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Analysis of derivation of the transportation and production cost minimum place

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1.Introduction

The spatial range of an economic activity has spread from the late 20th century, and the influence of the spread has expanded all over the world and has entered people life deeply. This economic expansion is caused by not only a drastic decrease in transportation costs and but also by the decline in office processing costs that is generated by various kinds of innovations. Indeed, technological innovation in transportation reduced shipping costs, decreased its economic influence, and increased the effects of other economic factors relatively. It should be noted, however, that the decline in transportation costs does not mean that the essential function of transportation costs is eliminated in economic world. While the manner of impact of transportation costs on economic activity is different from before globalization economy. The expenses for the movement of goods and services play a decisive role in not only location of economic activities but also behavior style of economic agents even as economic activity becomes globalized.　Therefore, when individual companies strategically plan the location of a factory, naturally they first investigate the level of transportation cost, labour cost and infrastructure in the location candidates.

When an individual firm decides the location of a factory, the firm selects a place that maximizes its profit. If there is little relation between revenue and factory location, the place where production cost is minimized is selected by the firm. Furthermore, when the production cost of goods has little influence on the factory location, the location of the factory is determined at the place where transportation costs are minimized. First, this chapter takes up the transportation cost which has been played an important role in determining the location of the factory. The two methods of derivation of the place of minimum transportation cost of a factory are explained and the place is derived by the methods. Secondly, this paper offers the concept of the *area* of minimum production costs; the analysis is enlarged by incorporating the factory's production function to search the area of minimum production costs including transportation cost, and the analysis explains the meaning the area has in determination of factory's location. Eventually, there are two purposes in the paper: One is to find out the place of the minimum transportation cost of a factory. Another is to determine the area of minimum production cost of a factory

This paper is organized as follow. Numerous methods for deriving the place that minimizes factory's transportation cost have been made so far. The scholars who systematically considered this method for the first time are Launhardt (1882) and Weber (1909). they devised a geometric method to obtain the place of minimum transportation cost for a factory. Thus, the section 2 explains their traditional geometric method. The section 3 clarifies this geometric method by analytical approach: The validity of the geometric method developed by Launhardt and Weber is clearly verified by the analytical approach. The section 4 explains the concept of the *area* of minimum production costs including transportation cost. The section 5 summarizes the results derived in the paper.

2. Derivation of minimum transportation cost place by geometric method

According to the framework of Launhardt (1882) and Weber (1909), this section explains the method of deriving the place that minimizes factory's transportation cost by using the Weight triangle.

2.1. Assumptions

The method of deriving the minimum transportation cost by Weight triangle is explained based on the following assumptions.

1) A factory uses the two kinds of raw materials M_1 and M_2 to manufacture the final product M_3.
2) The raw materials M_1 and M_2 are produced at the points of A_1 and A_2, respectively, and are transported to the factory. The final product a_3 is transported from the factory to the market place A_3.
3) The weights of raw materials M_1 and M_2 are fixed and are given by m_1 and m_2, respectively. The weight of the product M_3 is fixed as m_3.
4) The freight rates of the raw materials a_1 and a_2 and the product a_3 are constant and they are all set to 1. Therefore, the transportation cost borne by the factory is proportional to the weight of these raw materials and products and their transport distances.
5) The factory should be located at the place that minimizes the transportation cost borne by the factory.

2.2. Explanation of method of weight triangle

Now, suppose that the sites of the raw material sites M_1 and M_2 and the market place M_3 are specified by Figure 1. A figure which is formed by connecting the three sites of the market place and the raw materials is named as location figure. The figure shown in Figure 1 is also called a location triangle. In Figure 1 r_1, r_2, and r_3 note the apex angles at A_1, A_2 and A_3. In such location triangle, the derivation of point minimizing factory shipping costs is one of the important tasks in traditional location theory. The traditional theory uses the method of the weight triangle to find a point where factory's transportation cost is minimized. This method is explained as follows. As explained in above assumptions, the two raw materials M_1 and M_2 used by the factory are produced at points A_1 andA_2, respectively, and their weights are m_1 and m_2. And then, the product M_3 is sold at the point A_3 and its weight is m_3. According to the weights m_1, m_2, m_3, let us give the force attracting the factory, the traction force, to the points A_1, A_2, A_3 shown in Figure1.

