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Stability of Business Cycles and Criteria for an Optimum Currency Area: A Kaldorian Two-country Model

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Abstract

In this study, we focus on the relationship between the stability of business cycle and the criteria of optimum currency area. We do so by investigating the effect of satisfying the criterion of the degree of an economic openness on the stability of the business cycle, using a Kaldorian two-country model with a monetary union and imperfect capital mobility. We find that an increase in capital mobility is a destabilizing factor, whereas an increase in the degree of openness of an economy and a counter-cyclical fiscal policy are a stabilizing factors. Furthermore, we obtain the result that a high degree of economic openness can adjust a shock in the monetary union regardless of whether the shock is asymmetric. The criterion of degree of economic openness serves as one of the criteria for optimum currency area, even if an asymmetric shock tends to occur by regional concentration of industry due to high degree of openness of the economy.

Keyword: Optimum currency area, Kaldorian two-country model, Stability of business cycle, Euro area

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1 Introduction

The theory of optimum currency area (OCA) considers what needs to be done for successful currency integration. Many economists (e.g., Mundell (1961), McKinnon (1963), and Kenen (1969)) have considered the OCA theory and have suggested the criteria for an OCA.

In particular, McKinnon (1963) argues the criteria should include "a high degree of openness of the economy," in other words, countries that are very open to trade and trade heavily with each other form an OCA (Baldwin and Wyplosz (2015)). Domestic prices of countries with a higher degree of openness are likely to be affected by international price changes, because the cost of the countries is strongly affected by import prices. In addition, domestic prices are sensitive to fluctuations in exchange rates. Therefore, a nominal exchange rate adjustment by currency devaluation is not very useful because it changes domestic prices. Countries with a higher degree of openness have low costs, which makes it impossible to devalue a currency by monetary union.

The criteria for an OCA can be classified into two viewpoints. The first is whether it is possible to prevent the occurrence of shock. The second viewpoint is whether an adjustment is possible after a shock occurs. The criterion of openness of the economy is compatible with the viewpoint of prevention of the occurrence of asymmetric shock.[1)]

European Commission (1990) indicates that a demand shock has similar impacts on countries participating in a monetary union. The reason is that a monetary union leads to structure of trade that trades products of the same category with each other occurs owing to economies of scale. Therefore, a shock tends to be symmetrical as integration progresses. This view on the criterion of openness of the economy is called the European Commission view.

However, there is another view on the relationship between openness of an economy and asymmetric shock, the Krugman view. Krugman (1991) points out that a negative relationship exists between economic integration and symmetry of shock, because economies of scale caused by a high degree of economic openness bring regional concentration of industry, so that industrial-specific shocks tend to become a country-specific shock.

The analysis of asymmetric shock is a central tenet of the OCA theory. Thus, the issue is whether openness of an economy causes an asymmetric shock. However, if the business cycles of countries in a monetary union are stabilized by satisfying the criterion of openness of the economy regardless of whether the shock is asymmetrical when countries depart from equilibrium by the shock, then the monetary union area satisfying the criterion becomes an OCA, even if Krugman's view is correct and an asymmetric shock tends to occur.

Gächter, Riedl and Ritzberger-Grünwald (2012) point out the relationship between business cycles and the OCA theory as well as problems with the theory. According to the authors, a criticism of the OCA theory is that its different criteria could not be integrated within a uniform framework. Moreover, some listed criteria are difficult to measure (Robson (1998)) or compare (e.g., Tavlas (1994)). In the end, the discussion led to the

[1)] See De Grauwe (2016) and Mongelli (2002) for a detailed discussion on openness of an economy.

development of a few "metacriteria" that implicitly subsume some of the individual conditions. In particular, the synchronization of business cycles has become established as a key OCA metacriterion.

Regarding the synchronization of business cycles, De Grauwe and Ji (2016) argues that the best possible way to deal with business cycle movements whose amplitude is unsynchronized is by introducing a budgetary union. The authors point out that the underlying assumption of the OCA prescription for structural reform is that asymmetric shocks are permanent, and that it does not follow that more flexibility is the answer when the shocks are temporary. Nakao (2017) proves this result theoretically and argues that an increase in capital mobility between countries in a capital markets union is a destabilizing factor, whereas an increase in fiscal transfers between such countries is a stabilizing factor.

However, if the business cycles of countries in a monetary union are stabilized by satisfying the criteria for an OCA regardless of whether the business cycles are synchronized when countries depart from equilibrium by the shock, they are synchronized at the equilibrium point finally. Therefore, it is important from the perspective of OCA metacriteria that business cycles are stabilized.

In this study, we investigate whether an increase in the degree of economic openness is a stabilizing factor. We use a Kaldorian two-country model with a monetary union and imperfect capital mobility. Although there are several studies on business cycles using a Kaldorian (or Keynesian dynamic) model (e.g., Asada, Inaba and Misawa (2001), Asada, Chiarella, Flaschel and Franke (2003), and Asada (2004)), little research has been undertaken to consider the OCA theory in Kaldorian terms. An analysis of the OCA theory using a Keynesian dynamic stability concept enables us to theoretically judge whether a monetary union has a shock adjustment function, and has the advantage of facilitating judgment about whether the Euro area can survive.

The rest of this paper is organized as follows. In Section 2, we formalize the Kaldorian model, which consists of a five-dimensional system of nonlinear differential equations. In Section 3, we analyze the uniqueness of the equilibrium solution of the model formulated in Section 2. In Section 4, we investigate the conditions for local stability of the equilibrium point. In Section 5, we present the results of some numerical simulations that support the theoretical analysis in Section 4. Section 6 concludes.

2 Formulation of the Kaldorian Two-Country Model

In this section, we consider the economy of a monetary union using a Kaldorian two-country model with a monetary union and imperfect capital mobility, based on Asada, Inaba and Misawa (2001) and Asada (2004).

To analyze a monetary union, such as the euro area, we represent the exchange rate as follows.

$$E = E^e = \bar{E} = 1, \tag{1}$$

where E is the exchange rate and E^e is the expected exchange rate of the near future. We assume that the exchange rate and expected exchange rate are one, because a single currency is used in a monetary union.

Furthermore, we assume a fixed price economy.

$$p_1 = p_2 = 1. \tag{2}$$

To simplify the analysis, we focus on a fixed price economy in the short run. This assumption eliminates price fluctuations. Therefore, we do not deal with the issues of inflation and deflation.

Under this assumption about the exchange rate and price, our model consists of the following system of dynamic equations in the medium run with flexible capital stock.

(1) Behavioral equations

$$\dot{Y}_i = \alpha_i \left[C_i + I_i + G_i + J_i - Y_i\right] \ ; \ \alpha_i > 0, \tag{3}$$

$$C_i = c_i \left(Y_i - T_i\right) + C_{0i} \ ; \ 0 < c_i < 1, C_{0i} \geq 0, \tag{4}$$

$$T_i = \tau_i Y_i - T_{0i} \ ; \ 0 < \tau_i < 1, T_{0i} \geq 0, \tag{5}$$

$$I_i = I_i \left(Y_i, K_i, r_i\right) \ ; \ I^i_{Y_i} = \frac{\partial I_i}{\partial Y_i} > 0, I^i_{K_1} = \frac{\partial I_i}{\partial K_i} < 0, I^i_{r_i} = \frac{\partial I_i}{\partial r_i} < 0, \tag{6}$$

$$\dot{K}_i = I_i \left(Y_i, K_i, r_i\right), \tag{7}$$

$$G_i = G_{0i} + \gamma_i \left(\bar{Y}_i - Y_i\right) \ ; \ \gamma_i > 0, \tag{8}$$

$$M_i = L_i \left(Y_i, r_i\right) \ ; \ \frac{\partial L_i}{\partial Y_i} > 0, L^i_{r_i} = \frac{\partial L_i}{\partial r_i} < 0, \tag{9}$$

$$J_1 = \delta H_1 \left(Y_1, Y_2\right) \ ; \ H^1_{Y_1} = \frac{\partial H_1}{\partial Y_1} < 0, H^1_{Y_2} = \frac{\partial H_1}{\partial Y_2} > 0, 0 \leq \delta \leq 1, \tag{10}$$

$$Q_1 = \beta\{r_1 - r_2\} \ ; \ \beta > 0 \tag{11}$$

(2) Definitional equations

$$A_1 = J_1 + Q_1, \tag{12}$$

$$J_1 + J_2 = 0, \tag{13}$$

$$Q_1 + Q_2 = 0, \tag{14}$$

$$A_1 + A_2 = 0, \tag{15}$$

$$\dot{M}_1 = A_1, \tag{16}$$

$$\bar{M} = M_1 + M_2, \tag{17}$$

where subscript i ($i = 1, 2$) is the index number of a country, and the definitions of the other symbols are as follows: Y_i is real net national income, C_i is real private consumption expenditure, I_i is real net private investment expenditure, G_i is real government expenditure, K_i is real capital stock, $\bar{Y}_i$ is the level of real national income that a government determine the counter-cyclical government expenditure (this is not necessarily natural output), T_i is the real income tax, T_{0i} is the negative income tax (or basic income), M_i is the nominal money supply, p_i is the price level, r_i is the nominal rate of interest,[2)] J_i is real net exports, Q_i is the real capital account balance,

[2)] In this study, for the sake of simplicity, public bonds and stock are treated as perfect substitute goods.

and A_i is the real total balance of payments. The dots above the symbols represent derivatives with respect to time.

Eq. (3) is the disequilibrium quantity adjustment process in the goods market. Parameter α_i represents the adjustment speed of the goods market. Eq. (4) is the Keynesian consumption function indicating the behavior of the consumer. Eq. (5) is the standard tax function. Eq. (6) is the standard Kaldorian investment function. Eq. (7) is the dynamic equation of capital stock. Eq. (8) is the government expenditure function. Parameter γ_i represents the degree of counter-cyclical fiscal policy. The larger γ_i is, the larger is counter-cyclical government expenditure;, Eq. (9) represents the equilibrium condition in the monetary market. Eq. (10) is the real net export function of country 1. Parameter δ represents the degree of openness of the economy and H_i is real net exports discounted by δ. Eq. (11) is the real capital account balance function of country 1 in the model with imperfect capital mobility. Parameter β indicates the degree of mobility of international capital flows. The larger β is, the higher is the degree of mobility of international capital flows. The model of perfect capital mobility is a special case in which β is infinite, and the following equation is always established in the case of a fixed exchange rate system: $r_1 = r_2$. Eq. (12) is the definitional equation of the real total balance of payments of country 1. Eqs. (13), (14), and (15) imply that net export surplus, capital account balance surplus and the total balance of payments surplus of a country must be accompanied by the same amounts of the current account deficit, capital account balance deficit, and the total balance of payments deficit of another country, respectively; Eq. (16) means that the nominal money supply of country 1 increases (decreases) according to the total balance of payment surplus (deficit) of country 1. Eq. (17) indicates that the total nominal money supply of two countries is fixed by the European Central Bank.

