009; Cyrenne, 2009; Dietl *et al.*, 2009). Dietl *et al.* (2008) that is considered a more general n-team league model; however, it is based on the assumption that all teams have identical revenue generating potential and cost functions. Thus the sporting contest is symmetric. These restrictions most probably apply to the Nash equilibrium model in sports because of the difficulty in managing non-identical teams with respect to their market size or drawing potential by conventional means, which treat the Nash equilibrium as a fixed point of the best response mapping. This entails working in a dimension space equal to the number of teams. In the present study, we adopt an alternative approach introduced in Cornes and Hartley (2003, 2005), which allows us to work completely with functions of a single variable, considerably simplifying the analysis. We will prove that there exists a unique non-trivial Nash equilibrium in which at least two teams must be active in equilibrium.

In addition, this study demonstrates that at the non-trivial equilibrium, each team's winning percentage and playing talent are determined by its composite strength, its market size and win preference. The findings' implications are significant for the premise of competitive-balance rules such as revenue sharing and salary caps. It has been recognized that unrestricted competition between teams will lead to a league dominated by a few large-market teams with strong-drawing potential. In the theoretical literature on sports contests, however, this situation is not self-evident. Szymanski and Késenne (2004, p. 169) demonstrated that if there is no revenue sharing in equilibrium, a large-market team will dominate a small one in a two-team league. However, Késenne (2005, p. 103) observed that this result does not necessarily hold in an n-team model. Moreover, Késenne (2007, pp. 54-55) and Dietl *et al.* (2011) demonstrated that if team objectives maximize a combination of profits and wins, as introduced by Rascher (1997), a large-market team will not always dominate a small one in equilibrium, but these studies are restricted to two-team models.[1] The contribution of the present study is in unifying and clarifying the results of these studies by putting them into a more general n-team model.

The rest of the paper is organized as follows. Section 2 explains the basic model and the assumptions. In Section 3, we establish the existence of Nash equilibria in an n-team sporting contest. In this section, we also compare the winning percentage and playing talent of teams of different market sizes and win preferences. Concluding remarks are presented in Section 4.

2 The Model

We consider a professional sports league consisting of $n(\geq 2)$ teams where each team $i(=1, \cdots, n)$ independently chooses a level of talent, $t_i(\geq 0)$, to maximize the objective function. Our analysis of the sports league is formulated as a simultaneous-move game and the solution concept we use throughout the study is that of a pure-strategy Nash equilibrium.

By assuming a competitive labor market and following the sports economics literature, talent can be hired in the players' labor market at a constant marginal cost $c > 0$; hence, the cost function can be written as

$$C_i(t_i) = ct_i. \tag{1}$$

On the revenue side, the season revenue function of a team is defined as

$$R_i = R_i(w_i). \tag{2}$$

R_i is total season revenue of team i, w_i is the winning percentage of the team. It is common in the sports economics literature to assume the following.

Assumption 1. *For all i, the function R_i satisfies $R_i(0) = 0$ and $R_i(w_i) > 0$ for $w_i \in (0, 1]$. Moreover, R_i is twice differentiable and either satisfies $R_i' > 0$ and*

$R_i'' \leq 0$ *for all* $w_i \in (0,1]$, *or there exists a* $\bar{w}_i \in (1/n, 1]$ *such that if* $w_i \geq \bar{w}_i$, *then* $R_i' < 0$; *otherwise,* $R_i' > 0$, *and* $R_i'' < 0$ *elsewhere.*

This assumption reflects the uncertainty of outcome hypothesis (Rottenberg, 1956; Neal, 1964) that consumers in aggregate prefer a close match to one that is unbalanced in favor of one of the teams.