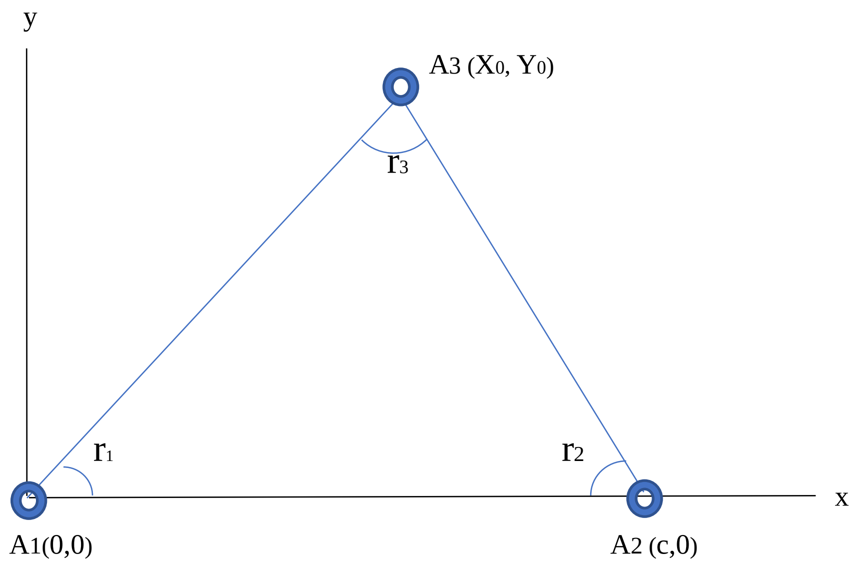

Figure 1. Location figure

The site that the factory's transportation cost is minimized is determined as point *P* where these three traction forces are balanced. The situation of point P is explained by using Figure 2: Three vectors, V_1, V_2, and V_3 are assumed so as to be in proportion to weight m_1, m_2, m_3, which imply the traction forces of three points. Then, each vector is set from the point P toward each of points A_1, A_2, A_3 as shown in Figure 2.

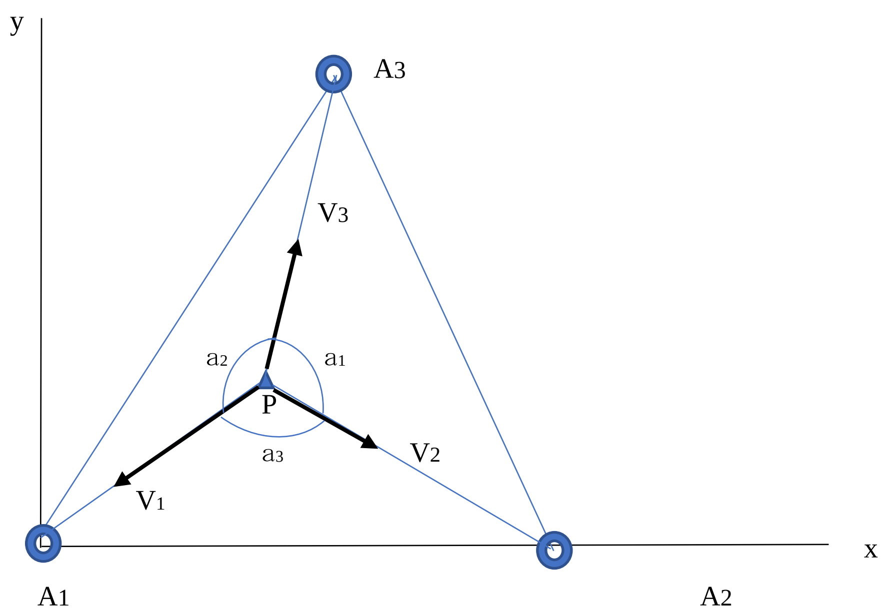

Figure 2 Balance of traction powers

The three vectors V_1, V_2, and V_3 are balanced at point P that the factory's transportation cost is minimized. The fact that these vectors are balanced at the point P means the following state: A parallelogram having two sides of vectors V_1 and V_2 shown in Figure 2 is created with the point P as a starting point. The length from point P to its diagonal is given by the sum of the above two vectors, V_1 and V_2. This new vector V_4 is the same in length as that of vector V_3 and its direction is opposite. That is, the sum of this new vector V_4 and vector V_3 is zero. In Figure 2, the angle formed by the vectors V_1 and V_2 around the point P is noted as a_3, the angle formed by the Vectors V_2 and V_3 is noted as a_1, and the angle made by the Vectors V_1 and V_3 is noted as a_2, respectively.

Now, from the arrangement of the three vectors under the location triangle as shown in Figure 2, it is possible to form a so-called weight triangle by the three vectors. Figure 3 shows this weight triangle.

