Discussion Paper No.284

Regulation of Non-point Source Pollution under n-firm Bertrand Competition

Toshiharu Ishikawa
Chuo University

Akio Matsumoto
Chuo University

Ferenc Szidarovszky
University of Pécs

October 2017

INSTITUTE OF ECONOMIC RESEARCH
Chuo University
Tokyo, Japan

Regulation of Non-point Source Pollution under n-firm Bertrand Competition*

Toshiharu Ishikawa,† Akio Matsumoto,‡ Ferenc Szidarovszky§

Abstract

This study demonstrates the conditions under which an increase in the ambient charge positively or negatively affects the total level of non-point source pollutions. For this purpose, an n-firm Bertrand framework is used in which goods are differentiated and the corresponding price functions are linear. It is shown first that the effect is definitely negative in duopoly and triopoly and second that, for $n \geq 4$, the sign of the effect depends on the number of the firms involved and the degree of substitutability.

Keywords: Non-point source pollution, Environmental polity, Ambient charge, n-firm Bertrand competition, Good-natured effect, Perverse effect.

---

*The authors highly acknowledge the financial supports from the MEXT-Supported Program for the Strategic Research Foundation at Private Universities 2013-2017, the Japan Society for the Promotion of Science (Grant-in-Aid for Scientific Research (C), 16K03556 and 17k03712) add Chuo University (Joint Research Grant). The usual disclaimer applies.

†Professor, Department of Economics, Senior Researcher, International Center for further Development of Dynamic Economic Research, Chuo University, 742-1, Higashi-Nakano, Hachioji, Tokyo, 192-0393, ishiy@tamacc.chuo-u.ac.jp

‡Professor, Department of Economics, Senior Researcher, International Center for further Development of Dynamic Economic Research, Chuo University, 742-1, Higashi-Nakano, Hachioji, Tokyo, 192-0393, akiom@tamacc.chuo-u.ac.jp

§Professor, Department of Applied Mathematics, University of Pécs, Ifjúság u. 6., H-7624, Pécs, Hungary, szidarka@gmail.com

1 Introduction

Concerning non-point source (NPS) pollution, the regulator can measure the ambient concentration of pollutants, knows all potential polluters involved in emissions, however, is unable to identity individual contributions to the total pollution. It could be very difficult or costly to monitor at the point of origin as well as to measure the individual emissions with sufficient accuracy. In consequence of the informational asymmetries between the regulator and the polluters, traditional environmental policy instruments such as emission taxes or tradable quotas cannot be used to regulate NPS pollution. An ambient charge or tax, advocated by Segerson (1988), is an environmental policy instrument designed to link with observation of NPS pollution. The regulator announces the charge or tax and a cut-off level of ambient concentration. If the deviation between the observed level and the cut-off level is positive, then all potential polluters pay a tax proportional to the deviation. If negative, then they receive a subsidy of a fixed proportion of the deviation. This study focuses attention on whether the ambient charge scheme can control NPS pollution under imperfect competition.

In a Cournot framework, it has been demonstrated that an increase of the ambient charge decreases the total level of NPS pollution, which will be called a "good-natured" effect. See Raju and Ganguli (2012) for a duopoly case and Matsumoto et al. (2017a) for an n-firm case. This finding indicates that environmental policy such as the ambient charge can control NPS pollutions in a Cournot market or industry. Furthermore, Ganguli and Raju (2013) study the ambient charge effect in a Bertrand duopoly and numerically exhibit a "perverse" effect that an increase in the ambient charge may lead to larger NPS pollutions in ála Cournot market. On the other hand, Matsumoto et al. (2017b) analytically show an opposite result in the same duopoly setting that a higher charge can decrease the total level of pollutions. In the existing literature, however, it has not been revealed yet whether the ambient charge can be effective in a n-firm Bertrand market.

The main purpose of this study is to demonstrate the economic circumstances under which the ambient charge effect can be good-natured or perverse. As in the standard literature, we employ a n-firm linear model with product differentiation. Despite the simplicity, however, it is not easy to determine the direction of the effect because derivatives of the total pollution level with respect to the ambient charge have cumbersome forms. In order to deal with this difficulty, we introduce a special function in parameters expressing the efficiency of the abatement technology of each firm. We then demonstrate the conditions under which the ambient charge effect is positive or negative. In particular, the followings are the main results:

(1) the effect is negative in duopoly and triopoly

(2) in the Bertrand market, the sign of the effect depends on the number of the firms involved and the degree of substitutability in oligopoly with $n \geq 4$.

In what follows, we introduce a basic model in Section 2 and discuss our main results in Section 3. Concluding remarks are given in Section 4.