The win percentage is characterized by the contest success function (CSF). The most widely used functional form in sporting contests is the logit that can be written as

$$w_i(t_1, \cdots, t_n) = \begin{cases} \frac{n}{2}\frac{t_i}{t_i+T_{-i}} & \text{if } t_i > 0 \text{ and } T_{-i} > 0, \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

where $T_{-i} = \sum_{j \neq i}^{n} t_j$.[2] The factor $n/2$ results from the fact that winning percentages must average to $1/2$ within a league during any one year; that is, $\frac{1}{n}\sum_{i=1}^{n} w_i = 1/2$. Notice that for the two-team models, the logit CSF (3) does not place a restraint on the teams' choices. However, for the n-team models this is not the case with the logit CSF (3). More precisely, the winning percentage can be larger than one if a team holds more than $2/n$ per cent of total league talent (with normalization of $\sum_{j=1}^{n} t_j$ to one). To avoid this, we can define the winning percentage as

$$w_i(t_1, \cdots, t_n) = min\Big\{\frac{n}{2}\frac{t_i}{t_i + T_{-i}}, 1\Big\}. \tag{4}$$

Consequently, the profit of team i is described by

$$\pi_i(t_1, \cdots, t_n) = R_i(w_i) - ct_i. \tag{5}$$

As in Rascher (1997), Késenne (2007, p.5), and Dietl *et al.* (2011), the objective function of team i is given by a linear combination of profits and wins, which can be written as

$$u_i(t_1, \cdots, t_n) = \pi_i + \gamma_i w_i, \tag{6}$$

where $\gamma_i \geq 0$ is the weight parameter that characterizes the weight team i places on winning in the objective function. Thus, the objective function allows teams to be more profit oriented or more win oriented because the weight parameter γ_i can be different for every team. The objective of each team is to maximize u_i with respect to t_i. We refer to this objective function as the payoff function of team i.

Finally, it is occasionally assumed that the total supply of talent is fixed in the analysis of sports leagues. Researchers who have made this assumption have used a non-Nash conjecture to reflect this scarcity in each team's first-order condition (Fort and Quirk, 1995; Vrooman, 1995). In this case and for a two-team league, we have $\frac{dt_2}{dt_1} = -1$. Indeed, although opinion is divided among sports economists on this subject, we use the Nash conjecture in this study (see e.g., Eckard, 2006; Szymanski, 2004, 2006).

3 Existence Analysis

We can now calculate the best response of team i. Assume first that $T_{-i} = 0$ in order that the other teams do not spend any resources on playing talent. Then, if $t_i > 0$, the payoff is negative in light of Assumption 1 and CSF (3). If team i sets $t_i = 0$, the payoff becomes zero. Therefore, this game always has a trivial equilibrium point $t_1^* = t_2^* = \cdots = t_n^* = 0$. Our concern is with non-trivial equilibria (i.e., $\sum_{i=1}^{n} t_i^* > 0$) and thus no further consideration is given to the trivial point.

If $T_{-i} > 0$, it follows from payoff function (6) that we have

$$\frac{\partial}{\partial t_i} u_i(t_1, \cdots, t_n) = \left(R_i'(w_i) + \gamma_i\right) \frac{n}{2} \frac{T_{-i}}{(t_i + T_{-i})^2} - c. \tag{7}$$

As the second-order condition we obtain

$$\frac{\partial^2}{\partial t_i^2} u_i(t_1, \cdots, t_n) = \frac{n}{2} \frac{T_{-i}}{(t_i + T_{-i})^2} \left(R_i''(w_i) - \left(R_i'(w_i) + \gamma_i\right) \frac{2}{t_i + T_{-i}} \right) < 0. \tag{8}$$

Under Assumption 1, the second-order condition (8) is satisfied. Hence, it follows from equation (7) that given $T_{-i} > 0$, team i's best response function $t_i = \phi_i(T_{-i})$ is given by

$$\phi_i(T_{-i}) = \begin{cases} 0 & \text{if } \left(R_i'(0) + \gamma_i\right) \frac{n}{2T_{-i}} - c \leq 0, \\ x_i & \text{otherwise,} \end{cases} \tag{9}$$

where x_i is the unique solution of the strictly monotonic equation

$$\left(R_i'(w_i) + \gamma_i\right)\frac{n}{2}\frac{T_{-i}}{(x_i + T_{-i})^2} - c = 0. \tag{10}$$

Observe that because of Assumption 1, the left-hand side of equation (10) strictly decreases and is continuous in x_i and positive at $x_i = 0$; therefore there is a unique solution. It is well known that a strategy profile $(t_1^*, \cdots, t_n^*)$ is an equilibrium if and only if for all i, t_i^* is the best response with fixed values of T_{-i}^*.

