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Abstract
Due to the reduction of the transportation costs, a retailer can increase the number of items stocked at a store. The customers are attracted to the retailer that provides the more verities of goods. The number of varieties may influence the retailer's market situation. This paper examines the effects of the number of items on the retailer's market situation. Dividing the retailing stores into three categories, convenience store, supermarket store, and department store, the paper analyses their market situation in spatial free-entry equilibrium: it derives the number of items, the average price of goods dealt with a store, and the market area size in the competitive equilibrium. It is shown in this analysis that the number of items of the convenience stores decreases and their market size shrinks as the transportation costs reduce, while, in the categories of the supermarket store and the department store, the number of items increases and the market area expands due to the reduction of transportation costs. Although the directions of the change of the number of items and market area size are different between the three kinds of stores, there is a common trend on the retailer's capacity to pull customer between the three kinds of retailers: the difference of the capacities at store's location and the market area boundary becomes shorter as the transportation costs decrease.

Keywords: Number of items, Capacity of pulling customers, Market situation
Spatial free-entry equilibrium,

JEL: R30.

I Introduction

Reviewing the literature on the firms' markets, it is found that many analyses paid attention to the influence of the price and the freight rates of the retailing goods on the retailers' market areas[1]. A few theories inquired the influence of the variety of goods on the retailers' markets[2]. This paper analyzes the relationships between the variety of goods dealt with a store and its market situation: Incorporating the number of items and their average price into the consideration, it theoretically analyzes the market situation; the number of items stocked at a store, the average price of goods, and market area size in spatial free-entry equilibrium. In addition, the effects on the retailer's capacity to pull customers of the reduction of the freight rates are analyzed.

The paper is organized as follow: In the next section the assumptions and the framework of the analysis are explained, and the retailer's profit function is derived. Section III examines the retailer's market situation in the equilibrium; it derives the number of items and the average price of goods, and the market area size of a retailer. And then, it is clarified that the changes of the number of items and the market area size, which are raised by the reduction of the transportation cost per mile, are greatly different according to the retailing types; convenience store, supermarket store, and department store. Finally, focusing the retailer's capacity to pull customers, it is found that there is a common trend on the capacity between the three retailing types: the difference of the retailer's capacity to pull customers at store's location and the market area boundary becomes shorter as the transportation costs decrease. Section IV summarizes the conclusions obtained in the above sections.

II Effect of the variety of items on the market area boundary

1. Assumptions of the analysis and derivation of retailer's profit function

According to the model of Baumol-Ide (1956) who systematically analyzed the relationships between the variety of items and the retailer's market area, the following assumptions are made:

1) Consumers evenly live in a plain market field with density K
2) Retailers located on the market field provide the number of varieties to consumers.
3) The probability that the consumer visits the retailer becomes higher as the number of items increases. This provability, Z, is expressed by equation (1) [3],

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[1] For example, see Parr(1995)

[2] Baumol-Ide (1956) incorporates the variety of the retailing goods into the analysis of the firm's market size and quantity demanded in its market area.

[3] Baumol-Ide did not use a concrete function like equation (1).

$$z(N)=\left(\frac{N}{A}\right)^{\alpha}, \qquad (1)$$

where *N* is the number of items and *A* and α is positive parameter, respectively. And z(N) belongs to the range $0 \leqq z(N) \leqq 1$.

4) While, in going to a retailer the customer incurs some costs: the difficulty of shopping increases with the congestion of the store and with the transportation costs to the store. The congestion level, *J*, increases with the number of items, and the transportation costs, *S*, increase with the distance to the retailer. The congestion and transportation costs are represented by equation (2) and (3), respectively,

$$J(N)=C_n N^{\beta}, \qquad (2)$$
$$S = C_d u, \qquad (3)$$

where C_n is marginal congestion, β is positive parameter, C_d is transportation costs per mile, and *u* is distance from a customer to the retailer's store.

5) The willingness of a customer to visit a store is influenced negatively by the average price, *p*, of items of the store. This influence, Pa, is expressed by equation (4)[4].

$$Pa(p) = \varphi p^{\emptyset}, \qquad (4)$$

where φ and ∅ are positive parameters.

Based on the above assumptions, the retailer's capacity of pulling customer, *f*, can be determined by the three factors, the number of items *N*, average price *p*, and distance *u*. And the retailer's capacity of pulling customers, that is, the retailer's traction power can be represented by equation (5),

$$f(N,p,u)= \omega\left(\frac{N}{A}\right)^{\alpha} - v(C_d u + C_n N^{\beta} + \varphi p^{\emptyset}). \qquad (5)$$

where ω and ν are positive parameters.