2 The model

The direct demand function of firm k is

$$q_k = a - p_k + \gamma \sum_{i \neq k} p_i \text{ for } k = 1, 2, ..., n, \tag{1}$$

where $a > 0$ and γ is a parameter measuring substitutability between good k and other goods. Goods are supposed to be homogenous or perfectly substitutes if $\gamma = 1$ and independent if $\gamma = 0$. To get rid of these two extreme cases, we impose the following condition.

Assumption 1. $0 < \gamma < 1$.

The profit function of firm k is

$$\pi_k = p_k q_k - c q_k - t \left(\phi_k q_k + \sum_{i \neq k} \phi_i q_i - \bar{E} \right) \tag{2}$$

where $c > 0$ is the identical marginal cost of production, ϕ_k is pollution abatement technology for firm k. It is assumed that $0 \leq \phi_k \leq 1$ where $\phi_k = 1$ means that firm k has the worst technology and discharges 100% pollution associated with its production and $\phi_k = 0$ means that it has the best technology and discharges no (0%) pollution. In addition, t is the ambient charge with $0 \leq t \leq 1$ and $\bar{E}$ is the ambient standard level, both are exogenously determined by the regulator. The total level of pollution E generated by all firms is given by

$$E = \sum_{i=1}^{n} \phi_i q_i.$$

According to the spirit of the ambient charge, although each firm's emission is different, each firm will pay the same fine $t\left(E - \bar{E}\right)$ if $E - \bar{E} > 0$ and receive the same subsidy $t\left(\bar{E} - E\right)$ if $E - \bar{E} < 0$. Substituting (1) into (2) presents the profit in terms of prices,

$$\pi_k = (p_k - c)\left(a - p_k + \gamma \sum_{i \neq k} p_i \right) - t \left[\phi_k \left(a - p_k + \gamma \sum_{i \neq k} p_i \right) + \sum_{i \neq k} \phi_i \left(a - p_i + \gamma \sum_{j \neq i} p_j \right) - \bar{E} \right]. \tag{3}$$

Differentiating (3) with respect to p_k and assuming an interior solution yields the first order condition for the profit-maximizing firm k for $k = 1, 2, ..., n$,

$$\frac{d\pi_k}{dp_k} = a + c - 2p_k + \gamma \sum_{i \neq k} p_i + t \left[\phi_k - \gamma \sum_{i \neq k} \phi_i \right] = 0$$

which can be rewritten as

$$2p_k - \gamma \sum_{i \neq k} p_i = a + c + t \left[\phi_k - \gamma \sum_{i \neq k} \phi_i \right]. \tag{4}$$

The FOCs for all firms are summarized in vector form,

$$\boldsymbol{B}\boldsymbol{p} = \boldsymbol{A}$$

where, for $i, j = 1, 2, ..., n$,

$$\boldsymbol{B} = (B_{ij})_{(n,n)} \text{ with } B_{ii} = 2 \text{ and } B_{ij} = -\gamma \ (i \neq j)$$

and

$$\boldsymbol{p} = (p_k)_{(n,1)} \text{ and } \boldsymbol{A} = \left(a + c + t\left[\phi_k - \gamma \sum_{i \neq k} \phi_i\right]\right)_{(n,1)}.$$

Here we assume the following, otherwise $|\boldsymbol{B}| = 0$ and we lose invertibility of matrix $\boldsymbol{B}$.

Assumption 2. $\gamma \neq \dfrac{2}{n-1}$

Under Assumption 2, the Bertrand price vector is given by

$$\boldsymbol{p} = \boldsymbol{B}^{-1}\boldsymbol{A}$$

where the diagonal and off-diagonal elements of inverse matrix $\boldsymbol{B}^{-1} = (b_{ij})$ are

$$b_{ii} = \frac{(n-2)\gamma - 2}{(2+\gamma)\left[(n-1)\gamma - 2\right]} \text{ and } b_{ij} = -\frac{\gamma}{(2+\gamma)\left[(n-1)\gamma - 2\right]}.$$

Hence the Bertrand price of firm k for $k = 1, 2, ..., n$ is

$$p_k^B = \frac{-(2+\gamma)(a+c) + t\left\{\left[(n-1)\gamma^2 + (n-2)\gamma - 2\right]\phi_k + \gamma \sum_{i \neq k} \phi_i\right\}}{(2+\gamma)\left[(n-1)\gamma - 2\right]}.$$

Introducing new variables to simplify the expression of the Bertrand prices

$$p_k^B = \frac{1}{K}\left\{-(2+\gamma)(a+c) + t\left[N\phi_k + \gamma \sum_{i \neq k} \phi_i\right]\right\} \tag{5}$$

with

$$K = (2+\gamma)\left[(n-1)\gamma - 2\right] \text{ and } N = (n-1)\gamma^2 + (n-2)\gamma - 2.$$

Substituting (5) into (1) presents the Bertrand output of firm k,

$$q_k^B = a - p_k^B + \gamma \sum_{i \neq k} p_i^B \text{ for } k = 1, 2, ..., n \tag{6}$$

and the total emission level at the Bertrand equilibrium is

$$E^B = \sum_{k=1}^{n} \phi_k q_k^B. \tag{7}$$

We discussed the conditions to guarantee that both p_k^B and q_k^B are positive in the Appendix.