Then, we transform this system into a more compact system. We obtain the following LM equation by solving Eq. (9) with respect to r_i.

$$r_i = r_i\left(Y_i, M_i\right)\ ;\ r^i_{Y_i} = \frac{\partial r_i}{\partial Y_i} = -\frac{L^i_{Y_i}}{L^i_{r_i}} > 0, r^i_{Mi} = \frac{\partial r_i}{\partial M_i} = \frac{1}{L^i_{r_i}} < 0. \tag{18}$$

Furthermore, we obtain the following money supply equation of country 2 from Eq. (17)

$$M_2 = \bar{M} - M_1 = M_2\left(M_1\right) \tag{19}$$

Combining Eqs. (3)–(19), we obtain the following five-dimensional system of nonlinear differential equations, given policy parameters $\bar{M}$, G_i, and τ_i $(i = 1, 2)$.

$$\begin{aligned}\dot{Y}_1 &= \alpha_1\left\{c_1\left(1-\tau_1\right)Y_1 + C_{01} + c_1T_{01} + G_{01} + \gamma_1\left(\bar{Y}_1 - Y_1\right) + I_1\left(Y_1, K_1, r_1\left(Y_1, M_1\right)\right) + \delta H_1\left(Y_1, Y_2\right) - Y_1\right\}\\ &= V_1\left(Y_1, K_1, Y_2, M_1; \alpha_1, \gamma_1, \mu\right),\end{aligned} \tag{20}$$

$$\dot{K}_1 = I_1\left(Y_1, K_1, r_1\left(Y_1, M_1\right)\right) = V_2\left(Y_1, K_1, M_1\right), \tag{21}$$

$$\begin{aligned}\dot{Y}_2 &= \alpha_2\left\{c_2\left(1-\tau_2\right)Y_2 + C_{02} + c_2T_{02} + G_{02} + \gamma_2\left(\bar{Y}_2 - Y_2\right) + I_2\left(Y_2, r_2\left(Y_2, M_2\left(M_1\right)\right)\right) - \delta H_1\left(Y_1, Y_2\right) - Y_2\right\}\\ &= V_3\left(Y_1, Y_2, K_2, M_1; \alpha_2, \gamma_2, \mu\right),\end{aligned} \tag{22}$$

$$\dot{K}_2 = I_2\left(Y_2, r_2\left(Y_2, M_2\left(M_1\right)\right)\right) = V_4\left(Y_2, K_2, M_1\right), \tag{23}$$

$$\dot{M}_1 = \delta H_1\left(Y_1, Y_2\right) + \beta\{r_1\left(Y_1, M_1\right) - r_2\left(Y_2, M_2\left(M_1\right)\right)\} = V_5\left(Y_1, Y_2, M_1; \beta\right) \tag{24}$$

3 Uniqueness of the Equilibrium Solution

In this section, we show that the equilibrium point of Eqs. (20)–(24) is unique if it exists.[3)]

We define an equilibrium point of Eqs. (20)–(24) as a point $(Y_1^*, K_1^*, Y_2^*, K_2^*, M_1^*) > (0,0,0,0,0)$ that satisfies $\dot{Y}_1 = \dot{K}_1 = \dot{Y}_2 = \dot{K}_2 = \dot{M}_1 = 0$. Then, it follows from Eqs. (20)–(24) that an equilibrium point is a solution of the following system of simultaneous equations.

$$0 = c_1\left(1-\tau_1\right)Y_1 + C_{01} + c_1 T_{01} + G_{01} + \gamma_1\left(\bar{Y_1} - Y_1\right) + I_1\left(Y_1, K_1, r_1\left(Y_1, M_1\right)\right) + \delta H_1\left(Y_1, Y_2\right) - Y_1, \tag{25}$$

$$0 = I_1\left(Y_1, K_1, r_1\left(Y_1, M_1\right)\right), \tag{26}$$

$$0 = c_2\left(1-\tau_2\right)Y_2 + C_{02} + c_2 T_{02} + G_{02} + \gamma_2\left(\bar{Y_2} - Y_2\right) + I_2\left(Y_2, r_2\left(Y_2, M_2\left(M_1\right)\right)\right) - \delta H_1\left(Y_1, Y_2\right) - Y_2, \tag{27}$$

$$0 = I_2\left(Y_2, r_2\left(Y_2, M_2\left(M_1\right)\right)\right), \tag{28}$$

$$0 = \delta H_1\left(Y_1, Y_2\right) + \beta\{r_1\left(Y_1, M_1\right) - r_2\left(Y_2, M_2\left(M_1\right)\right)\}. \tag{29}$$

In what follows, we assume that at least one equilibrium point of Eqs. (20)–(24) exist.

However, adding some assumptions defined hereafter, we prove the uniqueness of this equilibrium point. We set the following nonempty, compact, rectangular domain Ω, where $\overline{Y_1}$, $\overline{K_1}$, $\overline{Y_2}$, $\overline{K_2}$ and $\overline{M_1}$ are sufficiently large values.

$$\Omega \equiv \left\{\left(Y_1, K_1, Y_2, K_2, M_1\right) \,\middle|\, 0 \le Y_1 \le \overline{Y_1},\ 0 \le K_1 \le \overline{K_1},\ 0 \le Y_2 \le \overline{Y_2},\ 0 \le K_2 \le \overline{K_2},\ 0 \le M_1 \le \overline{M_1}\right\}.$$

We write the Jacobian matrix of the system of Eqs. (20)–(24) on Ω that are evaluated at the equilibrium point as follows:

$$J = \begin{bmatrix} V_{11} & V_{12} & V_{13} & 0 & V_{15} \\ V_{21} & V_{22} & 0 & 0 & V_{25} \\ V_{31} & 0 & V_{33} & V_{34} & V_{35} \\ 0 & 0 & V_{43} & V_{44} & V_{45} \\ V_{51} & 0 & V_{53} & 0 & V_{55} \end{bmatrix} = \begin{bmatrix} \alpha_1\Phi_{11} & \alpha_1\Phi_{12} & \alpha_1\Phi_{13} & 0 & \alpha_1\Phi_{15} \\ V_{21} & \Phi_{12} & 0 & 0 & \Phi_{15} \\ \alpha_2\Phi_{31} & 0 & \alpha_2\Phi_{33} & \alpha_2\Phi_{34} & \alpha_2\Phi_{35} \\ 0 & 0 & V_{43} & \Phi_{34} & \Phi_{35} \\ V_{51} & 0 & V_{53} & 0 & V_{55} \end{bmatrix}, \tag{30}$$

where

$$\Phi_{11} = -\underbrace{\{1 - c_1\left(1-\tau_1\right)\}}_{(+)} + \underset{(+)}{I^1_{Y_1}} + \underset{(-)}{I^1_{r_1}}\underset{(+)}{r^1_{Y_1}} + \underset{(-)}{\delta H^1_{Y_1}} - \gamma_1,\ \Phi_{12} = \underset{(-)}{I^1_{K_1}} < 0,$$

$$\Phi_{13} = \underset{(+)}{\delta H^1_{Y_2}} > 0,\ \Phi_{15} = \underset{(-)}{I^1_{r_1}}\underset{(-)}{r^1_{M_1}} > 0, V_{21} = \underset{(+)}{I^1_{Y_1}} + \underset{(-)}{I^1_{r_1}}\underset{(+)}{r^1_{Y_1}},\ \Phi_{31} = -\underset{(-)}{\delta H^1_{Y_1}} > 0,$$

$$\Phi_{33} = -\underbrace{\{1 - c_2\left(1-\tau_2\right)\}}_{(+)} + \underset{(+)}{I^2_{Y_2}} + \underset{(-)}{I^2_{r_2}}\underset{(+)}{r^2_{Y_2}} - \underset{(+)}{\delta H^1_{Y_2}} - \gamma_2,$$

[3)] The explanation on the uniqueness of the equilibrium point in this section is based on Murakami and Asada (2018).

$$\Phi_{34} = \underset{(-)}{I^2_{K_2}} < 0,\ \Phi_{35} = -\underset{(-)}{I^2_{r_2}}\,\underset{(-)}{r^2_{M_2(M_1)}} < 0,\ V_{43} = \underset{(+)}{I^2_{Y_2}} + \underset{(-)}{I^2_{r_2}}\,\underset{(+)}{r^2_{Y_2}},$$

$$V_{51} = \delta \underset{(-)}{H^1_{Y_1}} + \beta \underset{(+)}{r^1_{Y_1}},\ V_{53} = \delta \underset{(+)}{H^1_{Y_2}} - \beta \underset{(+)}{r^2_{Y_2}},\ V_{55} = \beta(\underset{(-)}{r^1_{M_1}} + \underset{(-)}{r^2_{M_2(M_1)}}) < 0.$$

Now, we assume as follows.

Assumption 1. In the region Ω, the following inequations hold.

$$I^1_{Y_1} > |I^1_{r_1} r^1_{Y_1}|,\ I^2_{Y_2} > |I^2_{r_2} r^2_{Y_2}|.$$

Remark 1. Assumption 1 implies $V_{21} > 0$ and $V_{43} > 0$ in the region Ω.

Assumption 2. Parameters δ and γ_i are sufficiently large.

Remark 2. Assumption 2 implies $V_{11} < 0$ ($\Phi_{11} < 0$) and $V_{33} < 0$ ($\Phi_{33} < 0$) in the region Ω.