6) The quantity demanded in the retailer's market area is proportional to the traction power of attracting customers to the store. When the retailer's market area is circle, the quantity demanded, Q, in the market area is expressed by equation (6),

[4] The average price of items is not incorporated into the analysis by Baumol-Ide.

$$Q = 2\pi K \int_0^U (\omega(\frac{N}{A})^{\alpha} - v(C_d u + C_n N^{\beta} + \varphi p^{\emptyset}))udu, \qquad (6)$$

where U is the radius of the circular market area.

The retailer's revenue, Rv, is derived by equation (7),

$$Rv=pQ. \qquad (7)$$

7) Inventory costs of the retailer are shown by equation (8),

$$IC=(E/I)r+(I/2+R)T, \qquad (8)$$

where *E* is expected sales volume of all commodities per period, *r* is handling costs of reordering. *I* is the quantity ordered for inventory each time stocks are replaced. *T* is the warehousing costs per item per period and R is stock level at which inventory is replaced, that is, the stocks on hand fall to *R*, and the retailer replaces inventory.

From equation (8), the optimal inventory, ICI, per item is derived by equation (9),

$$ICI= (2rTQ/N)^{0.5} + RT. \qquad (9)$$

8) The fixed costs of the warehouse and the retail facility are shown by F. The dealing costs of goods are proportional to the number of items, and the costs are expressed as $aN^{0.5}$.The total costs of retailer are given by equation (10),

$$TC=N (2rTQ/N)^{0.5} + NRT+ aN^{0.5} + F. \qquad (10)$$

From the equation (7) and (10), the retailer's profits, *Y*, is derived by equation (11),

$$Y=p(2\pi KU^2(0.5(\omega(N/A)^{\alpha} - vC_n N^{\beta} - v\varphi p^{\emptyset}) - vC_dU/3)) - N^{0.5}((2rT(2\pi K U^2(0.5(\omega(N/A)^{\alpha} - vC_n N^{\beta} - v\varphi p^{\emptyset}) - vC_dU/3)))^{0.5} + a) - NRT-F$$

$$. \qquad (11)$$

III The retailer's market situation appeared in spatial free-entry equilibrium

1 The influence of the competition style on the equilibrium market situation

This subsection, assuming the free-entry competition is prevailed in the market field, derives the number of items, their average price and the market area size in spatial free-entry equilibrium.

There are three conditions for the free-entry equilibrium to be established in spatial free-entry market: Every retailer determines the number of items and the average price to maximize its profit. And new retailer comes into the market until the retailer's profits is just zero. These conditions are shown by equations (12), (13), and (14).

$$\partial Y/dN = \partial Y/\partial N + \partial Y/\partial U \cdot \partial U/\partial N = 0 \quad (12)$$

$$\partial Y/dp = \partial Y/\partial p + \partial Y/\partial U \cdot \partial U/\partial p = 0 \quad (13)$$

$$Y = 0 \quad (14)$$

$\partial U/\partial N$ equation (12) represents the variation of the radius of the market area when a retailer changes the number of items by one unite, $\partial U/\partial p$ in equation (13) the variation of the radius of the market area when a retailer changes the price by one unite. $\partial U/\partial N$ and $\partial U/\partial p$ are given by equation (15) and (16), respectively.

$$\partial U/\partial N = (((\omega/v)(\alpha/A^{\alpha})\, N^{\alpha-1} - \beta C_n\, N^{\beta-1})/2C_d)(1 - dN'/dN) \quad (15)$$

$$\partial U/\partial p = ((\frac{1}{2Cd})\varphi \emptyset p^{\emptyset-1})(1 - dp'/dp) \quad (16)$$

where N' in equation (15) is the rival retailer's number of items, and p' in equation (16) is the rival retailer's price. dN'/dN is the conjectural variation of the variety and dp'/dp is the conjectural variation of the price.

When the values of dN'/dN is equals 1, the Lösch competition type is indicated in terms of items. In this case, when a retailer increases an item at the store, the retailer conjectures that the rival also increases one item at its store. Similarly the dp'/dp is equals to 1, the Lösch competition type is indicated in terms of average price. When a retailer lowers average price by unite, it conjectures that the retailer conjectures that the rival lowers its price by unite. Assuming that the values of dN'/dN and dp'/dp are 1, and solving the simultaneous equations (12), (13), and (14) with respect to N, p, U, the number of items, the average price and market size in the Lösch equilibrium are obtained. When both values of dN'/dN and dp'/dp are assumed 0, it indicates Nash competition, by using the same way the values are obtained in the Nash equliribrium.