3 Ambient Charge Effects

The effect of an increase in the ambient charge on q_k^B is obtained by differentiating (6) with respect to t,

$$\frac{dq_k^B}{dt} = -\frac{dp_k^B}{dt} + \gamma \sum_{i \neq k} \frac{dp_i^B}{dt}$$

where

$$\frac{dp_k^B}{dt} = \frac{1}{K}\left(N\phi_k + \gamma \sum_{i \neq k} \phi_i\right) \text{ and } \frac{dp_i^B}{dt} = \frac{1}{K}\left(N\phi_i + \gamma \sum_{j \neq i} \phi_j\right).$$

Arranging the terms yields the following form of the ambient charge effect on the Bertrand production:

$$\frac{dq_k^B}{dt} = \frac{1}{K}\left(N_1\phi_k + N_2 \sum_{i \neq k} \phi_i\right) \tag{8}$$

with

$$N_1 = 2 - (n-2)\gamma \text{ and } N_2 = (n-1)\gamma^3 + 2(n-2)\gamma^2 - 3\gamma.$$

Hence differentiating (7) with respect to t and then substituting (8) into the derivative yields, after arranging the terms, the effect on the total pollution in the following form,

$$\frac{dE^B}{dt} = \frac{1}{K}\left[N_1 \sum\nolimits_{k=1}^{n} \phi_k^2 + N_2 \sum\nolimits_{k=1}^{n} \sum\nolimits_{i \neq k}^{n} \phi_k\phi_i\right] \tag{9}$$

Before proceeding to determine the sign of the ambient charge effect, we provide a useful relation about the relative order of magnitudes of

$$\sum_{k=1}^{n}\sum_{i \neq k}^{n} \phi_k\phi_i \text{ and } \sum_{k=1}^{n} \phi_k^2 \tag{10}$$

which can be proved as follows. Using the relation $2ab \leq a^2 + b^2$ due to $(a-b)^2 \geq 0$,

$$\begin{aligned}
& \sum_{k=1}^{n}\sum_{i \neq k}^{n} \phi_k\phi_i \\
= \; & (\phi_1\phi_2 + \cdots + \phi_1\phi_n) + (\phi_2\phi_1 + \phi_2\phi_3 \cdots + \phi_2\phi_n) + \cdots + (\phi_n\phi_1 + \cdots + \phi_n\phi_{n-1}) \\
\leq \; & \frac{1}{2}\left(\phi_1^2 + \phi_2^2 + \cdots + \phi_1^2 + \phi_n^2\right) + (\phi_2^2 + \phi_1^2 + \phi_2^2 + \phi_3^2 + \cdots + \phi_2^2 + \phi_n^2) + \cdots + (\phi_n^2 + \phi_1^2 + \cdots + \phi_n^2 + \phi_{n-1}^2) \\
= \; & \frac{1}{2} 2(n-1) \sum_{k=1}^{n} \phi_k^2 \\
= \; & (n-1) \sum_{k=1}^{n} \phi_k^2.
\end{aligned}$$

Therefore we have

$$\sum_{k=1}^{n}\sum_{i \neq k}^{n} \phi_k\phi_i \leq (n-1) \sum_{k=1}^{n} \phi_k^2 \tag{11}$$

where equality holds if and only if $\phi_1 = \phi_2 = \cdots = \phi_n$. Next dividing the first term in (10) by the second term gives the ratio that goes to zero if ϕ_k is fixed and all other $\phi_i \to 0$,

$$\frac{\sum_{k=1}^{n}\sum_{i\neq k}^{n}\phi_k\phi_i}{\sum_{k=1}^{n}\phi_k^2} \to 0. \tag{12}$$

Since this ratio is continuous in the ϕ_i values, we have the following result with (12) and (11).