Further, we can rewrite the best responses of the teams in terms of aggregate talent, which we will denote by $T = \sum_{i=1}^n t_i$. From equation (9), we have

$$\Phi_i(T) = \begin{cases} 0 & \text{if } \left(R_i'(0) + \gamma_i\right)\frac{n}{2T} - c \leq 0, \\ \tilde{x}_i & \text{otherwise,} \end{cases} \tag{11}$$

where $\tilde{x}_i$ is the unique solution of the equation

$$\frac{n}{2}\left(R_i'(w_i) + \gamma_i\right)\left(1 - \frac{\tilde{x}_i}{T}\right) - cT = 0. \tag{12}$$

Following Wolfstetter (1999, p. 91), we call $\Phi_i(T)$ the *inclusive reaction function* of team i.[3] Note that in the second case of (11), the left-hand side of equation (12) is positive at $\tilde{x}_i = 0$ and strictly decreasing, because it has a negative derivative given by

$$\frac{\partial}{\partial \tilde{x}_i}\left\{\frac{n}{2}\left(R_i'(w_i) + \gamma_i\right)\left(1 - \frac{\tilde{x}_i}{T}\right) - cT\right\} = \frac{n^2 R_i''(w_i)}{4T}\left(1 - \frac{\tilde{x}_i}{T}\right) - \frac{n}{2T}\left(R_i'(w_i) + \gamma_i\right) < 0,$$

where the sign comes from Assumption 1.

Rather than using the inclusive reaction function directly, we will examine the properties of player i's *share function* $s_i(T) = \frac{\Phi_i(T)}{T}$, proposed by Cornes and Hartley (2003, 2005). It can be readily verified that Nash equilibrium values of T occur where the *aggregate share function* equals unity. That is, $\sum_{i=1}^n s_i(T^*) = 1$. Given T^*, the corresponding equilibrium $(t_1^*, \cdots, t_n^*)$ is found by multiplying T^* by each team's share evaluated at T^*: $t_i^* = T^* s_i(T^*)$. This result enables us to prove the existence of a unique equilibrium by demonstrating that the aggregate share is equal to one at a single value of T. We define team i's share value as $\sigma_i = \frac{t_i}{T}$ and Appendix 1 proves the following lemma.

Lemma 1. *Under Assumption 1, there exists a share function: $s_i(T)$. $s_i(T)$ satisfies*

$$s_i(T) = \begin{cases} 0 & \text{if } T \geq \frac{n}{2c}\left(R_i'(0) + \gamma_i\right), \\ \sigma_i & \text{otherwise,} \end{cases} \tag{13}$$

where σ_i is the unique solution of

$$\frac{n}{2}(1 - \sigma_i)\left(R_i'(\sigma_i) + \gamma_i\right) = cT. \tag{14}$$

Proof. See Appendix 1. □

We may use this lemma to infer the crucial qualitative properties of the share function derived under Assumption 1. The full details are set out in the following lemma.

Lemma 2. *Under Assumption 1, the share function $s_i(T)$ has the following properties:*

1. *$s_i(T)$ is continuous,*

2. $\lim_{T \to 0} s_i(T) = 1$,

3. *$s_i(T)$ is strictly decreasing where positive,*

4. *if $R_i'(0) < \infty$, $s_i(T) > 0$ for $0 < T < \frac{n}{2c}\left(R_i'(0) + \gamma_i\right)$ and $s_i(T) = 0$ if $T \geq \frac{n}{2c}\left(R_i'(0) + \gamma_i\right)$, and*

5. *if $R_i'(0) = \infty$, $s_i(T) > 0$ for all $T > 0$ and $s_i(T) \to 0$ as $T \to \infty$.*

Proof. See Appendix 2. □

For notational simplicity, we now define $\bar{T} = \frac{n}{2c}\left(R_i'(0) + \gamma_i\right)$. It follows from Lemma 2 that if $R_i'(0)$ is finite, the share function decreases continuously from one to zero over the interval $(0, \bar{T})$ beyond which it takes the value zero. Following Cornes and Hartley (2005), we call $\bar{T}$ the dropout value of team i. In light of Lemma 2, it is interesting to note that if $R_i'(0) = \infty$, the dropout value is infinite and team i invests strictly positive amounts in playing talent in any equilibrium.