Let us equilibrium values in the Lösch and Nash equilibria, assigning the following numerical values to parameters; ω=45, v=5.6667, Cd=0.2, Cn=1.2, α=0.26, β=0.5, φ=0.2, $\emptyset$ =1.05,A=20, F=20, K=1.75 , R=5, T=2 a=0.15, r=1.6,

α=0.26,ω=45,ν=5.6667, C_d=1.2,φ=0.2, $\emptyset = 1.05$, β=0.5,C_n=1.2.

First, the retailer's profit function Y is shown by equation (17).

$$Y = p * \left(2 * 3.14159 * 1.75 * U^2 * \left(0.5 * \left(45 * (N/20)^{0.26} - (5.6667 * 1.2) * N^{0.5} - (5.6667 * 0.2 * p^{1.05})\right) - (5.6667 * 1.2/3) * U\right)\right) - 20 - N^{0.5} * \left(\left(\left(2 * 1.6 * 2 * \left(2 * 3.14159 * 1.75 * U^2 * \left(0.5 * \left(45 * (N/20)^{0.26} - 6.8 * N^{0.5} - (5.6667 * 0.2 * p^{1.05})\right) - (5.6667 * 1.2/3) * U\right)\right)\right)^{0.5} + 0.15\right) - N * 5 * 2 \quad (17)$$

And $\partial Y/\partial N$ and $\partial Y/\partial p$ are given by equations (18) and (19),

$$\partial Y/\partial N = -10+5.49778(5.3693/N^{0.74}-3.40002/N^{0.5})pU^2-(2.09719 (5.3693/N^{0.74}-3.4/N^{0.5}) N^{0.5} U^2)/((0.5 (20.6512 N^{0.26}-6.8 N^{0.5}-1.13334 p^{1.05})-2.26668 U) U^2)^{0.5}-(0.5 (0.15 +8.38878 ((0.5 (20.6512 N^{0.26}-6.8 N^{0.5}-1.13334p^{1.05})-2.26668 U) U^2)^{0.5}))/N^{0.5} \quad (18)$$

$$\partial Y/\partial p = -6.5424 p^{1.05} U^2+10.9956 (0.5 (20.6512 N^{0.26}-6.80004 N^{0.5}-1.13334 p^{1.05})-2.26668 U) U^2+(2.49568 N^{0.5} p^{0.05} U^2)/((0.5 (20.6512 N^{0.26}-6.8 N^{0.5}-1.13334 p^{1.05})-2.26668 U) U^2)^{0.5} \quad (19)$$

Lastly, the term of $\partial Y/\partial U$ in equations (12) and (13) is derived as equation (20),

$$\partial Y/\partial U = 21.9911 p (0.5 (20.6512 N^{0.26}-6.80004 N^{0.5}-1.13334 p^{1.05})-2.26668 U) U-24.9234 p U^2-(4.19439 N^{0.5} (2 (0.5 (20.6512 N^{0.26}-6.8 N^{0.5}-1.13334 p^{1.05})-2.26668U) U-2.26668 U^2))/((0.5 (20.6512 N^{0.26}-6.8 N^{0.5}-1.13334 p^{1.05})-2.26668 U) U^2)^{0.5} \quad (20)$$

Now, by using the above four equations (17) ~ (20), equations (12), (13), and (14) are rewritten to be calculable. Solving the simultaneous equations system (12), (13), and (14) with respect to N, p, U gives the equilibrium number of items, average price, and market area size. The equilibrium values are shown in Table 1. If the transportation costs per mile lower from 1.2 to 0.2, the equilibrium values of the retailer are changed. In this case the equilibrium values are shown in Table 2.

Table 1 The equilibrium number of items, average price, and market size

Style	item, N	average price, p	market size, U
Lösch	1.081	4.894	0.506
Nash	2.822	7.011	0.659

Table 2 The equilibrium values when the transportation costs is low

Style	item, N	average price, p	market size, U
Lösch	0.881	5.491	0.422
Nash	5.188	10.941	1.035

By comparing the figures shown in Table 1 and 2, it is found that the number of items in Nash equilibrium is higher than that of Lösch equilibrium, the average price in Nash is lower than Lösch, and the market area size in Nash is larger than Lösch. In addition, the changes of the equilibrium values due to the reduction of transportation costs per mile are different according to the competition style between the retailers. When the Lösch competition is prevailed between the retailers, as the transport costs is the lower, the number of items decreases and the market size shrinks, while in Nash competition the number of items increases and the market area size expands.