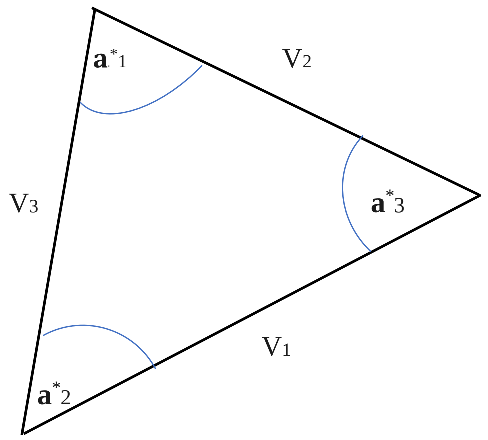

Figure 3 Weight triangle

The length of the three Vectors of this weight triangle follows the ratio of weight m_1, m_2, m_3. From the weight triangle the diagonal of each side V_1, V_2, V_3 is derived as a^*_1, a^*_2, a^*_3. Each angle is supplementary angle for each of angles, a_1, a_2, a_3 formed around point P in Figure 2. Hence, if the angles at vertexes of the weight triangle formed by the three vectors V_1, V_2, V_3 are known as shown in Figure3, the angles a_1, a_2, a_3 around point P in Figure 2 can be obtained. Eventually it is concluded that point P minimizing the transport cost of the factory is a place at which the straight lines connecting each of points A_1, A_2, A_3 and point P forms three angles a_1, a_2, a_3, which are derived from the supplementary angles a^*_1, a^*_2, a^*_3 in the weight triangle.

2.3. Derivation of minimum transportation cost place by location circles

Let us geometrically derive the place of minimum transportation cost based on the location triangle and the weight triangle revealed in the above subsection. When drawing a circle passing through points A_1 and A_2 in Figure 2 and having a central angle twice the angle a_3 which is the circumferential angle formed on the circle, this circle passes through point P. This circle is appropriate to be named as location circle. In a similar way, when drawing a circle passing through points A1 and A3 and having a central angle twice as large as the angle a2 which is the circumferential angle, this circle passes through point P. This circle also is the location circle. Therefore, point where such two location circles intersect indicates the place that minimizes the factory's transportation cost.

Now, suppose that the coordinates of points A_1 and A_2 are specifically (0, 0), (5, 0), and let the coordinates (X_0, Y_0) of point A3 be (2.5, 5): And the weights m_1 and m_2 of the raw materials M_1 and M_2 handled by points A1 and A2 are assumed to be 1.5 tons and 1 ton, respectively: The weight m_3 of the product M_3 is 1 ton. In this situation, by using the location triangle and the weight triangle, two location circles, C_1 and C_2, are drawn in Figure 4. The two location circles intersect at point P which is shown by the mark, ▲. Point P indicates the place that factory's transportation cost is minimized.

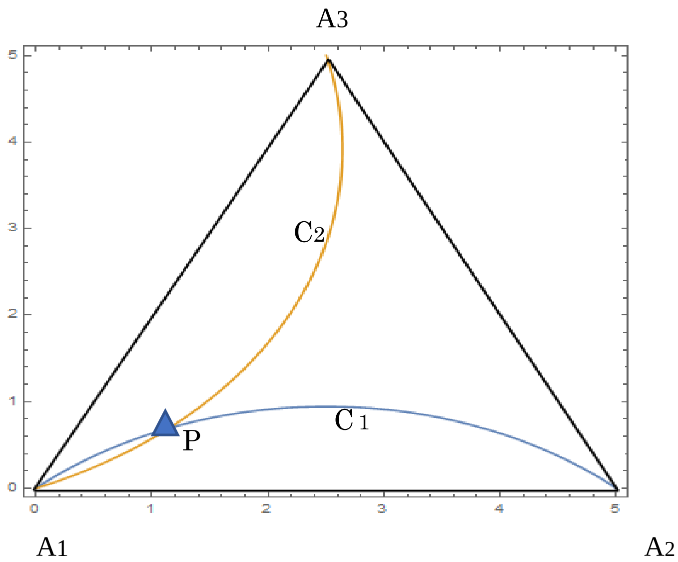

Figure 4 Two location circles and the place of minimum transportation cost

3. Analytical explanation of the weight triangle

3.1. Derivation of minimum transportation cost place by analytical geometrics

This section analyzes the method which depicts two location circles to find out the place of minimum transportation cost. It is assumed based on the assumptions in the previous section that the location figure is given by the triangle like Figure 4, and the weight assigned to each point A1, A2, and A3 is given as m_1, m_2, m_3. Factory's transportation cost T is shown by equation (1). Thus, the task in this section is the analytical geometric derivation of the point P that minimizes this transportation cost T.

$$T= m_1(x^2+y^2)^{0.5} \quad + \quad m_2((x - c)^2+y^2)^{0.5} \quad + \quad m_3((x -X_0)^2+(y-Y_0)^2)^{0.5} . \qquad (1)$$

Since each weight m_1, m_2, m_3 is given, transportation cost T is a function of the factory's coordination, x and y. The place P that minimizes transportation cost must the following two necessary conditions,

$$\partial T/\partial x = 0, \qquad (2)$$

$$\partial T/\partial y = 0. \qquad (3)$$

And point P must satisfy the sufficient condition, Z, as indicated by the following equation (4),

$$Z = \begin{vmatrix} \partial^2 T/\partial x^2 & \partial^2 T/\partial x\, \partial y \\ \partial^2 T/\partial x\, \partial y & \partial^2 T/\partial y^2 \end{vmatrix} \rangle \quad 0. \qquad (4)$$