Assumption 3. In the region Ω, the following inequations hold.

$$|\delta H^1_{Y_1}| > |\beta r^1_{Y_1}|,\ |\delta H^1_{Y_2}| > |\beta r^2_{Y_2}|.$$

Remark 3. Assumption 3 implies $V_{51} < 0$ and $V_{53} > 0$ in the region Ω.

Assumption 1 means that the sensitivities of investment with respect to the change of national incomes of country i are sufficiently large at the equilibrium point. This assumption is nothing but the standard hypothesis of the Kaldorian business cycle model (c.f., Kaldor (1940), Asada, Inaba and Misawa (2001), and Asada (2004)).

Now, we show that the sign of the principal minors of Jacobian matrix J become as follows in region Ω, in the case in which these assumptions are satisfied.

$$V_{11} = \alpha_1 \underset{(-)}{\Phi_{11}} < 0,\ V_{22} = \underset{(-)}{\Phi_{12}} < 0,\ V_{33} = \alpha_2 \underset{(-)}{\Phi_{33}} < 0,\ V_{44} = \underset{(-)}{\Phi_{34}} < 0,\ V_{55} = \underset{(-)}{V_{55}} < 0,$$

$$\begin{vmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{vmatrix} = \alpha_1 \underset{(-)}{\Phi_{11}}\,\underset{(-)}{\Phi_{12}} - \alpha_1 \underset{(-)}{\Phi_{12}}\,\underset{(+)}{V_{21}} > 0,\quad \begin{vmatrix} V_{11} & V_{13} \\ V_{31} & V_{33} \end{vmatrix} = \alpha_1 \underset{(-)}{\Phi_{11}} \alpha_2 \underset{(-)}{\Phi_{33}} - \alpha_1 \underset{(+)}{\Phi_{13}} \alpha_2 \underset{(+)}{\Phi_{31}} > 0,$$

$$\begin{vmatrix} V_{11} & 0 \\ 0 & V_{44} \end{vmatrix} = \alpha_1 \underset{(-)}{\Phi_{11}}\,\underset{(-)}{\Phi_{34}} > 0,\quad \begin{vmatrix} V_{11} & V_{15} \\ V_{51} & V_{55} \end{vmatrix} = \alpha_1 \underset{(-)}{\Phi_{11}}\,\underset{(-)}{V_{55}} - \alpha_1 \underset{(+)}{\Phi_{15}}\,\underset{(-)}{V_{51}} > 0,$$

$$\begin{vmatrix} V_{22} & 0 \\ 0 & V_{33} \end{vmatrix} = \underset{(-)}{\Phi_{12}} \alpha_2 \underset{(-)}{\Phi_{33}} > 0,\quad \begin{vmatrix} V_{22} & 0 \\ 0 & V_{44} \end{vmatrix} = \underset{(-)}{\Phi_{12}}\,\underset{(-)}{\Phi_{34}} > 0,$$

$$\begin{vmatrix} V_{22} & V_{25} \\ 0 & V_{55} \end{vmatrix} = \underset{(-)}{\Phi_{12}}\,\underset{(-)}{V_{55}} > 0,\quad \begin{vmatrix} V_{33} & V_{34} \\ V_{43} & V_{44} \end{vmatrix} = \alpha_2 \underset{(-)}{\Phi_{33}}\,\underset{(-)}{\Phi_{34}} - \alpha_2 \underset{(-)}{\Phi_{34}}\,\underset{(+)}{\Phi_{43}} > 0,$$

$$\begin{vmatrix} V_{33} & V_{35} \\ V_{53} & V_{55} \end{vmatrix} = \alpha_2 \underset{(-)}{\Phi_{33}}\,\underset{(-)}{V_{55}} - \alpha_2 \underset{(-)}{\Phi_{35}}\,\underset{(+)}{V_{53}} > 0,\quad \begin{vmatrix} V_{44} & V_{45} \\ 0 & V_{55} \end{vmatrix} = \underset{(-)}{\Phi_{34}}\,\underset{(-)}{V_{55}} > 0,$$

$$\begin{vmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & 0 \\ V_{31} & 0 & V_{33} \end{vmatrix} = \alpha_1\alpha_2 \underset{(-)}{\Phi_{11}} \underset{(-)}{\Phi_{12}} \underset{(-)}{\Phi_{33}} - \underset{(+)}{V_{21}} \alpha_1\alpha_2 \underset{(-)}{\Phi_{12}} \underset{(-)}{\Phi_{33}} - \alpha_1\alpha_2 \underset{(+)}{\Phi_{31}} \underset{(+)}{\Phi_{13}} \underset{(-)}{\Phi_{12}} < 0,$$

$$\begin{vmatrix} V_{11} & V_{12} & 0 \\ V_{21} & V_{22} & 0 \\ 0 & 0 & V_{44} \end{vmatrix} = \alpha_1 \underset{(-)}{\Phi_{11}} \underset{(-)}{\Phi_{12}} \underset{(-)}{\Phi_{34}} - \alpha_1 \underset{(+)}{V_{21}} \underset{(-)}{\Phi_{12}} \underset{(-)}{\Phi_{34}} < 0,$$

$$\begin{vmatrix} V_{11} & V_{12} & V_{15} \\ V_{21} & V_{22} & V_{25} \\ V_{51} & 0 & V_{55} \end{vmatrix} = \alpha_1 \underset{(-)}{\Phi_{11}} \underset{(-)}{\Phi_{12}} \underset{(-)}{V_{55}} - \alpha_1 \underset{(+)}{V_{21}} \underset{(-)}{\Phi_{12}} \underset{(-)}{V_{55}} + \alpha_1 \underset{(-)}{V_{51}} \underset{(-)}{\Phi_{12}} \underset{(+)}{\Phi_{15}} - \alpha_1 \underset{(-)}{V_{51}} \underset{(+)}{\Phi_{15}} \underset{(-)}{\Phi_{12}} < 0,$$

$$\begin{vmatrix} V_{11} & V_{13} & 0 \\ V_{31} & V_{33} & V_{34} \\ 0 & 0 & V_{44} \end{vmatrix} = \alpha_1\alpha_2 \underset{(-)}{\Phi_{11}} \underset{(-)}{\Phi_{33}} \underset{(-)}{\Phi_{34}} - \alpha_1\alpha_2 \underset{(+)}{\Phi_{31}} \underset{(+)}{\Phi_{13}} \underset{(-)}{\Phi_{34}} < 0,$$

$$\begin{vmatrix} V_{11} & V_{13} & V_{15} \\ V_{31} & V_{33} & V_{35} \\ V_{51} & V_{53} & V_{55} \end{vmatrix} = \alpha_1\alpha_2 \underset{(-)}{\Phi_{11}} \underset{(-)}{\Phi_{33}} \underset{(-)}{V_{55}} - \alpha_1\alpha_2 \underset{(-)}{\Phi_{11}} \underset{(-)}{\Phi_{35}} \underset{(+)}{V_{53}} - \alpha_1\alpha_2 \underset{(+)}{\Phi_{31}} \underset{(+)}{\Phi_{13}} \underset{(-)}{V_{55}} + \alpha_1\alpha_2 \underset{(+)}{\Phi_{31}} \underset{(+)}{\Phi_{15}} \underset{(+)}{V_{53}}$$

$$+ \alpha_1\alpha_2 \underset{(-)}{V_{51}} \underset{(+)}{\Phi_{13}} \underset{(-)}{\Phi_{35}} - \alpha_1\alpha_2 \underset{(-)}{V_{51}} \underset{(+)}{\Phi_{15}} \underset{(-)}{\Phi_{33}} < 0,$$

$$\begin{vmatrix} V_{11} & 0 & V_{15} \\ 0 & V_{44} & V_{45} \\ V_{51} & 0 & V_{55} \end{vmatrix} = \alpha_1 \underset{(-)}{\Phi_{11}} \underset{(-)}{\Phi_{34}} \underset{(-)}{V_{55}} - \alpha_1 \underset{(-)}{V_{51}} \underset{(+)}{\Phi_{15}} \underset{(-)}{\Phi_{34}} < 0,$$

$$\begin{vmatrix} V_{22} & 0 & 0 \\ 0 & V_{33} & V_{34} \\ 0 & V_{43} & V_{44} \end{vmatrix} = \alpha_2 \underset{(-)}{\Phi_{12}} \underset{(-)}{\Phi_{33}} \underset{(-)}{\Phi_{34}} - \alpha_2 \underset{(-)}{\Phi_{12}} \underset{(-)}{\Phi_{34}} \underset{(+)}{\Phi_{43}} < 0,$$

$$\begin{vmatrix} V_{22} & 0 & V_{25} \\ 0 & V_{33} & V_{35} \\ 0 & V_{53} & V_{55} \end{vmatrix} = \alpha_2 \underset{(-)}{\Phi_{12}} \underset{(-)}{\Phi_{33}} \underset{(-)}{V_{55}} - \alpha_2 \underset{(-)}{\Phi_{12}} \underset{(-)}{\Phi_{35}} \underset{(+)}{V_{53}} < 0,$$

$$\begin{vmatrix} V_{22} & V_{24} & V_{25} \\ 0 & V_{44} & V_{45} \\ 0 & 0 & V_{55} \end{vmatrix} = \underset{(-)}{\Phi_{12}} \underset{(-)}{\Phi_{34}} \underset{(-)}{V_{55}} < 0,$$

$$\begin{vmatrix} V_{33} & V_{34} & V_{35} \\ V_{43} & V_{44} & V_{45} \\ V_{53} & 0 & V_{55} \end{vmatrix} = \alpha_2 \underset{(-)}{\Phi_{33}} \underset{(-)}{\Phi_{34}} \underset{(-)}{V_{55}} - \alpha_2 \underset{(+)}{\Phi_{43}} \underset{(-)}{\Phi_{34}} \underset{(-)}{V_{55}} < 0,$$