It is possible from the results obtained in the above analysis to characterize the two competition styles: The Lösch competition is a style to invite the new retailers to the market field since the market area of the retailer becomes small and the variety of goods at a store decreases due to the reduction of the transportation costs. While the Nash completion is a style to expel the existing retailers from the market field since the retailer's market area becomes large and the variety of goods increases due to the reduction of the transportation costs.

2 Changes of the number of items and market area size by decreasing transport costs

Now, the retailing types in the real world can be roughly classified into the three categories, convenience store, supermarket store, and department store. It would be possible to assign the conjectural variation of item and price to the competition style prevailed in each category of the retailers:

The distances among convenience stores are short and they deal with small and light goods with low prices. Thus the conjectural variation of item and price would be assumed as $dp'/dp=1$ and $dN'/dN=1$, that is Lösch competition style.

The distances among supermarkets are relatively long and they deal with daily goods with low and medium prices. Then it would be possible to assume that the conjectural variation of the average price is dp'/dp=1, and the conjectural variation of the item is dN'/dN=0.75. Supermarket store assumes that when it changes the number of items by one unit, it assumes that the rival store changes the number of items by 0.75 since the rival cannot immediately change the number of items to the same level.

Lastly, it would be assumed in the case of department store that the conjectural variations are dp'/dp= - 0.5 and dN'/dN= - 0.25. Because the customers to visit a department store to purchase the goods with high prices and they attach the great importance to their taste. The department store attempts to provide many varieties of goods and plans to sell the goods differentiated from rivals' ones. Thus, the department store assumes that against the change of the number of items and the average price of the store, the rival does not respond like supermarket store: Conversely, the rival takes the opposite response. Hence in this analysis it is assumed that the department store set the conjectural variations of items and price as dp'/dp= - 0.5 and dN'/dN= - 0.25 in order to maximize its profits[5].

Now, since the fixed costs, the number of the varieties and some parameters' values seems to be different between the three retailing types, this section supposes that for the convenient stores F=5, A=20, R=5, r=1.5, ω=40; for supermarket stores F=27,A=20,R=20,r=1.5,ω=42; and for the department stores F=30, A=16, R=10, r=1.7, ω=46. Other values of parameters are assumed to be the same for simplicity of the analysis. Lowering the transportation costs per mile from 0.6 to 0.2, the equilibrium market values are derived for the three retailing types at each level of the transportation costs. The results are shown by Table 3A, B, C, respectively.

The comparison of the figures shown in these Tables gives the following interesting facts: the market area of the convenience store becomes smaller and the number of variety stocked in the store decreases as the transportation costs per mile lowers. The market area of the supermarket store becomes smaller until transportation costs per mile lower to 0.4 and it begins to expand by the reduction of the costs, and the number of items stocked in the store increases by the decreasing transportation costs. The number of the items stocked in the department store increases and its market area expands as the transportation costs per mile lowers.

It is interesting to know that the changes of the number of the items and the market area size of retailers, which are raised by the reduction of the transportation costs, are different between the retailing types.

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[5] Schöler, K. (1993) shows the cases that the conjectural variation of price takes minus value.

Table 3A The changes of the equilibrium values of the convenience store

Cd	N	p	U
0.6	0.140	1.883	0.391
0.5	0.136	1.914	0.382
0.4	0.132	1.945	0.374
0.3	0.129	1.974	0.367
0.2	0.126	2.002	0.360

Table 3B The changes of the equilibrium values of the supermarket store

Cd	N	p	U
0.6	0.689	4.211	0.738
0.5	0.822	4.550	0.731
0.4	0.847	4.585	0.712
0.3	1.056	4.831	0.720
0.2	1.389	5.097	0.745

Table 3C The changes of the equilibrium values of the department store

Cd	N	p	U
0.6	5.085	10.763	0.979
0.5	5.471	11.229	1.036
0.4	5.912	11.724	1.113
0.3	6.424	12.262	1.224
0.2	7.042	12.864	1.402

3 The difference of the traction powers at a store and the market area boundary

This subsection examines the retailer's capacity to pull customers, traction power, at a store and the market area boundary. Figure 1 illustrates the traction powers for the three kinds of retailers to the six transport cost levels. The supermarket store's traction power, which is shown by brawn square, is the highest of three kinds of the stores. Its traction power changes like U shape as the transport costs decrease. The department store's traction power, which is shown by green triangle, is the lowest of them. The convenience store's power, blue diamond, is in between the supermarket store and the department store. The traction power at a store of the convenience store and the department store decrease as the transport costs reduce.