These equations (2), (3), and (4) are developed and shown as following equations (2a), (3a), and (4a),

$$\partial T/\partial x = m_1 x / (x^2+y^2)^{0.5} + m_2(x - c) / ((x - c)^2+y^2)^{0.5} + m_3(x - X_0) / ((x - X_0)^2+(y-Y_0)^2)^{0.5} = 0 \qquad (2a)$$

$$\partial T/\partial y = m_1 y / (x^2+y^2)^{0.5} + m_2 y / ((x - c)^2+y^2)^{0.5} + m_3(x - Y_0) / ((x - X_0)^2+(y-Y_0)^2)^{0.5} \qquad = 0 \qquad (3a)$$

$$\begin{aligned} Z = {} & m_1^2\, m_2^2(xy - (x - c)y)^2 / ((x^2+y^2)^{1.5}((x - c)^2+y^2)^{1.5}) \\ & + m_1^2\, m_3^2(x(y-Y_0) - (x - X_0)y)^2 / ((x^2+y^2)^{1.5}((x - X_0)^2+(y-Y_0)^2)^{1.5}) \\ & + m_2^2\, m_3^2((x-c)(y-Y_0) - (x - X_0)y)^2 / (((x-c)^2+y^2)^{1.5}((x - X_0)^2+(y-Y_0)^2)^{1.5}) \quad \rangle\ 0 \end{aligned} \qquad (4a)$$

Since equation (4a) is positive, solving the simultaneous equations of equations (2a) and (3a) for x and y gives the coordinates of point that minimizes transportation cost of the factory[1]. This section obtains point that minimizes transportation cost by developing equation (2a) and (3a) as follows. First, by transferring the third term of equations (2a) and (3a) to the right side, the following equations (5) and (6) are derived,.

$$m_1 x / (x^2+y^2)^{0.5} + m_2(x - c) / ((x - c)^2+y^2)^{0.5} = - m_3(x - X_0) / ((x - X_0)^2+(y-Y_0)^2)^{0.5} \qquad (5)$$

$$m_1 y / (x^2+y^2)^{0.5} + m_2 y / ((x - c)^2+y^2)^{0.5} = - m_3(x - Y_0) / ((x - X_0)^2+(y-Y_0)^2)^{0.5}. \qquad (6)$$

By squaring both sides of equations (5) and (6), and adding these two equations, and further

[1] Point of minimum transportation cost can be directly derived by numerical calculation method. For example, the method is introduced by Kuhn and Kunne (1962).

transforming, equation (7) is derived,

$$2m_1\ m_2(x(x - c)+y^2) / ((x^2+y^2)^{0.5}((x - c)^2+y^2)^{0.5}) = m_3^2 - m_1^2 - m_2^2\ . \qquad (7)$$

By squaring equation (7), and then transforming it, equation (8) is obtained,

$$4\ (m_1\ m_{2)}{}^2(x(x - c)^2+y^2)^2 = (x^2+y^2)(\ (x - c)^2+y^2)\ (m_3^2 - m_1^2 - m_2^2\ )^2\ . \qquad (8)$$

Equation (8) can be transformed to equation (8a),

$$4\ (m_1\ m_{2)}{}^2 / ((m_3^2 - m_1^2 - m_2^2\ )^2)\ (x(x - c)^2+y^2)^2 = (x^2+y^2)(\ (x - c)^2+y^2)\ . \qquad (8a)$$

Considering the following relationship shown by equation (9), equation (8a) can be transformed as equation (10),

$$((x^2+y^2)(x - c)^2+y^2)\text{-}\ (x(x - c)^2+y^2)^2 = (x\text{-}(x - c)^2)^2y^2 = c^2y^2, \qquad (9)$$

$$(4\ (m_1\ m_{2)}{}^2 / (m_3^2 - m_1^2 - m_2^2\ )^2 – 1)\ (x\ (x - c)^2+y^2)^2 = c^2y^2. \qquad (10)$$

Equation (10) can be transformed to equation (11),

$$x(x - c)^2+y^2 = \mp\ c/(K^2\text{-}1)^{0.5}y, \qquad (11)$$

where $K= 2m_1\ m_2 / (m_3^2 - m_1^2 - m_2^2\ )$. And from equations (2a) and (3a), it is known that x and y are positive. And the sign of $(x\ (x - c)+y^2)$ of the left side of equation (11) depends on $(m_3^2 - m_1^2 - m_2^2\ )$ in the right side of equation (7). Thus, sign "$\mp$" is added in the right hand side of equation (11). Then, equation (11) is transformed to equation (12),

$$(x – c/2)^2 + (\ y \mp\ c/(2(K^2\text{-}1)^{0.5}))^2 = (cK/((2(K^2\text{-}1)^{0.5}))^2\ . \qquad (12)$$

Equation (12) indicates that the place of minimum transportation cost is located on the circle, which is called the location circle and is described as follows[2]: This location circle has its center on the perpendicular bisector of the line connecting the points A_1 and A_2 and passes through points A_1 and A_2.