$$
\begin{aligned}
\begin{vmatrix} V_{11} & V_{12} & V_{13} & 0 \\ V_{21} & V_{22} & 0 & 0 \\ V_{31} & 0 & V_{33} & V_{34} \\ 0 & 0 & V_{43} & V_{44} \end{vmatrix}
&= \alpha_1\alpha_2 \underset{(-)}{\Phi_{11}}\underset{(-)}{\Phi_{12}}\underset{(-)}{\Phi_{33}}\underset{(-)}{\Phi_{34}} - \alpha_2 \underset{(-)}{\Phi_{12}}\underset{(-)}{\Phi_{34}}\underset{(+)}{\Phi_{43}} - \alpha_1\alpha_2 \underset{(+)}{V_{21}}\underset{(-)}{\Phi_{12}}\underset{(-)}{\Phi_{33}}\underset{(-)}{\Phi_{34}} \\
&\quad + \alpha_1\alpha_2 \underset{(+)}{V_{21}}\underset{(-)}{\Phi_{12}}\underset{(-)}{\Phi_{34}}\underset{(+)}{\Phi_{43}} + \alpha_1\alpha_2 \underset{(+)}{\Phi_{31}}\underset{(-)}{\Phi_{12}}\underset{(+)}{\Phi_{13}}\underset{(-)}{\Phi_{34}} > 0,
\end{aligned}
$$

$$
\begin{aligned}
\begin{vmatrix} V_{11} & V_{12} & V_{13} & V_{15} \\ V_{21} & V_{22} & 0 & V_{25} \\ V_{31} & 0 & V_{33} & V_{35} \\ V_{51} & 0 & V_{53} & V_{55} \end{vmatrix}
&= \alpha_1\alpha_2 \underset{(-)}{\Phi_{11}}(\underset{(-)}{\Phi_{12}}\underset{(-)}{\Phi_{33}}\underset{(-)}{V_{55}} - \underset{(-)}{\Phi_{12}}\underset{(-)}{\Phi_{35}}\underset{(+)}{V_{53}}) - \alpha_1\alpha_2 \underset{(+)}{V_{21}}\underset{(-)}{\Phi_{12}}\underset{(-)}{\Phi_{33}}\underset{(-)}{V_{55}} \\
&\quad + \alpha_1\alpha_2 \underset{(+)}{V_{21}}\underset{(-)}{\Phi_{12}}\underset{(-)}{\Phi_{35}}\underset{(+)}{V_{53}} - \alpha_1\alpha_2 \underset{(+)}{\Phi_{31}}\underset{(-)}{\Phi_{12}}\underset{(+)}{\Phi_{15}}\underset{(+)}{V_{53}} + \alpha_1\alpha_2 \underset{(+)}{\Phi_{31}}\underset{(-)}{\Phi_{12}}\underset{(+)}{\Phi_{13}}\underset{(-)}{V_{55}} \\
&\quad - \alpha_1\alpha_2 \underset{(+)}{\Phi_{31}}\underset{(-)}{\Phi_{12}}\underset{(+)}{\Phi_{15}}\underset{(+)}{V_{53}} > 0,
\end{aligned}
$$

$$
\begin{aligned}
\begin{vmatrix} V_{11} & V_{12} & 0 & V_{15} \\ V_{21} & V_{22} & 0 & V_{25} \\ V_{31} & 0 & V_{44} & V_{35} \\ V_{51} & 0 & 0 & V_{55} \end{vmatrix}
&= \alpha_1 \underset{(-)}{\Phi_{11}}\underset{(-)}{\Phi_{12}}\underset{(-)}{\Phi_{34}}\underset{(-)}{V_{55}} - \alpha_1 \underset{(+)}{V_{21}}\underset{(-)}{\Phi_{12}}\underset{(-)}{\Phi_{34}}\underset{(-)}{V_{55}} + \alpha_1 \underset{(-)}{V_{51}}\underset{(-)}{\Phi_{12}}\underset{(+)}{\Phi_{15}}\underset{(-)}{\Phi_{34}} \\
&\quad + \alpha_1 \underset{(-)}{V_{51}}\underset{(-)}{\Phi_{12}}\underset{(+)}{\Phi_{15}}\underset{(-)}{\Phi_{34}} > 0,
\end{aligned}
$$

$$
\begin{aligned}
\begin{vmatrix} V_{11} & V_{13} & 0 & V_{15} \\ V_{31} & V_{33} & V_{34} & V_{35} \\ 0 & V_{43} & V_{44} & V_{45} \\ V_{51} & V_{53} & 0 & V_{55} \end{vmatrix}
&= \alpha_1\alpha_2 \underset{(-)}{\Phi_{11}}(\underset{(-)}{\Phi_{33}}\underset{(-)}{\Phi_{34}}\underset{(-)}{V_{55}} - \underset{(+)}{\Phi_{43}}\underset{(-)}{\Phi_{34}}\underset{(-)}{V_{55}}) - \alpha_1\alpha_2 \underset{(+)}{\Phi_{31}}(\underset{(+)}{\Phi_{13}}\underset{(-)}{\Phi_{34}}\underset{(-)}{V_{55}} - \underset{(+)}{V_{53}}\underset{(+)}{\Phi_{15}}\underset{(-)}{\Phi_{34}}) \\
&\quad - \alpha_1\alpha_2 \underset{(-)}{V_{51}}(\underset{(-)}{\Phi_{33}}\underset{(+)}{\Phi_{15}}\underset{(-)}{\Phi_{34}} - \underset{(+)}{\Phi_{43}}\underset{(+)}{\Phi_{15}}\underset{(-)}{\Phi_{34}} > 0,
\end{aligned}
$$

$$
\begin{vmatrix} V_{22} & 0 & 0 & V_{25} \\ 0 & V_{33} & V_{34} & V_{35} \\ 0 & V_{43} & V_{44} & V_{45} \\ 0 & V_{53} & 0 & V_{55} \end{vmatrix}
= \alpha_2 \underset{(-)}{\Phi_{12}}\underset{(-)}{\Phi_{34}}(\underset{(-)}{\Phi_{33}}\underset{(-)}{V_{55}} - \underset{(+)}{\Phi_{43}}\underset{(-)}{V_{55}} + \underset{(+)}{V_{53}}\underset{(-)}{\Phi_{35}} - \underset{(+)}{V_{53}}\underset{(-)}{\Phi_{35}}) > 0,
$$

$$
\begin{aligned}
\begin{vmatrix} V_{11} & V_{12} & V_{13} & 0 & V_{15} \\ V_{21} & V_{22} & 0 & 0 & V_{25} \\ V_{31} & 0 & V_{33} & V_{34} & V_{35} \\ 0 & 0 & V_{43} & V_{44} & V_{45} \\ V_{51} & 0 & V_{53} & 0 & V_{55} \end{vmatrix}
&= \alpha_1 \underset{(-)}{\Phi_{11}} (\underset{(-)}{\Phi_{12}} (\alpha_2 \underset{(-)}{\Phi_{33}} \underset{(-)}{\Phi_{34}} \underset{(-)}{V_{55}} - \underset{(+)}{\Phi_{43}} \alpha_2 \underset{(-)}{\Phi_{34}} \underset{(-)}{V_{55}} - \underset{(+)}{\Phi_{43}} \alpha_2 \underset{(-)}{\Phi_{34}} \underset{(-)}{V_{55}} \\
&\quad + \underset{(+)}{V_{53}} \alpha_2 \underset{(-)}{\Phi_{34}} \alpha_2 \underset{(-)}{\Phi_{35}} - \underset{(+)}{V_{53}} \alpha_2 \underset{(-)}{\Phi_{35}} \underset{(-)}{\Phi_{34}}) - \underset{(+)}{V_{21}} \alpha_1 \underset{(-)}{\Phi_{12}} (\alpha_2 \underset{(-)}{\Phi_{33}} \underset{(-)}{\Phi_{34}} \underset{(-)}{V_{55}} \\
&\quad - \underset{(+)}{\Phi_{43}} \alpha_2 \underset{(-)}{\Phi_{34}} \underset{(-)}{V_{55}} - \underset{(+)}{\Phi_{43}} \alpha_2 \underset{(-)}{\Phi_{34}} \underset{(-)}{V_{55}} + \underset{(+)}{V_{53}} \alpha_2 \underset{(-)}{\Phi_{34}} \alpha_2 \underset{(-)}{\Phi_{35}} - \underset{(+)}{V_{53}} \alpha_2 \underset{(-)}{\Phi_{35}} \underset{(-)}{\Phi_{34}}) \\
&\quad - \alpha_2 \underset{(+)}{\Phi_{31}} \alpha_1 \underset{(-)}{\Phi_{12}} \underset{(+)}{V_{53}} \underset{(-)}{\Phi_{34}} \underset{(+)}{\Phi_{15}} + \alpha_2 \underset{(+)}{\Phi_{31}} \underset{(-)}{\Phi_{12}} (\alpha_1 \underset{(+)}{\Phi_{13}} \underset{(-)}{\Phi_{34}} \underset{(-)}{V_{55}} - \underset{(+)}{V_{53}} \alpha_1 \underset{(+)}{\Phi_{15}} \underset{(-)}{\Phi_{34}}) \\
&\quad - \underset{(-)}{V_{51}} \alpha_1 \underset{(-)}{\Phi_{12}} \underset{(+)}{V_{53}} \underset{(+)}{\Phi_{15}} \underset{(-)}{\Phi_{34}} - \underset{(-)}{V_{51}} \underset{(-)}{\Phi_{12}} \alpha_1 \underset{(+)}{\Phi_{13}} \underset{(-)}{\Phi_{34}} \underset{(-)}{V_{55}} + \underset{(-)}{V_{51}} \underset{(-)}{\Phi_{12}} \underset{(+)}{V_{53}} \alpha_1 \underset{(+)}{\Phi_{15}} \underset{(-)}{\Phi_{34}} < 0.
\end{aligned}
$$

Then, Jacobian matrix J is termed an N-P-matrix from the following definition.

Definition 1 (Nikaido (1968, p.361, Definition 20.3)). A real square matrix A of order n is termed an N-P-matrix if it has all the principal minors of odd orders negative and those of even orders positive.

Therefore, we conclude from Nikaido (1968, p.371, Corollary (i)) that the system of Eqs. (20)–(24) is univalent on Ω. Thus, we have proved the following proposition.

Proposition 1. At most one equilibrium point of Eqs. (20)–(24) exists in the region Ω under Assumptions 1–3.