Figure 1. Retailers' capacity to pull customers at a store's location

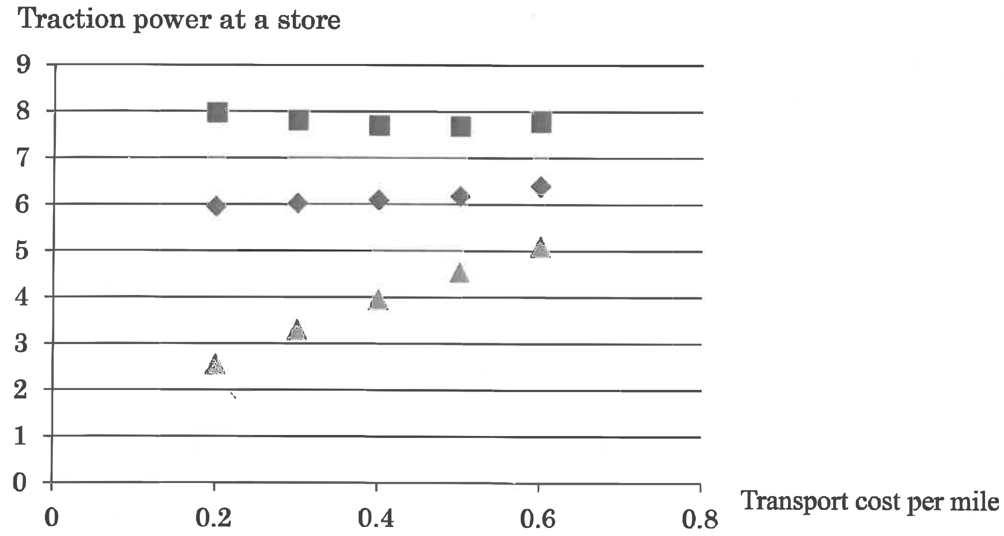

Figure 2 shows the traction powers at the market area boundary for the three kinds of retailers. The supermarket store's traction power is the highest of three kinds of stores. Its traction power increases as the transport costs decrease. The department store's traction power is the lowest of them The traction powers of supermarket store and convenience store increase as the transport costs decreases. The department store's power decreases as the transport costs reduce.

Figure 2. Retailers' capacity to pull customers at the market area boundary

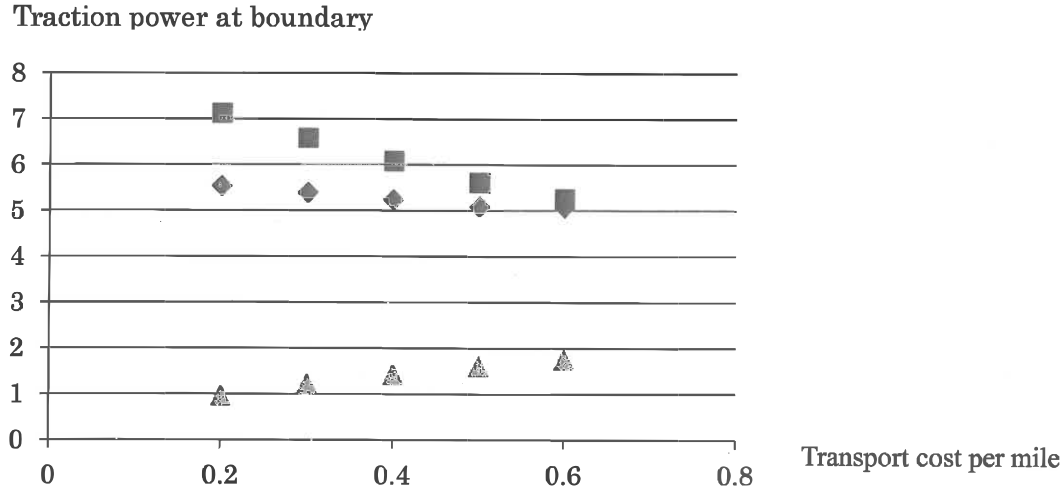

Figure 3. Difference of the traction powers at a store and the market area boundary