Recall that a Nash equilibrium T^* corresponds to the solution to $\sum_{i=1}^{n} s_i(T^*) = 1$. It follows from Lemma 2 that the aggregate share function is continuous, exceeds one for sufficiently small T, is less than one for sufficiently large T, and is strictly decreasing when positive. Therefore, the equilibrium value is unique. Finally, recall that a unique T^* implies a unique strategy profile $(t_1^*, \cdots, t_n^*)$, and we have the following result.

Proposition 1. *Under Assumption 1, the sporting contest has a unique non-trivial Nash equilibrium in pure strategies.*

The approach to share function used in this study is also useful for deriving certain properties of the equilibrium win percentage and playing talent of team i. We can establish the following results.

Proposition 2. *Suppose Assumption 1 holds for all teams. Then at the non-trivial Nash equilibrium, we have*

$$w_i^* \ [t_i^*] \ \lesseqqgtr \ w_j^* \ [t_j^*] \quad \textit{if and only if} \quad R_i'(w_i^*) + \gamma_i \ \lesseqqgtr \ R_j'(w_j^*) + \gamma_j.$$

Proof. See Appendix 3. □

It follows from Proposition 2 that in the non-trivial equilibrium, the teams winning percentages are determined by their composite strength—the marginal revenue of the winning percentage (R_i') and the weight parameter (γ_i)—. Following Quirk and Fort (1992, p. 272), we define the marginal revenue of a win for team i as the market size or drawing potential for the team. In line with most of the existing literature, if $R_i' > R_j' (i \neq j)$ for any given win percentage, we will refer to team i as the *large-market* (or *strong-drawing*) team and team j as the *small-market* (or *weak-drawing*) team.[4]

First, we consider a special case in which all teams are pure profit-maximizers. Thus, the following corollary follows from Proposition 2.

Corollary 1. *Suppose all teams are pure profit-maximizers and satisfy Assumption 1. Then, at the non-trivial Nash equilibrium we have*

$$w_i^* \ [t_i^*] \ \lesseqqgtr \ w_j^* \ [t_j^*] \quad \textit{if and only if} \quad R_i'(w_i^*) \ \lesseqqgtr \ R_j'(w_j^*).$$

This corollary implies that if all teams are assumed to be profit-maximizers, the large-market team hires more talent than the small one in the non-trivial equilibrium. Thus, the large-market team will always dominate competition in a league with (pure) profit-maximizing teams. This agrees with the result of Szymanski and Késenne (2004, p.169) for a two-team model. Késenne (2005, p. 103) observed that this result does not necessarily hold in an n-team model. However, it follows from Corollary 1 that Szymanski and Késenne's results still hold in the general n-team setting. Therefore, Corollary 1 significantly extends the result of Szymanski and Késenne.[5]

Second, in view of Proposition 2, it is interesting to note that weak-drawing teams that are more win-oriented can dominate strong-drawing teams that are more profit-oriented, as the following examples demonstrate.

Example 1. Suppose $R_i = m_i w_i$ with $m_i > 0$. The parameter m_i represents the market size of team i. Using Proposition 2, it is easily seen that $w_i^* \lesseqqgtr w_j^*$ iff $m_i + \gamma_i \lesseqqgtr m_j + \gamma_j$. Thus, if $m_i > m_j$, $\gamma_j > \gamma_i$, and $m_i + \gamma_i < m_j + \gamma_j$ $(i \neq j)$, then the small-market team j dominates the large-market team i.

Example 2. Let $R_i = m_i w_i - \frac{b}{2} w_i^2$ with $m_i > 0$ and $b > 0$. The parameter b characterizes the effect of competitive balance on team revenues. Then, in view of Proposition 2, it can be easily demonstrated that in the non-trivial Nash equilibrium $w_i^* \lesseqqgtr w_j^*$ iff $m_i - bw_i^* + \gamma_i \lesseqqgtr m_j - bw_j^* + \gamma_j$; clearly, this is equivalent to $w_i^* \lesseqqgtr w_j^*$ iff $m_i + \gamma_i \lesseqqgtr m_j + \gamma_j$. Therefore, the result is same as given in Example 1 above.