4 Local Stability Analysis

In this section, we assume that a unique equilibrium solution $(Y_1^*, K_1^*, Y_2^*, K_2^*, M_1^*) > (0, 0, 0, 0, 0)$ exists, and we analyze the local stability of this equilibrium solution. Although we can write the Jacobian matrix of the system of Eqs. (20)–(24) that are evaluated at the equilibrium point, it is the same as Eq. (30). Now, we rewrite Eq. (30).

$$
J = \begin{bmatrix} V_{11} & V_{12} & V_{13} & 0 & V_{15} \\ V_{21} & V_{22} & 0 & 0 & V_{25} \\ V_{31} & 0 & V_{33} & V_{34} & V_{35} \\ 0 & 0 & V_{43} & V_{44} & V_{45} \\ V_{51} & 0 & V_{53} & 0 & V_{55} \end{bmatrix} = \begin{bmatrix} \alpha_1\Phi_{11} & \alpha_1\Phi_{12} & \alpha_1\Phi_{13} & 0 & \alpha_1\Phi_{15} \\ V_{21} & \Phi_{12} & 0 & 0 & \Phi_{15} \\ \alpha_2\Phi_{31} & 0 & \alpha_2\Phi_{33} & \alpha_2\Phi_{34} & \alpha_2\Phi_{35} \\ 0 & 0 & V_{43} & \Phi_{34} & \Phi_{35} \\ V_{51} & 0 & V_{53} & 0 & V_{55} \end{bmatrix}; \tag{30}
$$

$$
\Phi_{11} = -\underbrace{\{1 - c_1(1-\tau_1)\}}_{(+)} + \underset{(+)}{I_{Y_1}^1} + \underset{(-)}{I_{r_1}^1} \underset{(+)}{r_{Y_1}^1} + \delta \underset{(-)}{H_{Y_1}^1} - \gamma_1, \ \Phi_{12} = \underset{(-)}{I_{K_1}^1} < 0,
$$

$$
\Phi_{13} = \delta \underset{(+)}{H_{Y_2}^1} > 0, \ \Phi_{15} = \underset{(-)}{I_{r_1}^1} \underset{(-)}{r_{M_1}^1} > 0, V_{21} = \underset{(+)}{I_{Y_1}^1} + \underset{(-)}{I_{r_1}^1} \underset{(+)}{r_{Y_1}^1}, \ \Phi_{31} = -\delta \underset{(-)}{H_{Y_1}^1} > 0,
$$

$$
\Phi_{33} = -\underbrace{\{1 - c_2(1-\tau_2)\}}_{(+)} + \underset{(+)}{I_{Y_2}^2} + \underset{(-)}{I_{r_2}^2} \underset{(+)}{r_{Y_2}^2} - \delta \underset{(+)}{H_{Y_2}^1} - \gamma_2,
$$

$$\Phi_{34} = \underset{(-)}{I^2_{K_2}} < 0,\ \Phi_{35} = -\underset{(-)}{I^2_{r_2}}\underset{(-)}{r^2_{M_2(M_1)}} < 0,\ V_{43} = \underset{(+)}{I^2_{Y_2}} + \underset{(-)}{I^2_{r_2}}\underset{(+)}{r^2_{Y_2}},$$

$$V_{51} = \delta \underset{(-)}{H^1_{Y_1}} + \beta \underset{(+)}{r^1_{Y_1}},\ V_{53} = \delta \underset{(+)}{H^1_{Y_2}} - \beta \underset{(+)}{r^2_{Y_2}},\ V_{55} = \beta(\underset{(-)}{r^1_{M_1}} + \underset{(-)}{r^2_{M_2(M_1)}}) < 0.$$

In this section, we adopt only Assumption 1 in the previous section but do not adopt Assumption 2 and 3. We express the characteristic equation of this system as follows.

$$f(\lambda) = |\lambda I - J| = \lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_5 = 0 \tag{31}$$

where

$$a_1 = -traceJ = -\alpha_1 \underset{(?)}{\Phi_{11}} - \underset{(-)}{\Phi_{12}} - \alpha_2 \underset{(?)}{\Phi_{33}} - \underset{(-)}{\Phi_{34}} - \underset{(-)}{V_{55}}, \tag{32}$$

$$\begin{aligned}
a_2 = {} & \alpha_1 \begin{vmatrix} \Phi_{11} & \Phi_{12} \\ V_{21} & \Phi_{12} \end{vmatrix} + \alpha_1\alpha_2 \begin{vmatrix} \Phi_{11} & \Phi_{13} \\ V_{31} & \Phi_{33} \end{vmatrix} + \alpha_1 \begin{vmatrix} \Phi_{11} & 0 \\ 0 & \Phi_{34} \end{vmatrix} + \alpha_1 \begin{vmatrix} \Phi_{11} & \Phi_{15} \\ V_{51} & V_{55} \end{vmatrix} \\
& + \alpha_2 \begin{vmatrix} \Phi_{12} & 0 \\ 0 & \Phi_{33} \end{vmatrix} + \begin{vmatrix} \Phi_{12} & 0 \\ 0 & \Phi_{34} \end{vmatrix} + \begin{vmatrix} \Phi_{12} & \Phi_{15} \\ 0 & V_{55} \end{vmatrix} + \alpha_2 \begin{vmatrix} \Phi_{33} & \Phi_{34} \\ V_{43} & \Phi_{34} \end{vmatrix} \\
& + \alpha_2 \begin{vmatrix} \Phi_{33} & \Phi_{35} \\ V_{53} & V_{55} \end{vmatrix} + \begin{vmatrix} \Phi_{34} & \Phi_{35} \\ 0 & V_{55} \end{vmatrix} \\
= {} & \alpha_1(\underset{(?)}{\Phi_{11}}\underset{(-)}{\Phi_{12}} - \alpha_2\underset{(-)}{\Phi_{12}}\underset{(+)}{V_{21}}) + \alpha_1\alpha_2(\underset{(?)}{\Phi_{11}}\underset{(?)}{\Phi_{33}} - \underset{(+)}{\Phi_{13}}\underset{(+)}{\Phi_{31}}) + \alpha_1\underset{(?)}{\Phi_{11}}\underset{(-)}{\Phi_{34}} \\
& + \alpha_1(\underset{(?)}{\Phi_{11}}\underset{(-)}{V_{55}} - \underset{(+)}{\Phi_{15}}\underset{(?)}{V_{51}}) + \alpha_2\underset{(-)}{\Phi_{12}}\underset{(?)}{\Phi_{33}} + \underset{(-)}{\Phi_{12}}\underset{(-)}{\Phi_{34}} + \underset{(-)}{\Phi_{12}}\underset{(-)}{V_{55}} + \alpha_2(\underset{(?)}{\Phi_{33}}\underset{(-)}{\Phi_{34}} - \underset{(-)}{\Phi_{34}}\underset{(+)}{\Phi_{43}}) \\
& + \alpha_2(\underset{(?)}{\Phi_{33}}\underset{(-)}{V_{55}} - \underset{(-)}{\Phi_{35}}\underset{(?)}{V_{53}}) \\
& + \underset{(-)}{\Phi_{34}}\underset{(-)}{V_{55}},
\end{aligned} \tag{33}$$

$$\begin{aligned}
a_3 = {} & -\alpha_1\alpha_2 \begin{vmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} \\ V_{21} & \Phi_{12} & 0 \\ \Phi_{31} & 0 & \Phi_{33} \end{vmatrix} - \alpha_1 \begin{vmatrix} \Phi_{11} & \Phi_{12} & 0 \\ V_{21} & \Phi_{12} & 0 \\ 0 & 0 & \Phi_{34} \end{vmatrix} - \alpha_1 \begin{vmatrix} \Phi_{11} & \Phi_{12} & \Phi_{15} \\ V_{21} & \Phi_{12} & \Phi_{15} \\ V_{51} & 0 & V_{55} \end{vmatrix} \\
& - \alpha_1\alpha_2 \begin{vmatrix} \Phi_{11} & \Phi_{13} & 0 \\ \Phi_{31} & \Phi_{33} & \Phi_{34} \\ 0 & V_{43} & \Phi_{34} \end{vmatrix} - \alpha_1\alpha_2 \begin{vmatrix} \Phi_{11} & \Phi_{13} & \Phi_{15} \\ \Phi_{31} & \Phi_{33} & \Phi_{35} \\ V_{51} & V_{53} & V_{55} \end{vmatrix} \\
& - \alpha_1 \begin{vmatrix} \Phi_{11} & 0 & \Phi_{15} \\ 0 & \Phi_{34} & \Phi_{35} \\ V_{51} & 0 & V_{55} \end{vmatrix} - \alpha_2 \begin{vmatrix} \Phi_{12} & 0 & 0 \\ 0 & \Phi_{33} & \Phi_{34} \\ 0 & V_{43} & \Phi_{34} \end{vmatrix} - \alpha_2 \begin{vmatrix} \Phi_{12} & 0 & \Phi_{15} \\ 0 & \Phi_{33} & \Phi_{55} \\ 0 & V_{53} & V_{55} \end{vmatrix}
\end{aligned}$$