Theorem 1 *For any number $M \in (0, n-1]$ that depends on the ϕ_k values, we have*

$$\sum_{k=1}^{n}\sum_{i\neq k}^{n}\phi_k\phi_i = M\sum_{k=1}^{n}\phi_k^2. \tag{13}$$

Now with this result, we proceed to determine the sign of (9). For sake of notational simplicity,

$$A = \sum_{k=1}^{n}\sum_{i\neq k}^{n}\phi_k\phi_i > 0 \text{ and } B = \sum_{k=1}^{n}\phi_k^2 > 0 \tag{14}$$

with which (9) can be written as

$$\frac{dE^B}{dt} = \frac{B(N_1 + MN_2)}{K}. \tag{15}$$

Solving $N_2 = 0,\ K = 0$ and $N_1 = 0$ for γ presents the following threshold values,

$$\gamma_n^1 = \frac{(2-n)+\sqrt{n^2-n+1}}{n-1},\ \gamma_n^2 = \frac{2}{n-1} \text{ and } \gamma_n^3 = \frac{2}{n-2}$$

where $N_2,\ K$ and N_1 were defined earlier. The followings in the (n,γ) plane are clear,

$$N_2 \gtreqless 0 \text{ according to } \gamma \gtreqless \gamma_n^1,$$

$$K \gtreqless 0 \text{ according to } \gamma \gtreqless \gamma_n^2,$$

$$N_1 \gtreqless 0 \text{ according to } \gamma \lesseqgtr \gamma_n^1$$

and

$$0 < \gamma_n^1 < \gamma_n^2 < \gamma_n^3.$$

Furthermore,

$$\gamma_n^2 < 1 \text{ as } n > 3 \text{ and } \gamma_n^3 < 1 \text{ as } n > 4.$$

Consider now $N_1 + MN_2$ as function of M and γ and denote it as $g(M,\gamma)$. It can be checked that

$$g(0,\gamma) = N_1 \text{ at } M = 0$$

and

$$g(n-1,\gamma) = (2+\gamma)\left[(n-1)\gamma - 1\right]^2 \text{ at } M = n-1,$$

where $\gamma_n^0 = 1/(n-1)$ is the minimum point of $g(n-1,\gamma)$ for $\gamma > 0$ and therefore

$$g(n-1,\gamma) \begin{cases} > 0 \text{ if } \gamma > 0 \text{ and } \gamma \neq \gamma_n^0, \\ = 0 \text{ if } \gamma = \gamma_n^0. \end{cases}$$

It is easy to see that $\gamma_n^1 > \gamma_n^0$. We denote the feasible region of γ by I that is, under Assumptions 1 and 2, defined as

$$I = (0, \gamma_n^2) \cup (\gamma_n^2, 1)$$

and divide it to four subintervals with the threshold values of γ defined above:

$$I_1 = (0, \gamma_n^1],\ I_2 = \left(\gamma_n^1, \gamma_n^2\right),\ I_3 = (\gamma_n^2, \gamma_n^3] \text{ and } I_4 = \left(\gamma_n^3, 1\right)$$

with $(0, \gamma_n^2) = I_1 \cup I_2$ and $(\gamma_n^2, 1) = I_3 \cup I_4$.

In each interval, the direction of the ambient charge effect is determined as follows.

(I). In interval I_1, $N_1 > 0$, $N_2 < 0$ and $K < 0$. $g(M,\gamma)$ decreases in M, $g(0,\gamma) > 0$ and $g(n-1,\gamma) > 0$ except $\gamma = \gamma_n^0$. Hence

$$\frac{dE^B}{dt} \begin{cases} = 0 \text{ if } M = n-1 \text{ and } \gamma = \gamma_n^0, \\ < 0 \text{ otherwise.} \end{cases}$$

The equality holds on the dotted downward curve shown in Figure 2, the curve on which $\gamma = \gamma_n^0$ holds. At the right end point, $\gamma = \gamma_n^1$ that is on the lowest solid downward curve, $N_1 > 0$ and $N_2 = 0$, so with all values of M, $N_1 + N_2 M = N_1$. Hence

$$\frac{dE^B}{dt} = \frac{BN_1}{K} < 0.$$

(II). In interval I_2, $N_1 > 0$, $N_2 > 0$ and $K < 0$. For all values of $M \in (0, n-1]$, $N_1 + MN_2 > 0$ implying that

$$\frac{dE^B}{\partial t} = \frac{B(N_1 + MN_2)}{K} < 0,$$

since the numerator is positive and the denominator is negative. We already exclude, by Assumption 2, the value of $\gamma_n^2 = 2/(n-1)$, which is on the middle solid downward curve that is also the boundary between the yellow and orange regions in Figure 2.

(III). In interval I_3, $N_1 > 0$, $N_2 > 0$, $K > 0$ and $M > 0$, so

$$\frac{dE^B}{dt} = \frac{B(N_1 + MN_2)}{K} > 0,$$

since both the numerator and denominator are positive. At the right end point $\gamma = \gamma_n^3$, which is on the upper solid downward curve, $N_1 = 0$ and $N_2 > 0$, so for all positive values of M,

$$\frac{dE^B}{dt} = \frac{BMN_2}{K} > 0.$$

(IV). In interval I_4, $N_1 < 0$, $N_2 > 0$ and $K > 0$, so $g(0,\gamma) < 0$ and $g(n-1,\gamma) > 0$. Therefore there is a unique value of $M = M_0$ such that $g(M_0,\gamma) = 0$. Then

$$\frac{dE^B}{dt} \begin{cases} > 0 \text{ if } M > M_0, \\ = 0 \text{ if } M = M_0, \\ < 0 \text{ if } M < M_0, \end{cases}$$

where

$$M_0 = \frac{(n-2)\gamma - 2}{[(n-1)\gamma^2 + 2(n-2)\gamma - 3]\,\gamma}. \tag{16}$$