Késenne (2004) called this phenomenon the "good" competitive imbalance because sports will be much more attractive, at least for the neutral spectator, when a small-market team succeeds in beating large-market teams. However, Examples 1 and 2 also suggest that if the win preference of large-market teams is larger than or equal to that of the small-market teams, a good imbalance will not occur in a professional sports league. This follows in general from the observation of Proposition 2.

Corollary 2. *Suppose Assumption 1 holds for all teams. Then, if the win preference* *of the large-market team is larger than or equal to that of the small-market team, the large-market team has a higher winning percentage than the small-market team in the non-trivial Nash equilibrium.*

Proof. See Appendix 4. □

Késenne (2004) called this scenario the "bad" competitive imbalance because a few large-market teams with strong drawing potential dominate the competition year after year. Competitive-balance rules, such as revenue sharing and salary caps, usually attempt to prevent the bad type of imbalance. Although Késenne (2007, pp. 54-55) and Dietl *et al.* (2011) demonstrated Corollary 2 , these studies are restricted to two-team models. Therefore, the results of Késenne and Dietl *et al* can be extended to a more general n-team model by Corollary 2.

4 Conclusions

This study has proven that under general conditions, a unique non-trivial Nash equilibrium exists in a contest model of an n-team sports league in which teams maximize a linear combination of profits and wins. Further, we have demonstrated that if the win preference of the large-market team is larger than (or equal to) the small-market team, then the former will dominate the latter in the non-trivial equilibrium. We also demonstrated that if all teams are pure profit-maximizers, then at the non-trivial equilibrium, the large market team will always dominate competition in a league.

Over the past few years, the Nash equilibrium concept has been used in the analysis of professional team sports. A particularly great deal of attention has been focused on revenue sharing's effects on competitive balance. However, when the number of teams exceeds by two, revenue sharing's effects on the competitive balance are not clearly described. This study applies the share function approach to a general n-team professional sports model, an approach that avoids the dimensionality problem associated with the best response function approach. We believe that the present study may serve as a basis for further research on the effects of the revenue sharing policy.

Appendix 1

Proof of Lemma 1

Using $\sigma_i = t_i/T$, we can rewrite (12) as (14). Recall that a team's winning percentage in (3) is determined by the ratio of its talent to all the talent in the league. Therefore, team i's revenue can be written as a function of σ_i.

Let us denote the left-hand side of (14) by $h_i(\sigma_i)$ and the right-hand side by $z_i(\sigma_i)$. An intersection of these two functions, if any, which is a solution of (14), determines share values. The function $h_i(\sigma_i)$ is strictly decreasing if and only if Assumption 1 holds. It is bounded from above (i.e., $h_i(0) = \frac{n}{2}\left(R_i'(0) + \gamma_i\right) > 0$) and below (i.e., $h_i(1) = 0$). In contrast, the function $z_i(\sigma_i)$ is a constant function whose value remains the same (i.e., $z_i = cT$) regardless of the value of σ_i. Thus, we may conclude that there is a unique share value for any $T > 0$ which is zero if and only if $\frac{n}{2}\left(R_i'(0) + \gamma_i\right) \leq cT$.

This completes the proof.

Appendix 2

Proof of Lemma 2

First, note that the shares are continuous (indeed differentiable where positive) by the implicit function theorem, establishing Part 1. Second, letting $T \to 0$ on both sides of (14) demonstrates that the share must approach one as T approaches zero, giving Part 2. To justify Part 3, we investigate the slope of s_i. The total differential of (14) has the following form:

$$\left(\frac{n}{2}R_i''(1-\sigma_i) - \frac{n}{2}(R_i' + \gamma_i)\right)d\sigma_i = cdT.$$