[2] As can be seen from the structure of equation (10), equation (12) is not defined in the following two cases: (1) $4\ (m_1\ m_{2)}{}^2 = (m_3^2 - m_1^2 - m_2^2\ )^2$, (2) $(m_3^2 - m_1^2 - m_2^2\ )=0$.

By using the same method, another location circle is derived which has its center on the perpendicular bisector of the line connecting points A_1 and A_3 and it passes through the points A_1 and A_3. It can be said, therefore, that the place of minimum transportation cost is indicated by the intersection of these two location circles; the place is derived by solving simultaneous equations representing the two location circles for x and y.

Let us find the place of minimum transportation cost by using the above analytical geometry with reference to Figure 4. It is assumed as follows; the coordinates of point A_1 are (0, 0), point A_2 are (5, 0), and point A_3 are (2.5, 5). The weight of goods handled by each point is m_1= 1.5 tons, $m_2 = m_3$ = 1 ton. From equation (12), the place of minimum transportation cost in Figure 4 is on the location circle, C_1, which passes point A_1 and A_2. This location circle C_1 is indicated by the next equation,

$$(x-2.5)^2 + (y+2.8347)^2 = (3.7796)^2 \qquad (13)$$

The equation of the location circle C_2 in Figure 4 which passes point A_1 and A_3 is derived as follows: First, the distance between the points A_1 to A_3 in Figure 4 is 5.5902. Thus, as shown in Figure 5, in another coordinate system X - Y, A^*_1 is placed at (0, 0) and A^*_3 is placed at (-5.5902, 0). By using the equation (12) to describe a location circle C^*_2 in this coordinate system X - Y.

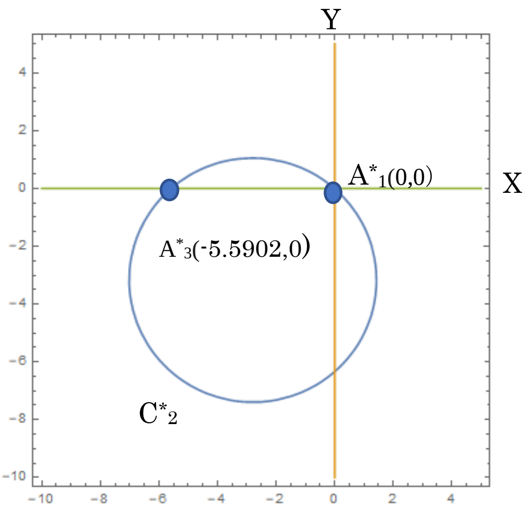

Figure 5. Location circle in X-Y coordination

Second, since the angle at point A is 63.43°, this coordinate is rotated by -116.57°. As a result, the relationship between the coordinate axes X and Y and the coordinate axes x and y is expressed by the following equation,

$$X = x \cdot \cos(-116.57^\circ) + y \cdot \sin(-116.57^\circ), \qquad (14)$$

$$Y = x \cdot \sin(-116.57^{\circ}) + y \cdot \cos(-116.57^{\circ}). \tag{15}$$

Therefore, the equation of the location circle C_2 in Figure 4 is given as equation (16),

$$(x+1.5843)^2 + (y-3.9175)^2 = (4.2258)^2. \tag{16}$$

Point P that minimizes the transportation cost is obtained by solving the simultaneous equations of equations (13) and (16) representing the location circle for x and y. The coordinates of point P are derived as X = 1.1499, y = 0.695, and are displayed at point P at which the two location circles intersect in Figure 4.

3.2 Derivation of minimum transportation cost place at a vertex

There are cases that point where transportation is minimized is a vertex of the location figure. If location figure is triangle, under what conditions it is known for some time that such a case arises[3].

As shown in Figure 1, vertex angles at points A_1, A_2, A_3 are indicated by r_1, r_2, and r_3. And the weights of the raw materials and products in charge of each point are noted as m_1, m_2, and m_3.
In this situation, the condition for point A_1 to minimize the transportation cost is expressed by the equation (17),

$$\cos(r_1) \leq (m_1^2 - m_2^2 - m_3^2) / (2\, m_2\, m_3). \tag{17}$$

The conditions for point A_2 and point A_3 to be the places to minimize the transportation cost are given by the equation (18) and (19),

$$\cos(r_2) \leq (m_2^2 - m_1^2 - m_3^2) / (2\, m_1\, m_3), \tag{18}$$

$$\cos(r_3) \leq (m_3^2 - m_1^2 - m_2^2) / (2\, m_1\, m_2). \tag{19}$$

Furthermore, when the weight of the goods in charge of a certain point, for instance point A_1, is the same as or larger than the total of the weights in charge of other points, that is, when equation (20) holds, that point A_1 minimizes the transportation cost[4],