If $dE^B/dt < 0$, then E^B strictly decreases in t, if the inequality is reversed, then E^B strictly increases in t and if $dE^B/dt = 0$, then the total emission output does not depend on t, that is, the ambient charge has no effect on the total emission output of the industry. We can summarize the above derivations as follows:

Theorem 2 *Under Assumptions 1 and 2, we have the following ambient charge effect on the total level of pollution:*

(*a*) E^B *strictly increases in* t *if* $\gamma_n^2 < \gamma \leq \gamma_n^3$ *or* $\gamma_n^3 < \gamma < 1$ *and* $M > M_0$,

(*b*) E^B *strictly decreases in* t *if* $0 < \gamma < \gamma_n^2$ *except* $\gamma = \dfrac{1}{n-1}$ *and* $M = n-1$ *or* $\gamma_n^3 < \gamma < 1$ *and* $M < M_0$,

(*c*) E^B *does not depend on* t *if* $\gamma = \dfrac{1}{n-1}$ *and* $M = n-1$ *or* $\gamma_n^3 < \gamma < 1$ *and* $M = M_0$.

The first condition of Theorem 2(b) holds in the interior of the yellow region except the dotted curve and so does the first half of Theorem 2(a) in the orange region in Figure 1. As will be discussed in the Appendix, p_k^B and q_k^B can be negative for some combinations of (n,γ) under given specification of a, c, t and ϕ_k. To consider these non-negative conditions may introduce unnecessary complication into Figure 1. Thus, at the expense of accuracy, we eliminate these conditions from Figure 1.

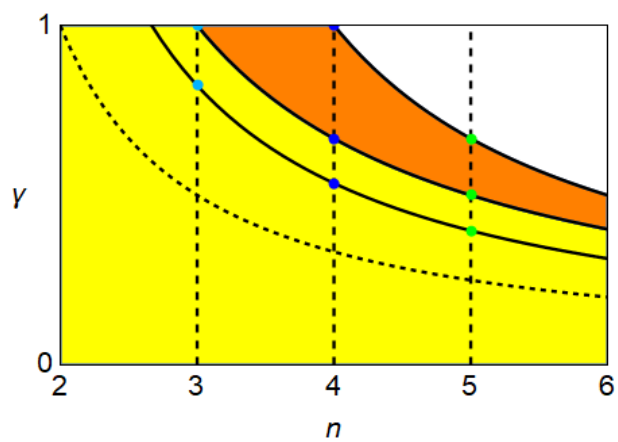


Figure 1. Division of the (n,γ) plane

From Theorem 2, we can easily derive the following results.

Corollary 1 *In Bertrand duopoly and triopoly, the ambient charge effect is always negative,*

$$\frac{dE^B}{dt} < 0.$$

Proof. For $n = 2$, since $\gamma_2^1 = \sqrt{3}$, $I \subset I_1$. For $n = 3$, since $\gamma_3^2 = 1$, $I = I_1 \cup I_2$. Theorem 2(b) implies that E^B strictly decreases in t. ■

Notice that point $(2, 1)$ is on the dotted curve as $\gamma_2^0 = 1$ in duopoly and that the vertical dotted line at $n = 3$ crosses the lower solid curve at the lower light blue point at $(3, \gamma_3^1)$ with $\gamma_3^1 \simeq 0.823$ and the middle solid curve at the upper light blue point $(3, \gamma_3^2)$ with $\gamma_3^2 = 1$ as seen in Figure 1.

Corollary 2 *In Bertrand quadropoly, the domain of γ is divided into two parts $I_1 \cup I_2$ and I_3 where*

$$\frac{dE^B}{dt} < 0 \text{ for } t \in I_1 \cup I_2 \text{ and } \frac{dE^B}{dt} > 0 \text{ for } t \in I_3.$$

Proof. For $n = 4$, the vertical dotted line at $n = 4$ crosses the middle solid curve (i.e., $K = 0$) at $\gamma_4^2 = 2/3$, the end point of I_2 and the upper solid curve at $\gamma_4^3 = 1$, the end point of I_3. Theorem 2(a) and (b) imply the positive effect for $t \in I_3$ and the negative effect for $t \in I_1 \cup I_2$, respectively, where $I_1 \cup I_2 \cup I_3 = I$. ■

This corollary indicates that the perverse effect shown by Ganguli and Raju (2012) is possible for $n = 4$. For $n \geq 5$, as is seen in Figure 1, the interval I_4 is not empty, implying a possibility of the perverse effect.