We can then express the slope of s_i as follows:

$$s_i'(T) = \frac{c}{\frac{n}{2}R_i''(1-\sigma_i) - \frac{n}{2}(R_i' + \gamma_i)} < 0.$$

The inequality follows because the denominator is negative by Assumption 1. We can deduce that the positive shares are strictly decreasing in T, establishing Part 3. The fourth part is an immediate consequence of Lemma 1. Finally, suppose that the marginal revenue $R_i'(0)$ is unbounded. We rewrite the equation (14) as

$$\frac{n(1-\sigma_i)}{2c} = \frac{T}{R_i'(\sigma_i) + \gamma_i}.$$

Then, an increase in T implies an increase in the right-hand side of the above equation. However, the left-hand side of it is bounded above (i.e., $\frac{n}{2c}$). Hence, as $T \to \infty$ for (14) to be satisfied, we must have $\sigma_i \to 0$.

This completes the proof.

Appendix 3

Proof of Proposition 2

Take two teams i and $j (i \neq j)$. If $\sigma_i^* = 0$, then $\sigma_i^* \leq \sigma_j^*$ necessarily holds in equilibrium. If $\sigma_i^* = 1$, then $\sigma_i^* > \sigma_j^*$ necessarily holds. Now consider the case where $0 < \sigma_i^*, \sigma_j^* < 1$. Then, in view of (14), the first-order conditions for teams i and j are

$$\begin{aligned} \frac{n}{2}(1-\sigma_i^*)(R_i' + \gamma_i) &= cT^*, \text{and} \\ \frac{n}{2}(1-\sigma_j^*)(R_j' + \gamma_j) &= cT^*, \end{aligned}$$

respectively. Dividing the first equation by the second and rearranging the terms, we get

$$\frac{1-\sigma_i^*}{1-\sigma_j^*} = \frac{R_j' + \gamma_j}{R_i' + \gamma_i}. \tag{A1}$$

From (A1), we can assert that $\sigma_i^* \lesseqgtr \sigma_j^*$ if and only if

$$R_i' + \gamma_i \lesseqgtr R_j' + \gamma_j.$$

The proof is completed by observing that $w_i^* = \frac{n}{2}\sigma_i^*$ in context to (3).

Appendix 4

Proof of Corollary 2.

Suppose that if the win preference of a large-market team i is larger than or equal to that of a small-market team j, then $w_i^* \leq w_j^*$ in the non-trivial equilibrium. Then, it must be true that $R_i'(w_i^*) + \gamma_i \leq R_j'(w_j^*) + \gamma_j$ in light of Proposition 2. However, if $w_i^* \leq w_j^*$, we know that $R_i'(w_i^*) + \gamma_i$ is greater than $R_j'(w_j^*) + \gamma_j$, because the marginal revenue curve for the large-market team, team i, lies above the marginal revenue curve for the small-market team, team j, for any given win percentage. Then $w_i^* > w_j^*$ by Proposition 2. This is a contradiction, since we assumed the winning percentage of team j is larger than or equal to that of team i in the non-trivial equilibrium. Therefore, if the win preference of a large-market team i is larger than or equal to that of a small-market team j, then $w_i^* > w_j^*$ in the non-trivial equilibrium.

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Notes

[1]Note that Sloane (1971) was the first to suggest that the owner of a sports team actually maximizes utility, which may include inter alia playing success and profits.

[2]The logit CSF was explicitly adopted in the seminal work of El-Hodiri and Quirk (1971). Groot (2008, pp. 97-100) has expressed the season winning percentage as follows: $w_i = \frac{t_i}{n-1}(\sum_{j \neq i}^{n} \frac{1}{t_i + t_j})$. Although this equation gives the correct relationship between winning percentage and team quality, it considerably complicates the derivative of the marginal product of talent. We therefore choose the simple approximation of the winning percentage (3).

[3]Szidarovszky and Yakowitz (1977) have adapted this function to prove that there exists a unique equilibrium in the Cournot oligopoly game.

[4]Burger and Walters (2003) and Krautmann (2009) empirically found that the marginal revenue of the win of a large-market team is larger than that of a small one in Major League Baseball.

[5]Corollary 1 will complement Késenne (2005)'s analysis of revenue sharing.

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