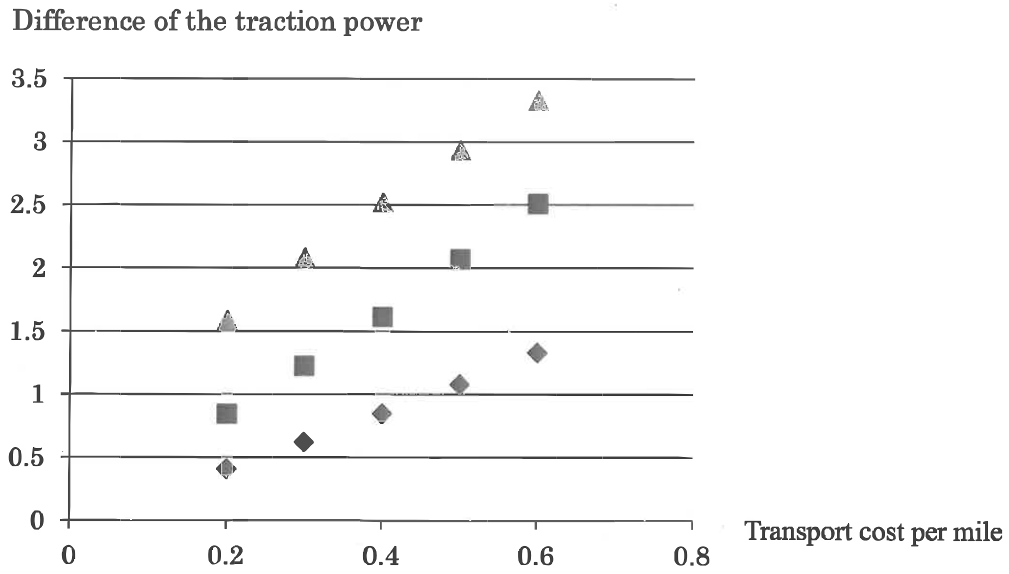

Figure 3 shows the difference between the traction powers at a store and its market area boundary for the three kinds of retailers to the six transport cost levels from 0.6 to 0.2. The difference of the traction powers of the every kind of store becomes shorter as the transport costs reduce. Although the directions of the change of the number of items and market size are different between the three kinds of stores, there is a common trend on the traction power between the three kinds of stores: the difference of the traction powers at store's location and the market area boundary becomes shorter as the transportation costs decrease.

IV Summary and conclusions

Due to the reduction of the transportation costs, the retailers become to be able to deal with many kinds of goods and to increase the number of items stocked at the stores. Corresponding to the increase of items stocked at stores, the customers are attracted to the retailer that provides the more verities of goods. Hence, besides of the price of goods, the number of items influences the retailer's market situation. Thus, this paper examines the effects of the number of items on the market situation.

Dividing the retailers into three categories, convenience store, supermarket store, and department store, the paper analyzes the number of items, the average price of goods dealt with a store, and the market area size in the free-entry equilibrium. It is shown in this analysis that in the category of the convenience store, the number of items decreases and the market area shrinks as the transportation costs reduce, while, in the categories of the supermarket and the department store, the number of items increases and the market area expands due to the reduction of transportation costs. Although the change of the number of items and market area size are different between the three kinds of retailers, there is a common trend on the traction power between these retailers: the difference of the traction powers at store's location and the market area boundary becomes shorter as the reduction of transportation costs decreases.

References

Arndt, S.W., and Kierzkowski, H. (2001) *Fragmentation*, Oxford University Press.

Baumol, W.J. and Ide, E.A. (1956) Variety in retailing, *Management Science* 3, pp.93-101.

Ishikawa, T. and Toda, M. (1998) An application of the frontier price concept in spatial equilibrium analysis, *Urban Studies*, 35, 8, pp.1345-1358.

Ishikawa, T. and Toda, M. (2002) On the optimal locational policy for the offshore firm entering a foreign market area, Canadian Journal of Regional Science,Vol.25,3, pp.355-375.

Lösch, A. (1940) *Die räumliche Ordnung der Wirtschaft*, G. Fischer.

Parr, J.B.(1995) The Economic Law of Market Areas: A Further Discussion, *Journal of Regional Science*, 35,599-615.

Schöler, K. (1993) Consistent conjectural variation in a two-dimensional spatial market, *Annals of Regional Science*, 23, pp.19-28.

Toyo Keizai (1998, 2008) *Chiikikeizai Soran* (Data Book, in Japanese).Tokyo.