Corollary 3 *For $n \geq 5$, the domain I is divided into four subintervals where*

$$\frac{dE^B}{dt} < 0 \text{ for } t \in I_1 \cup I_2 \text{ and } \frac{dE^B}{dt} > 0 \text{ for } t \in I_3$$

and for $t \in I_4$,

$$\frac{dE^B}{dt} < 0 \text{ for } M < M_0,\ \frac{dE^B}{dt} = 0 \text{ for } M = M_0 \text{ and } \frac{dE^B}{dt} > 0 \text{ for } M > M_0.$$

Proof. The first half is clear from Theorem 2. The second half is also clear from our earlier discussion about interval I_4. ■

The second result of Corollary 3 is illustrated in Figure 2 with $n = 5, 6, 7, 8, 12$ and infinity. Each M_0-curve is defined for $\gamma \geq \gamma_n^3$ and divides the (γ, M) plane into two parts: the ambient charge is good-nature (i.e., $dE^B/dt < 0$) below the curve and perverse (i.e., $dE^B/dt > 0$) above. These corollaries imply that the perverse effect discussed by Ganguli and Raju (2012) is impossible for

$n \leq 3$ and possible for $n \geq 4$. As n increases, the threshold value of γ_n^3 becomes smaller and the M_0-curve shifts upward, implying that a possibility of the perverse effect becomes less. The threshold values of γ are

$$\gamma_5^3 = \frac{2}{3},\ \gamma_6^3 = \frac{1}{2},\ \gamma_7^3 = \frac{2}{5},\ \gamma_8^3 = \frac{1}{3},\ \text{and}\ \gamma_{12}^3 \simeq= \frac{1}{5}.$$

Notice that the M_0-curve with $n = \infty$ is illustrated as the dotted curve. In Figure 2, 1/7 and 1/3 on the right vertical line are the M_0-values with $n = 5$ and $n = \infty$ for $\gamma = 1$, respectively. Any other M_0-values for $\gamma = 1$ are possible to obtain.

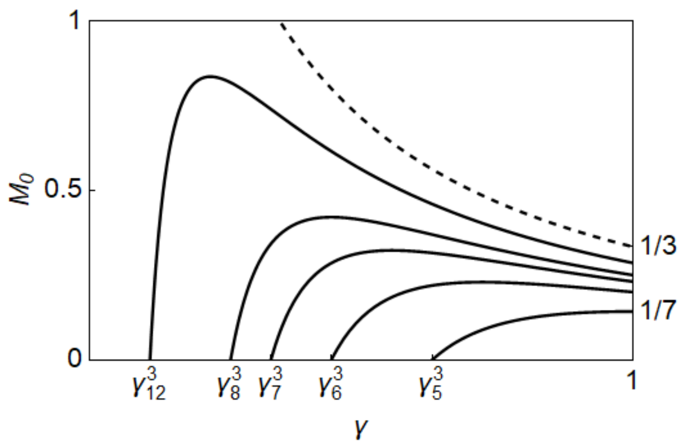


Figure 2. M_0-curves with $n = 5, 6, 7, 8, 12$ and ∞

4 Concluding Remarks

We have examined the ambient charge effect on the observable concentration of pollutants under Bertrand competition. The main result was summarized in Theorem 2. We found that the ambient charge definitely regulates the NPS pollutions in duopoly and triopoly markets. If the number of the firms in the Bertrand market increases, we demonstrated that controllability of the ambient charge depends on the substitutability of the differentiated goods.

Appendix

In this Appendix, we examine the conditions under which p_k^B and q_k^B are positive.

- Conditions for positive p_k^B

Equation (5) can be rewritten as

$$Kp_k^B = t(n-1)\phi_k\gamma^2 + \{t[(n-3)\phi_k + \phi] - (a+c)\}\gamma - 2[t\phi_k + (a+c)] \tag{A-1}$$

with $\phi = \sum_{i=1}^n \phi_i$. Notice that the right hand side of (A-1) is quadratic in γ. Since its leading term is positive and its constant term is negative, there are two roots for γ, one is negative and the other is positive that is denoted by $\bar{\gamma}$.

$$Kp_k^B < 0 \text{ if } \gamma < \bar{\gamma} \text{ and } Kp_k^B > 0 \text{ if } \gamma > \bar{\gamma}.$$