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[3] Nishioka (1976) explains the condition under which minimum transportation cost place is settled at a vertex.
[4] This fact is explained by Weber (1909,S.235).

$$m_1 \geq \sum_{i=2}^{3} m_i \quad . \tag{20}$$

4. Derivation of area of minimum production costs

4.1. Incorporation of the production function into the analysis

In the derivation of the above-mentioned transportation cost minimization point, the weight of the raw materials and products to be used is given. Such assumption is of course effective in theoretical analysis in a rather short period of time. Under a rather long period of time, however, the theoretical effectiveness of this assumption declines. The prices of raw materials fluctuate in a somewhat long period, and the amounts of raw materials used change accordingly. As a result, the ratio of the weights of the raw materials used change and the transportation cost minimizing point moves. The change of prices of the raw materials vary the combination of the weights of the raw materials used at the factory. The problem of how the combination of raw materials is changed with respect to price changes of raw materials is explained by incorporating the factory's production function into the analysis.

By introducing the production function into the analysis, the object of the analysis is led to the derivation of the point which minimizes the production cost including the transportation cost. Therefore, the assumption of analysis is expanded and the place where the production cost of the factory is minimized is derived.

The assumptions used in the previous section are expanded. (1) Prices of material M_1 and M_2 are given as p_1, p_2. (2) Distances between the factory and each point A_1, A_2, A_3 are denoted by d_1, d_2, and d_3. (3) The freight rates of each material M_1, M_2 and the final goods M_3 are given as t_{m1}, t_{m2} t_{m3}, respectively. (4) Production function of the factory is shown by equation (21),

$$Q = Am_1^{\alpha} m_2^{\beta} \quad . \tag{21}$$

where Q is production amount of the final goods M_3, *A* denotes parameter to indicate production efficiency of the factory, parameter α and β are assumed 0.4 for simplicity of calculation.

Making use of the law of equi-marginal productivity, that is, the ratio between the productivities of the two raw materials should be equal to the ratio between their delivered prices, quantities of them are derived as equations (22) and (23),

$$m_1 = Q^{1.25}A^{-1.25}(p_2+t_{m2}d_2)^{0.5}/(p_1+t_{m1}d_1)^{0.5}\,, \tag{22}$$

$$m_2 = Q^{1.25}A^{-1.25}(p_1+t_{m1}d_1)^{0.5}/(p_2+t_{m2}d_2)^{0.5} \tag{23}$$

The factory's production cost C_p is given by equation (24),

$$C_p = 2Q^{1.25}A^{-1.25}(p_1+t_{m1}d_1)^{0.5}(p_2+t_{m2}d_2)^{0.5}+Qt_{m3}d_3. \quad (24)$$

When the cost function is given as in equation (24), the production cost becomes a function of the factory's location. The factory's production cost is varied as a function of x and y as shown in Figure 6. It is assumed in Figure 6 that points A_1, A_2, A_3 in Figure 4 are specified as A_1= (0.0), A_2= (5.0), A_3= (2.5.5), the production amount of the final goods M_3,Q=50, production efficiency; A=1, the prices of the raw materials are given as p_1=0.25, p_2=0.75. the freight rates are given as t_{m1}=0.11**,** t_{m2}=0.15, t_{m3}=0.57. As shown in Figure 6, the production cost of the factory is minimized at point (2.65, 3.49) and rises as it goes away from that point.

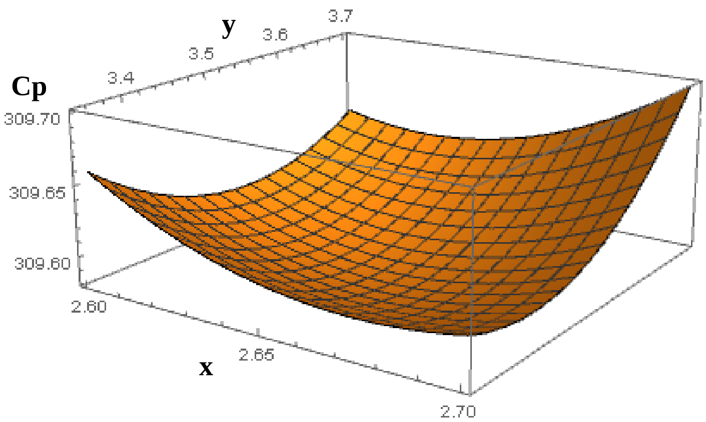

Figure 6. The relationship between the factory's location and production cost

Then, as the production cost becomes a function of the factory's location, by solving the simultaneous equations of (25) and (26) for x and y, the place at which production cost is minimized can be derived. In deviation of equation (25) and (26), A_1= (0.0), A_2= (5.0), A_3= (2.5.5) are assumed.