$$
\begin{aligned}
&- \begin{vmatrix} \Phi_{12} & 0 & \Phi_{15} \\ 0 & \Phi_{34} & \Phi_{35} \\ 0 & 0 & V_{55} \end{vmatrix} - \alpha_2 \begin{vmatrix} \Phi_{33} & \Phi_{34} & \Phi_{35} \\ V_{43} & \Phi_{34} & \Phi_{35} \\ V_{53} & 0 & V_{55} \end{vmatrix} \\
&= -\alpha_1\alpha_2(-\underset{(-)}{\Phi_{12}}\underset{(+)}{\Phi_{13}}\underset{(+)}{\Phi_{31}} - \underset{(+)}{V_{21}}\underset{(-)}{\Phi_{12}}\underset{(?)}{\Phi_{33}} + \underset{(?)}{\Phi_{11}}\underset{(-)}{\Phi_{12}}\underset{(?)}{\Phi_{33}}) - \alpha_1\underset{(-)}{\Phi_{34}}(\underset{(+)}{V_{21}}\underset{(-)}{\Phi_{12}} + \underset{(?)}{\Phi_{11}}\underset{(-)}{\Phi_{12}}) \\
&\quad - \alpha_1(-\underset{(+)}{V_{21}}\underset{(-)}{V_{55}}\underset{(-)}{\Phi_{12}} + \underset{(-)}{V_{55}}\underset{(?)}{\Phi_{11}}\underset{(-)}{\Phi_{12}}) - \alpha_1\alpha_2(-\underset{(+)}{V_{43}}\underset{(?)}{\Phi_{11}}\underset{(-)}{\Phi_{34}} - \underset{(+)}{\Phi_{13}}\underset{(+)}{\Phi_{31}}\underset{(-)}{\Phi_{34}} + \underset{(?)}{\Phi_{11}}\underset{(?)}{\Phi_{33}}\underset{(-)}{\Phi_{34}}) \\
&\quad - \alpha_1\alpha_2\{\underset{(-)}{V_{55}}(\underset{(?)}{\Phi_{11}}\underset{(?)}{\Phi_{33}} - \underset{(+)}{\Phi_{13}}\underset{(+)}{\Phi_{31}}) + \underset{(?)}{V_{53}}(\underset{(+)}{\Phi_{15}}\underset{(+)}{\Phi_{31}} - \underset{(?)}{\Phi_{11}}\underset{(-)}{\Phi_{35}}) + \underset{(?)}{V_{51}}(\underset{(+)}{\Phi_{13}}\underset{(-)}{\Phi_{35}} - \underset{(+)}{\Phi_{15}}\underset{(?)}{\Phi_{33}})\} \\
&\quad - \alpha_1(\underset{(?)}{V_{55}}\underset{(?)}{\Phi_{11}}\underset{(-)}{\Phi_{34}} - \underset{(?)}{V_{51}}\underset{(+)}{\Phi_{15}}\underset{(-)}{\Phi_{34}}) - \alpha_2\underset{(-)}{\Phi_{12}}(-\underset{(+)}{V_{43}}\underset{(-)}{\Phi_{34}} + \underset{(?)}{\Phi_{33}}\underset{(-)}{\Phi_{34}}) \\
&\quad - \alpha_2(\underset{(-)}{V_{55}}\underset{(-)}{\Phi_{12}}\underset{(?)}{\Phi_{33}} - \underset{(?)}{V_{53}}\underset{(-)}{\Phi_{12}}\underset{(-)}{\Phi_{35}}) - \underset{(-)}{V_{55}}\underset{(-)}{\Phi_{12}}\underset{(-)}{\Phi_{34}} - \alpha_2(-\underset{(+)}{V_{43}}\underset{(-)}{V_{55}}\underset{(-)}{\Phi_{34}} + \underset{(-)}{V_{55}}\underset{(?)}{\Phi_{33}}\underset{(-)}{\Phi_{34}}),
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
a_4 &= \alpha_1\alpha_2 \begin{vmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & 0 \\ V_{21} & \Phi_{12} & 0 & 0 \\ \Phi_{31} & 0 & \Phi_{33} & \Phi_{34} \\ 0 & 0 & V_{43} & \Phi_{34} \end{vmatrix} + \alpha_1\alpha_2 \begin{vmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{15} \\ V_{21} & \Phi_{12} & 0 & \Phi_{15} \\ \Phi_{31} & 0 & \Phi_{33} & \Phi_{35} \\ V_{51} & 0 & V_{53} & V_{55} \end{vmatrix} \\
&\quad + \alpha_1 \begin{vmatrix} \Phi_{11} & \Phi_{12} & 0 & \Phi_{15} \\ V_{21} & \Phi_{12} & 0 & \Phi_{15} \\ 0 & 0 & \Phi_{34} & \Phi_{35} \\ V_{51} & 0 & 0 & V_{55} \end{vmatrix} + \alpha_1\alpha_2 \begin{vmatrix} \Phi_{11} & \Phi_{13} & 0 & \Phi_{15} \\ \Phi_{31} & \Phi_{33} & \Phi_{34} & \Phi_{35} \\ 0 & V_{43} & \Phi_{34} & \Phi_{35} \\ V_{51} & V_{53} & 0 & V_{55} \end{vmatrix} \\
&\quad + \alpha_2 \begin{vmatrix} \Phi_{12} & 0 & 0 & \Phi_{15} \\ 0 & \Phi_{33} & \Phi_{34} & \Phi_{35} \\ 0 & V_{43} & \Phi_{34} & \Phi_{35} \\ 0 & V_{53} & 0 & V_{55} \end{vmatrix} \\
&= \alpha_1\alpha_2\{\underset{(+)}{V_{21}}\underset{(-)}{\Phi_{12}}\underset{(-)}{\Phi_{34}}(\underset{(+)}{V_{43}} - \underset{(?)}{\Phi_{33}}) + \underset{(?)}{\Phi_{11}}\underset{(-)}{\Phi_{12}}\underset{(-)}{\Phi_{34}}(\underset{(?)}{\Phi_{33}} - \underset{(+)}{V_{43}}) - \underset{(-)}{\Phi_{12}}\underset{(+)}{\Phi_{13}}\underset{(+)}{\Phi_{31}}\underset{(-)}{\Phi_{34}}\} \\
&\quad + \alpha_1\alpha_2\{\underset{(-)}{V_{55}}\underset{(-)}{\Phi_{12}}(\underset{(?)}{\Phi_{33}}\underset{(?)}{\Phi_{11}} - \underset{(+)}{\Phi_{13}}\underset{(+)}{\Phi_{31}} - \underset{(+)}{V_{21}}\underset{(?)}{\Phi_{33}}) + \underset{(-)}{\Phi_{12}}\underset{(-)}{\Phi_{35}}(\underset{(+)}{V_{21}}\underset{(?)}{V_{53}} - \underset{(?)}{V_{53}}\underset{(?)}{\Phi_{11}} + \underset{(?)}{V_{51}}\underset{(+)}{\Phi_{13}})\} \\
&\quad + \alpha_1\alpha_2\underset{(-)}{\Phi_{34}}\{\underset{(+)}{V_{43}}(\underset{(?)}{V_{51}}\underset{(+)}{\Phi_{15}} - \underset{(-)}{V_{55}}\underset{(?)}{\Phi_{11}}) + \underset{(-)}{V_{55}}(\underset{(?)}{\Phi_{11}}\underset{(?)}{\Phi_{33}} - \underset{(+)}{\Phi_{13}}\underset{(+)}{\Phi_{31}}) + \underset{(+)}{\Phi_{15}}(\underset{(?)}{V_{53}}\underset{(+)}{\Phi_{31}} - \underset{(?)}{V_{51}}\underset{(?)}{\Phi_{33}})\} \\
&\quad + \alpha_1\underset{(-)}{V_{55}}\underset{(-)}{\Phi_{12}}\underset{(-)}{\Phi_{34}}(\underset{(?)}{\Phi_{11}} - \underset{(+)}{V_{21}}) + \alpha_2\underset{(-)}{V_{55}}\underset{(-)}{\Phi_{12}}\underset{(-)}{\Phi_{34}}(\underset{(?)}{\Phi_{33}} - \underset{(+)}{V_{43}}),
\end{aligned} \tag{35}
$$

$$
\begin{aligned}
a_5 = -detJ = -\alpha_1\alpha_2 & \begin{vmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & 0 & \Phi_{15} \\ V_{21} & \Phi_{12} & 0 & 0 & \Phi_{15} \\ \Phi_{31} & 0 & \Phi_{33} & \Phi_{34} & \Phi_{35} \\ 0 & 0 & V_{43} & \Phi_{34} & \Phi_{35} \\ V_{51} & 0 & V_{53} & 0 & V_{55} \end{vmatrix} \\
= -\alpha_1\alpha_2 (& \underset{(?)}{\Phi_{11}}\underset{(-)}{\Phi_{12}}\underset{(?)}{\Phi_{33}}\underset{(-)}{\Phi_{34}}\underset{(-)}{V_{55}} - \underset{(?)}{\Phi_{11}}\underset{(-)}{\Phi_{12}}\underset{(-)}{\Phi_{34}}\underset{(+)}{V_{43}}\underset{(-)}{V_{55}} - \underset{(-)}{\Phi_{12}}\underset{(+)}{\Phi_{13}}\underset{(+)}{\Phi_{31}}\underset{(-)}{\Phi_{34}}\underset{(-)}{V_{55}} + \underset{(-)}{\Phi_{12}}\underset{(+)}{\Phi_{15}}\underset{(+)}{\Phi_{31}}\underset{(-)}{\Phi_{34}}\underset{(?)}{V_{53}} \\
& - \underset{(-)}{\Phi_{12}}\underset{(+)}{\Phi_{15}}\underset{(?)}{\Phi_{33}}\underset{(-)}{\Phi_{34}}\underset{(?)}{V_{51}} + \underset{(-)}{\Phi_{12}}\underset{(+)}{\Phi_{15}}\underset{(-)}{\Phi_{34}}\underset{(+)}{V_{43}}\underset{(?)}{V_{51}} - \underset{(-)}{\Phi_{12}}\underset{(+)}{\Phi_{15}}\underset{(+)}{\Phi_{31}}\underset{(-)}{\Phi_{34}}\underset{(?)}{V_{53}} + \underset{(-)}{\Phi_{12}}\underset{(+)}{\Phi_{15}}\underset{(?)}{\Phi_{33}}\underset{(-)}{\Phi_{34}}\underset{(?)}{V_{51}} \\
& - \underset{(-)}{\Phi_{12}}\underset{(+)}{\Phi_{15}}\underset{(-)}{\Phi_{34}}\underset{(+)}{V_{43}}\underset{(?)}{V_{51}} - \underset{(-)}{\Phi_{12}}\underset{(?)}{\Phi_{33}}\underset{(-)}{\Phi_{34}}\underset{(+)}{V_{21}}\underset{(-)}{V_{55}} + \underset{(-)}{\Phi_{12}}\underset{(-)}{\Phi_{34}}\underset{(+)}{V_{21}}\underset{(+)}{V_{43}}\underset{(-)}{V_{55}})
\end{aligned} \tag{36}
$$

Then, the Routh–Hurwitz terms $\Delta_i (i = 1, 2, ..., 5)$ are defined as follows in this five-dimensional case.

$$\Delta_1 = a_1, \tag{37}$$

$$\Delta_2 = \begin{vmatrix} a_1 & a_3 \\ 1 & a_2 \end{vmatrix} = a_1 a_2 - a_3, \tag{38}$$

$$\Delta_3 = \begin{vmatrix} a_1 & a_3 & a_5 \\ 1 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix} = a_1 a_2 a_3 - a_3^2 - a_1^2 a_4 + a_1 a_5, \tag{39}$$

$$\Delta_4 = \begin{vmatrix} a_1 & a_3 & a_5 & 0 \\ 1 & a_2 & a_4 & 0 \\ 0 & a_1 & a_3 & a_5 \\ 0 & 1 & a_2 & a_4 \end{vmatrix} = a_4 \Delta_3 + a_5 \left(a_1 a_4 - a_5 - a_2 \Delta_2 \right), \tag{40}$$

$$\Delta_5 = \begin{vmatrix} a_1 & a_3 & a_5 & 0 & 0 \\ 1 & a_2 & a_4 & 0 & 0 \\ 0 & a_1 & a_3 & a_5 & 0 \\ 0 & 1 & a_2 & a_4 & 0 \\ 0 & 0 & a_1 & a_3 & a_5 \end{vmatrix} = a_5 \Delta_4 \tag{41}$$

All the roots of the characteristic equation (31) have negative real parts if and only if the following Routh–Hurwitz conditions are satisfied.

$$\Delta_i > 0 \qquad \forall i \in \{1, 2, ..., 5\} \tag{42}$$

Then, the equilibrium point of the system (20)–(24) is locally stable.