By the definition of K,

$$K < 0 \text{ if } \gamma < \gamma^* = \frac{2}{n-1} \text{ and } K > 0 \text{ if } \gamma > \gamma^*$$

where γ^* is denoted by γ_n^2 in Section 3. Therefore we have

$$p_k^B > 0 \text{ if either } \gamma < \min[\bar{\gamma}, \gamma^*] \text{ or } \gamma > \max[\bar{\gamma}, \gamma^*] \tag{A-2}$$

and

$$p_k^B < 0 \text{ if } \min[\bar{\gamma}, \gamma^*] < \gamma < \max[\bar{\gamma}, \gamma^*]. \tag{A-3}$$

To check the relative magnitude between $\bar{\gamma}$ and γ^*, we denote the right hand side of (A-1) as $S(n,\gamma)$ and substitute γ^* into it,

$$S(n, \gamma^*) = -\frac{2}{n-1}[(a+c)n - t\phi].$$

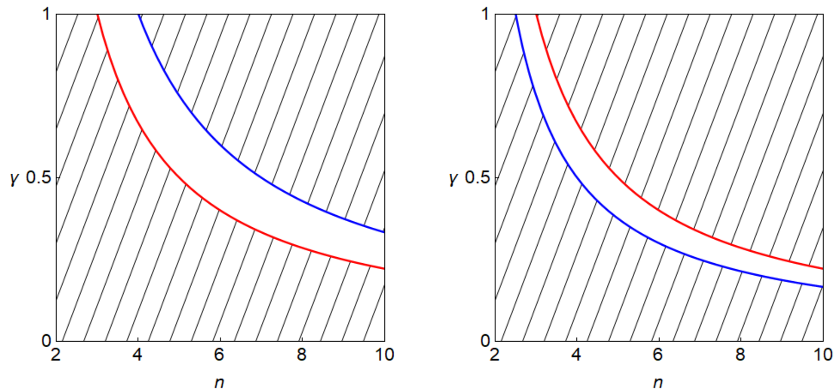
where two inequality conditions, $\phi \le n$ as $0 \le \phi_k \le 1$ and $0 \le t \le 1$, imply

$$(a+c)n - t\phi \ge (a+c)n - n = (a+c-1)n.$$

Hence, if $a+c > 1$, then $S(n,\gamma) > 0$ on the $K = 0$ locus, implying that the $S(n,\gamma) = 0$ locus is located above the $K = 0$ locus. It is also supposed that the $S(n,\gamma) = 0$ locus is located below if $a+c$ takes small values. In Figure A with $\phi_k = 2/3$ and $t = 1/2$, we visualize these relations in which the red curve corresponds to the $K = 0$ locus and the blue curve to the $S(n,\gamma) = 0$ locus. In Figure A(I) where $\gamma^* < \bar{\gamma}$ holds, $p_k^B > 0$ in the lower shaded region below the red curve and the upper shaded region above the blue curve. On the other hand, $p_k^B < 0$ in the white region between the red and blue curves. In the same way, $p_k^B > 0$ in the shaded regions and $p_k^B < 0$ in the white region in Figure A(II) where $\bar{\gamma} < \gamma^*$ holds. For the current specification of the parameters, the blue curve is descrived by

$$\gamma = \frac{1+3(a+c)}{n-1}$$

that becomes identical with the red curve if $a + c = 1/3$.



(I) $a + c = 2$ (II) $a + c = 1/6$

Figure A. Shaded regions for positive p_k^B and white region for negative p_k^B

- Conditions for positive q_k^B

Consider next the positivity of output. Multiplying K to both sides of (6) gives

$$Kq_k^B = Ka - Kp_k^B + \gamma \sum_{i \neq k} Kp_i^B \tag{A-4}$$

Substituting the terms in the bracket of (5) reduces the right hand side of (A-4) to a cubic equation in γ,

$$Kq_k^B = a_3\gamma^3 + a_2\gamma^2 + a_1\gamma + a_0$$

where

$$\begin{aligned} a_3 &= t(n-1)(\phi - \phi_k) > 0, \\ a_2 &= -(n-1)c + 2t(n-2)(\phi - \phi_k), \\ a_1 &= -(a+c) - 2c(n-2) - t\left[3(\phi - \phi_k) + (n-2)\phi_k\right] < 0 \text{ for } n \geq 2, \\ a_0 &= 2\left(c - a + t\phi_k\right). \end{aligned}$$

According to Descartes' rule of signs, this cubic equation, regardless of the sign of a_2, has one positive root if $a_0 < 0$ since the number of sign differences between consecutive coefficients is one while it has zero or two positive roots including the identical roots if $a_0 > 0$ since the number is two. Let us denote this function by $f(\gamma)$. In the case of $a_0 < 0$, suppose that $\tilde{\gamma}$ solves $f(\gamma) = 0$. Then

$$Kq_k^B = f(\gamma) \lesseqgtr 0 \text{ if } \gamma \lesseqgtr \tilde{\gamma}.$$