$$\begin{aligned}\partial C_p/\partial x =& Qt_{m3}(x-2.5)/(((x-2.5)^2+(y-5)^2)^{0.5})\\ &+Q^{1.25}t_{m1}x(p_2+t_{m2}((x-5)^2+y^2)^{0.5})^{0.5}/(A^{1.25}((x^2+y^2)^{0.5})(p_1+t_{m1}(x^2+y^2)^{0.5})^{0.5})\\ &+ Q^{1.25}t_{m2}(x-5)\ (p_1+t_{m1}(x^2+y^2)^{0.5})^{0.5}/(A^{1.25}((x-5)^2+y^2)^{0.5})(p_2+t_{m2}((x-5)^2+y^2)^{0.5})^{0.5})=0, \quad (25)\end{aligned}$$

$$\begin{aligned}\partial C_p/\partial y =& Qt_{m3}(y-5)/(((x-2.5)^2+(y-5)^2)^{0.5})\\ &+Q^{1.25}t_{m1}y(p_2+t_{m2}((x-5)^2+y^2)^{0.5})^{0.5}/(A^{1.25}((x^2+y^2)^{0.5})(p_1+t_{m1}(x^2+y^2)^{0.5})^{0.5})\\ &+ Q^{1.25}t_{m2}y\ (p_1+t_{m1}(x^2+y^2)^{0.5})^{0.5}/(A^{1.25}((x-5)^2+y^2)^{0.5})(p_2+t_{m2}((x-5)^2+y^2)^{0.5})^{0.5})=0. \quad (26)\end{aligned}$$

If the parameters are given the same value used in the derivation of Figure 6, the place of minimizing production costs is dived from equations of (25) and (26). The solution can be derived as (2.653,3.490).

4.2. The concept of area of minimum production costs

The curved surface shown in Figure 6 shows the relationship between the location of the factory and its production cost. Let's cut this curved surface several times at different height around the minimum value with a plane parallel to the bottom. Each cut is projected onto x-y plane and they are indicated by each closed curve in Figure 7. These curves may be called *equal production cost curves*. Among these equal production cost curves, the inside of the innermost equal cost curve is indicated by dark blue. If the factory is located in an area indicated by dark blue, the production costs of the factory is not higher than 309.7. That is, it can be said that it is not more than 0.1 higher than the minimum production cost. Point (2.65, 3.49) of minimum production cost is indicated by the white mark "*" in Figure 7.

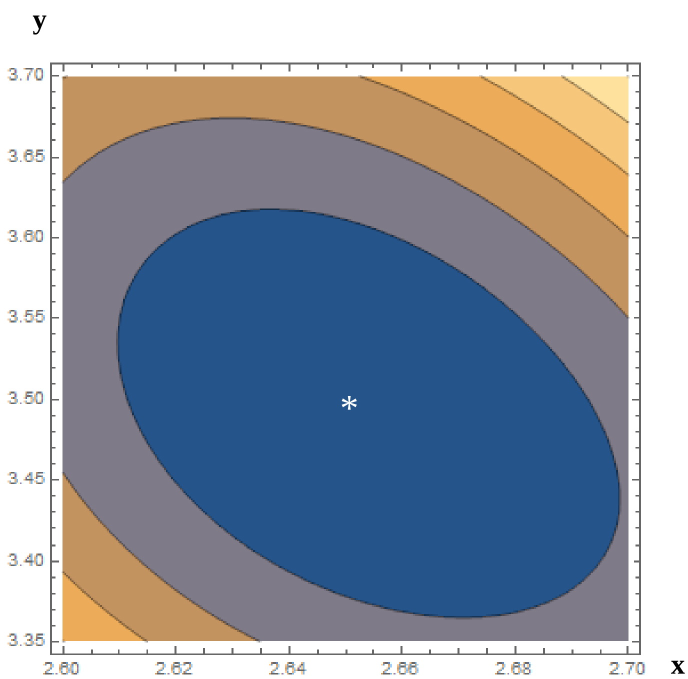

Figure 7. Derivation of area of minimum production costs

Based on the considerations of production costs using Figure4, Figure 6 and Figure7, the following inference is possible. If the amount of each of the raw materials used by the factory has been determined and it is stable over a relatively long period of time, deriving a point that minimizes the transportation expenditure is quite important in determining the location of the factory. In a situation, however, where the prices of raw materials change in relatively long term, even if one place that minimizes shipping costs is found, that place will shift quickly. Hence, it can be said that the importance of finding such a point is relatively low.