Proposition 2.

(i) Suppose that the parameter β is fixed at any level. Then, the equilibrium point of the system (20)–(24) is locally stable if at least one of the parameters γ_1, γ_2 and δ is sufficiently large.

(ii) Suppose that the parameter γ_1, γ_2 and δ are relatively small and inequalities $\Phi_{11} > 0$ and $\Phi_{33} > 0$ hold. Then, the equilibrium point of the system (20)–(24) is locally unstable if the parameter β is sufficiently large.

(Proof.) See Appendix A.

As a result, it is evident from the Kaldorian model that the increase in parameter δ stabilize the econony through a monetary union satisfies the criterion of openness of the economy. The difference in the viewpoints of European Commission and Krugman with regard to the criterion of openness of the economy relates to whether the criterion cause an asymmetric shock. However, the theoretical result in this section indicates that, even if the high degree of economic openness causes an asymmetric shock, the high degree of economic openness serves as a shock adjustment factor. In brief, the criterion of economic openness adapts to the OCA theory from the viewpoint of shock adjustment, even if not from the viewpoint of prevention of a shock.

5 Numerical Simulations

In this section, we present numerical simulations that support the theoretical analysis regarding relationships between β, δ and γ_i in the previous sections.

Based on Asada (2004), we assume the following parameter values.[4)]

$$\alpha_1 = 2,\ \alpha_2 = 2.5,\ c_i = 0.8,\ \tau_i = 0.2,\ T_{0i} = 10,\ C_{01} = 20,\ C_{02} = 40,$$

$$G_{01} = 30,\ G_{02} = 60,\ \bar{M} = 600,\ \bar{Y_1} = 200,\ \bar{Y_2} = 310$$

Furthermore, we assume the functional forms of the LM equation, the investment function and the current account function, as follows:

$$r_i = 10\sqrt{Y_i} - M_i + 160, \tag{43}$$

$$I_i = 25\sqrt{Y_i} - 0.3K_i - r_i + 160, \tag{44}$$

$$H_1 = -0.4Y_1 + 0.2Y_2. \tag{45}$$

Asada (2004) assumes that the coefficients of Y_1 and Y_2 are equal in the export function. Thus, a country with a larger national income automatically runs a current account deficit. However, Germany, which is representative of core countries in the Euro area, runs a current account surplus. Hence, we assume that the coefficients of Y_1 and Y_2 are not equal in the export function (45).

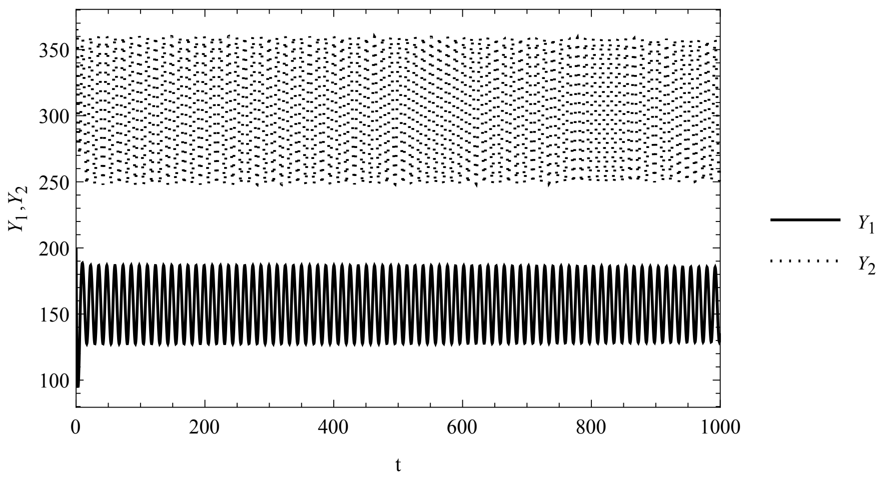
In this case, the five-dimensional dynamical system (20)–(24) becomes as follows:

$$\dot{Y_1} = 2\left\{-0.36Y_1 + 15\sqrt{Y_1} + \delta\left(-0.4Y_1 + 0.2Y_2\right) - 0.3K_1 + M_1 + \gamma_1\left(240 - Y_1\right) + 58\right\}, \tag{46}$$

$$\dot{K_1} = 15\sqrt{Y_1} - 0.3K_1 + M_1, \tag{47}$$

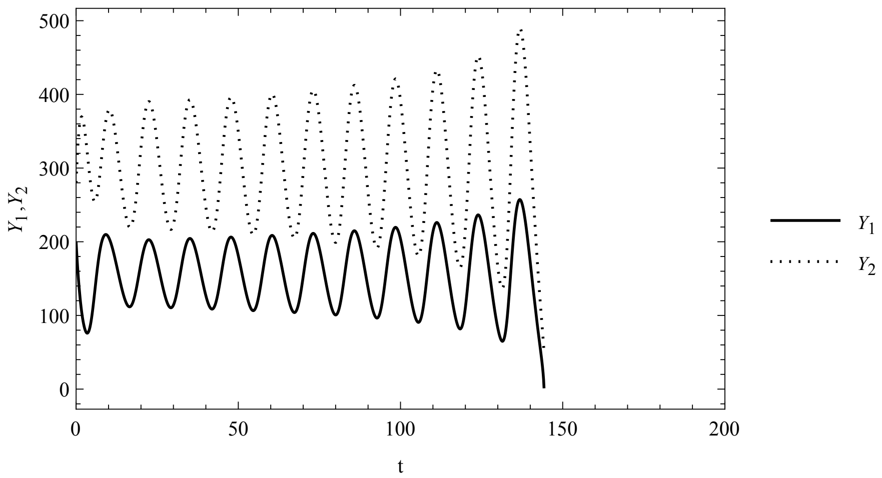
$$\dot{Y_2} = 2.5\left\{-0.36Y_2 + 15\sqrt{Y_2} - \delta\left(-0.4Y_1 + 0.2Y_2\right) - 0.3K_2 - M_1 + \gamma_2\left(310 - Y_2\right) + 708\right\}, \tag{48}$$

4) See Asada et al. (2001) and Asada (2004) for the effect of the size of parameters α_1 and α_2 on business cycles.



Note: $\beta = 0.5$, $\delta = 0.679$, $\gamma_1 = 0$, $\gamma_2 = 0$.

Figure 1: Cycle with constant amplitude



Note: $\beta = 0.8$, $\delta = 0.679$, $\gamma_1 = 0$, $\gamma_2 = 0$.

Figure 2: Increase in capital mobility

$$\dot{K}_2 = 15\sqrt{Y_2} - 0.3K_2 - M_1 + 600, \tag{49}$$

$$\dot{M}_1 = \delta\left(-0.4Y_1 + 0.2Y_2\right) + \beta\left(10\sqrt{Y_1} - 10\sqrt{Y_2} - 2M_1 + 600\right). \tag{50}$$

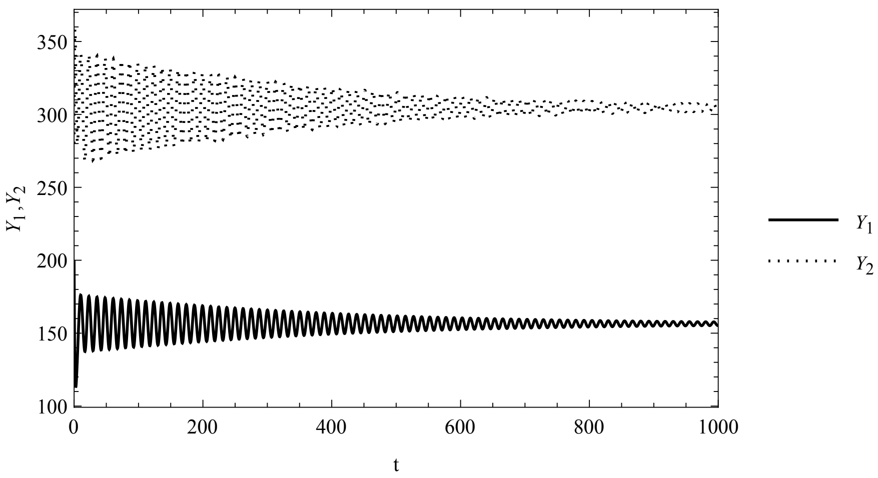
The equilibrium values of Eqs. (46)–(50) depend on the size of the parameter β, γ_1, γ_2 and δ.