Hence we have

$$q_k^B > 0 \text{ if either } \gamma < \min\left[\tilde{\gamma}, \gamma^*\right] \text{ or } \gamma > \max\left[\tilde{\gamma}, \gamma^*\right] \tag{A-5}$$

and

$$q_k^B < 0 \text{ if } \min\left[\tilde{\gamma}, \gamma^*\right] < \gamma < \max\left[\tilde{\gamma}, \gamma^*\right]. \tag{A-6}$$

In the case of $a_0 > 0$ and $f(\gamma) > 0$ for all $\gamma > 0$, we have

$$q_k^B > 0 \text{ if } \gamma > \gamma^* \text{ and } q_k^B < 0 \text{ if } \gamma < \gamma^*. \tag{A-7}$$

In the case of $a_0 > 0$ and $f(\gamma) = 0$ has two positive solutions, $0 < \tilde{\gamma}_1 < \tilde{\gamma}_2$,

$$Kq_k^B = f(\gamma) < 0 \text{ if } \tilde{\gamma}_1 < \gamma < \tilde{\gamma}_2 \text{ and } Kq_k^B = f(\gamma) > 0 \text{ if } \gamma < \tilde{\gamma}_1 \text{ or } \tilde{\gamma}_2 < \gamma.$$

Hence we have

$$q_k^B > 0 \text{ if } \gamma > \gamma^* \text{ and } \gamma < \tilde{\gamma}_1 \text{ or } \tilde{\gamma}_2 < \gamma \text{ or if } \gamma < \gamma^* \text{ and } \tilde{\gamma}_1 < \gamma < \tilde{\gamma}_2. \tag{A-8}$$

- p_k^B, q_k^B, π_k^B and E^B along $\gamma = 1/(n-1)$

  We can check how p_k^B, q_k^B, π_k^B and E^B look like if γ goes to γ^* where

$$\text{as } \gamma \to \gamma^*, \begin{cases} N = (n-1)\gamma^2 + (n-2)\gamma - 2 \to \gamma^*, \\ K = (2+\gamma)\left[(n-1)\gamma - 2\right] \to \begin{cases} +0 \text{ if } \gamma > \gamma^*, \\ -0 \text{ if } \gamma < \gamma^*. \end{cases} \end{cases}$$

From (5) and these results, we further have

$$Kp_k^B \to -\frac{2}{n-1}\left[(a+c)n - t\phi\right] = Z$$

implying that

$$p_k^B \to \begin{cases} +\infty \text{ if either } \gamma > \gamma^* \text{ and } Z > 0 \text{ or } \gamma < \gamma^* \text{ and } Z < 0, \\ -\infty \text{ if either } \gamma > \gamma^* \text{ and } Z < 0 \text{ or } \gamma < \gamma^* \text{ and } Z > 0, \end{cases}$$

that is, one sided limit is $-\infty$ and the other $+\infty$. Further,

$$K\left(q_k^B - a\right) \to \left[(n-1)\gamma^* - 1\right] Z = Z.$$

Thus Kq_k^B has same limit as Kp_k^B. Multiplying by K^2 both sides of (3) presents

$$K^2\pi_k^B = (Kp_k^B)(Kq_k^B) - cK^2q_k^B - t\left[\phi_k K^2 q_k^B + \sum_{i\neq k}\phi_i K^2 q_k^B - K^2\bar{E}\right].$$

Notice that $K \to 0$ implies $K\left(Kq_k^B\right)$, $K\left(Kq_i^B\right)$ and $K^2\bar{E} \to 0$. Thus $K^2\pi_k^B \to Z^2$. Therefore we have

$$\pi_k^B \to +\infty \text{ form both sides of } \gamma < \gamma^* \text{ and } \gamma > \gamma^*.$$

References

[1] Ganguli, S. and S. Raju, Perverse environmental effect on ambient charge in a Bertrand duopoly, *Journal of Environmental Economics and Policy*, 1-8, DOI:10.1080/21606544.2012.714972, 2012.

[2] Matsumoto, A., K. Nakayama and F. Szidarovszky, Environmental policy for non-point source pollutions under Bertrand duopoly. Mimeo, 2017a.

[3] Matsumoto, A., F. Szidarovszky and M. Yabuta, Environmental effect of ambient charge in Cournot duopoly, *Journal of Environmental Economics and Policy,* 1-16, DOI: 10.1080/21606544.2017.347527, 2017b.

[4] Raju, S. and S. Ganguli, Strategic firm interaction, return to scale, environmental regulation and ambient charges in a Cournot duopoly, *Technology and Investment*, 4, 113-122, 2013.

[5] Segerson, K., Uncertainty and incentives for nonpoint pollution control, *Journal of Environmental Economics and Management*, 15, 87-98, 1988.

[6] Xepapadeas, A., The economics of nonpoint source pollution, *Annual Review of Resource Economics*, 3, 355-374, 2011.