Instead, it is significant in the factory's location strategy for the factory to set an amount that allows deviation from the minimum production cost level, and the factory sets the spatial area in which the deviation of the production costs is allowed. And if the factory is in the area, the factory accepts its location from the viewpoint of production cost. This area can be called as *area of minimum production costs*. It could be said, thus, that the location issue of the factory is to set the area of minimum production cost in a geographical space.

As mentioned above, it is predicted that changes in raw material prices will often occur. In response to such raw material price change, it is thought that the idea of *area of minimum production costs* is effective for the firm's location strategy. For instance, if the factory sets 0.1 that can deviate from the minimum production cost level in the situation assumed in the above analysis, the spatial area in which the factory's location is allowed from the viewpoint of the production costs is indicated by the area of dark blue in Figure 7. This area is the area of minimum production costs.

Let us examine the significance of the area of minimum production costs. Suppose that the price p_2 of the material M_2 is changed by 12 percent from the 0.25: if the price is increased to 0.28, other parameters are kept the same level, point of minimum production costs shift from point (2.65,3.49) to point (2.63,3.56) which is shown by the white diamond mark. In this situation, the new area of minimum production costs is set as shown by the inside dark blue area of the innermost equal cost curve in Figure 8. The existing location of the factory, which is shown by the white mark "*", is included in the new area of minimum production costs in Figure 8. Thus, the factory does not need to consider the shifting its location according to the increase of price of the material.

In sum, when the factory allows the concept of area of minimum production costs, it can be said that even if the price of the raw material changes somewhat, the factory does not shift its location point according to the change.

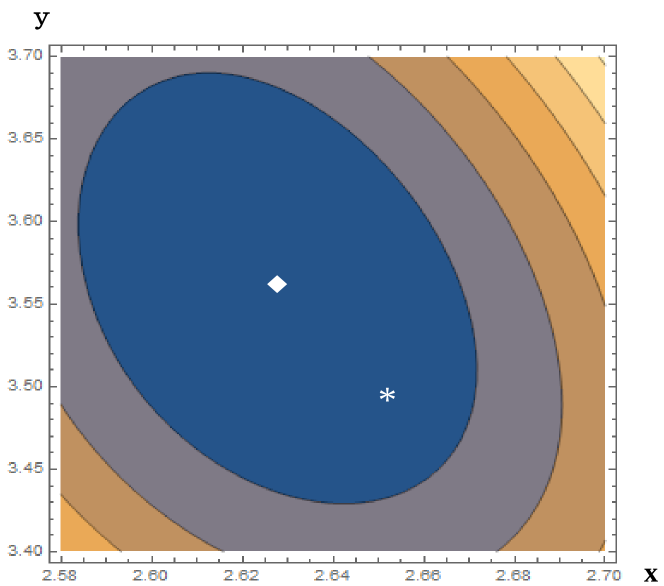

Figure 8　A new area of minimum production costs

In addition, since within area of minimum production costs the production costs of the factory are almost same irrelatively its location, the factory can have a lot of location selections: Within this area the factory incorporates many location factors in the determination of the factory's location such as security, health care, and dwellings of workers and so on. The concept of area of minimum production costs may offer usefulness to the factory's location determination: the factory can flexibly consider the its location and select a better location considering various location factors.

5. Concluding remarks

Factory's shipping costs tend to decline due to intermittent technological progress in transportation systems. In addition, the transportation costs have been greatly reduced by technological innovation in office processing. Due to the decline in transportation costs, many production processes become to be released from places where transportation costs are minimized. The locations of the production processes have become more influenced by labor costs and the agglomeration economies, and the locations of the factories are becoming diversified. Transportation costs have various impacts on the location determination of various economic activities in each region of the world. The economic role played by transportation expenses has been relatively declining, but the way of its work has become diversified as economic activity is geographically broadened.

This paper analyzes the location effect of transportation cost on factory: At the initial stage of economic development, the production activities are relatively simple, and it is assumed that the prices of the raw materials used by the factory are constant for a relatively long time. Under this assumption, first, the paper explains the traditional theory to derive the place that minimizes the transportation cost of the factory, and clarifies its validity by analytical method. Secondly, the globalized economy is assumed, in this stage of economic development production process is fragmented into small blocs, and some of them are spatially scattered from the existing place to various places which provide production condition suitable to the individual production blocs. In this stage, there are many raw materials and intermediate good are used to produce final goods, and it is assumed that the prices of these materials and intermediate goods are not constant, but they change in a relatively short period. Thus, this paper proposes the concept of *area of production cost minimization*. This concept implies as follows: In determining the location of a factory, deriving place that minimizes transportation costs is not very important as the prices of intermediate goods easily changes. Rather it is more important for the factory to allow some deviation from the minimum transportation cost and to determine an acceptable geographical range. By setting area of production cost minimization the factory does not need to consider the shifting its location in response to mild price changes of the materials and intermediate goods. And the factory can flexibly consider the its location within this area and select a better location by incorporating various location factors into the consideration.

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