Now, we compute the trajectories produced by Eqs. (46)–(50) by selecting several values of β, γ_1, γ_2 and δ and the following initial conditions of the variables:

$$Y_1(0) = 200,\ K_1(0) = 1680,\ Y_2(0) = 280,\ K_2(0) = 1830,\ M_1(0) = 280.$$

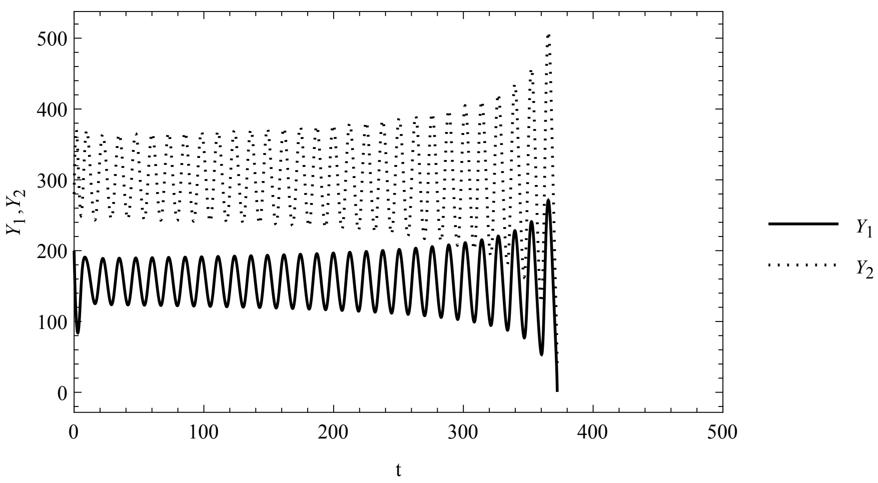
Figures 1–Figure 5 illustrate the trajectories of Y_1 and Y_2 by numerical simulation.

Figure 1 shows the case of business cycles with constant amplitude. In this case, the business cycles do not converge to the equilibrium point, whereas they do not diverge. If the parameters change only slightly, the



Note: $\beta = 0.8$, $\delta = 1$, $\gamma_1 = 0$, $\gamma_2 = 0$.

Figure 3: Increase in degree of economic openness



Note: $\beta = 5$, $\delta = 1$, $\gamma_1 = 0$, $\gamma_2 = 0$.

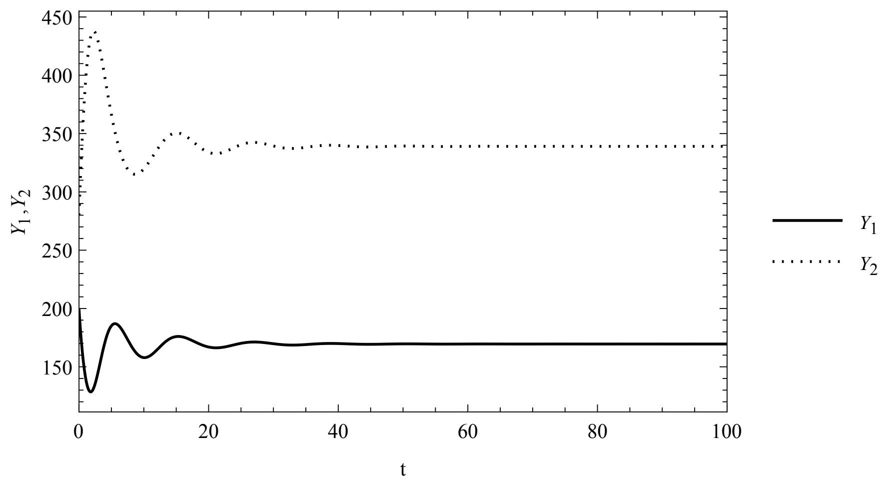
Figure 4: Greater increase in capital mobility than Figure 2

business cycles are stable or unstable.

Figure 2 indicates the case of an increase in capital mobility. This case can be conceived of as that in which parameter β increases by removing various barriers to capital movement in the Euro area. As Nakao (2017) points out, this case can also be conceived of as that in which β increases by the capital markets union in the European Union. If δ is relatively small in this situation, the business cycles are destabilized.

Figure 3 illustrates the case of an increase in the degree of economic openness under the same β in the case of Figure 2. As Proposition 2 clarifies, δ is a stabilizing factor, mitigating the instability caused by an increase in β.

Figure 4 shows the case of an increase in capital mobility. In this case, the effect of instability caused by an increase in β is larger than the effect of stability caused by large β. Moreover, countries in a monetary union cannot improve by an increase in the degree of economic openness, because δ reaches the maximum value.



Note: $\beta = 5$, $\delta = 1$, $\gamma_1 = 0.1$, $\gamma_2 = 0.1$.

Figure 5: Increase in degree of counter-cyclical fiscal policy

Table 1: Equilibrium point, money supply and net export

Case	Y_1^*	Y_2^*	M_1^*	J_2^*
Figure1 ($\beta = 0.5$, $\gamma_1 = 0$, $\gamma_2 = 0$, $\delta = 0.679$)	157.2	303.9	274.1	2.1
Figure2 ($\beta = 0.8$, $\gamma_1 = 0$, $\gamma_2 = 0$, $\delta = 0.679$)	157.2	303.9	274.6	2.1
Figure3 ($\beta = 0.8$, $\gamma_1 = 0$, $\gamma_2 = 0$, $\delta = 1$)	156.5	304.6	274.2	1.7
Figure4 ($\beta = 5$, $\gamma_1 = 0$, $\gamma_2 = 0$, $\delta = 1$)	156.5	304.6	275.1	1.7
Figure5 ($\beta = 5$, $\gamma_1 = 0.1$, $\gamma_2 = 0.1$, $\delta = 1$)	162.6	309.1	275.5	3.2

Figure 5 shows an effect of counter-cyclical fiscal policy under the same β in the case of Figure 4. This policy can stabilize business cycles fluctuations caused by an increase in β.

Table 1 summarizes equilibrium values of Figure 1–Figure 5. As shown in Table 1, the change of β and δ have little effect on the equilibrium value. In the case of Figure 5, the equilibrium value of Y_1 and Y_2 increase with counter-cyclical fiscal policy. In this model, γ_i is not limited by a ceiling, whereas δ is limited. Therefore, from the viewpoint of the stability of the business cycle and an increase in the equilibrium value, the influence of counter-cyclical fiscal policy is higher than that of the degree of economic openness.

6 Conclusion

In this study, we analyzed the effect that a high degree of openness of the economy—as one of the criteria for OCA—has on business cycles using a Kaldorian two-country model with a monetary union and imperfect capital mobility. The results of this study indicate that an increase in the capital mobility between two countries is a destabilizing factor, while a high degree of openness of the economy and a counter-cyclical fiscal policy is stabilizing factor. A monetary union causes an increase in capital mobility in the currency area. To offset the instability caused by an increase in capital mobility, other stabilizing factors are required.

The OCA theory has two key features. The first feature is whether prevention of the occurrence of shock is possible. The second feature is whether an adjustment is possible after a shock occurs. However, there is a competing view about the relationship between the degree of economic openness and prevention of asymmetric shock. European Commission (1990) consider that a high degree of economic openness prevents an asymmetric shock, whereas Krugman (1991) consider that it causes an asymmetric shock.

It is interpreted from the result of this study that a high degree of economic openness can adjust a shock in the monetary union regardless of whether the shock is asymmetric or not. Therefore, it seems natural to conclude that the criterion of degree of economic openness serves as one of the criteria fjor OCA, even if Krugman's view is correct and an asymmetric shock tends to occur.

However, there are upper limits to the degree of economic openness in previous simulations, as shown in the simulations. The Euro area increases the degree of economic openness by the creation of a customs union and a single market. Thus, it is difficult for the euro area to increase the degree of economic openness to deal with the euro crisis. It should be mentioned that a counter-cyclical fiscal policy is important to this problem in that such a policy is a stabilizing factor.

One of the limitations of this study is that we eliminate price fluctuations. Therefore, a further study of a model that includes the inflation rate and expected inflation rate should be constructed. Despite some remaining challenges, this study contributes to the literature on economic stability in economies with a high degree of openness.

Appendix A Proof of Proposition 2

Proof.

(i) Suppose that parameter β is fixed at any level. If parameter γ_1 is sufficiently large, then $a_1 > 0$. Second, if parameters γ_2 and δ are constant, we can express Eq. (32) as a function with respect to γ_1 as follows.

$$a_1 = b_1\gamma_1 + b_2 \tag{A.1}$$

Then, if parameter γ_1 is sufficiently large, we have $a_1 > 0$ because of $b_1 > 0$. We obtain $a_2 > 0$, $a_3 > 0$, $a_4 > 0$ and $a_5 > 0$ in a similar way.

Furthermore, $\Delta_i > 0\,(i = 1, 2, ..., 5)$ is a function with respect to γ_1. The coefficient of the highest order of each equation is positive. As a result, $\Delta_i > 0$ if parameter γ_1 is sufficiently large. In a similar way, $\Delta_i > 0$ in the case in what γ_2 is sufficiently large.

If the parameters γ_1 and γ_2 are constant, a_1 is a linear equation with respect to δ. The coefficient of δ is positive. Thus, $a_1 > 0$, if parameter δ is sufficiently large. For the same reason, we obtain $a_2 > 0$, $a_3 > 0$, $a_4 > 0$, and $a_5 > 0$, if the parameters δ are sufficiently large.

Moreover, Δ_1 is a linear equation, Δ_2 is a quadratic equation, Δ_3 is a quintic equation, Δ_4 is a seventh degree equation and Δ_5 is a ninth degree equation with respect to δ. The coefficient of the highest order of each equation is positive. As a result, we have $\Delta_i > 0\,(i = 1, 2, ..., 5)$ if parameter δ is sufficiently large.

From the above, the equilibrium point of the system (20)–(24) is locally stable, because the Routh-Hurwitz conditions are satisfied, if at least one of the parameters γ_1, γ_2, and δ is sufficiently large.

(ii) Suppose that the parameters γ_1, γ_2 and δ are relatively small and inequalities $\Phi_{11} > 0$ and $\Phi_{33} > 0$ hold. If parameter β is sufficiently large, $V_{51}(\beta) > 0$ and $V_{53}(\beta) < 0$. Then, Δ_2 is a quadratic equation with respect to β. The coefficient of β squared term is negative. Therefore, $\Delta_2 < 0$ if parameter β is sufficiently large. From the above, the equilibrium point of the system (20)–(24) is locally unstable, because the Routh–Hurwitz conditions are not satisfied in the case of sufficiently large β.

$\square$

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