Strategic Trade Policy under Isoelastic Demand and Asymmetric Production Costs

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Abstract

We demonstrate that whether a good of a rival firm is a strategic substitute or astrategic complement is endogenously determined when the market inverse demand is hyperbolic. The relative competitiveness, which is expressed by the ratio of firms' marginal costs, is the key determinant. We derive optimal trade policies, which are dependant upon the firms' form of strategic action. In particular, the optimal policy recommendation for the home government is to give an export subsidy to its firm if a choice variable of the foreign firm is a strategic substitute and to levy an export tax if it is a strategic complement.

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1 Introduction

The market is imperfectly competitive, either because the number of firms in that market is few, because the goods are differentiated, or because there is some kind of economies of scale. In reality, we see many imperfectly competitive industries, in which firms compete fiercely both domestically and internationally. In such an imperfectly competitive international market, governments may be motivated to introduce trade policies that includes tariffs, export subsidies and taxes, even if such policy intervention is not justified. If governments can affect, or more precisely, strategically alter market structure by introducing policies, how would behavior of the firms be impacted or what would the impacts on the market be. To address these issues and investigate their motive, researchers have contributed contributed numerous trade theory papers since 1980s. In many international oligopoly models, it has been shown that a government can introduce trade policy so as to increase its domestic welfare by shifting monopoly rent from foreign rival firms to its own firms. This is why such government policy is called strategic trade policy. But, this does not mean that there is a clear understanding to fill the gap between the normative and positive implications of economic activities among firms and governments and between firms and governments.

OF the vast amount of literature on strategic trade policy in imperfectly competitive markets, the pioneering Brander and Spencer [1985] show that when the home government intends to shift monopolistic rent from foreign to domestic firms an export subsidy is optimal in a model, in which one home firm and one foreign firm of constant-return-to-scale technology produce homogeneous goods and compete in quantities in a third market. An increase in the domestic export subsidy raises the domestic firm's output (market share) and its profit. Eaton and Grossman [1986], however, derive an opposite result in that an export tax for the home firm, which raises profits of the foreign firm, is optimal when two firms compete in prices. What is the source of this sharp contrast? Brander and Spencer [1985] assume that a firm's own marginal revenue declines with an increase in the output of the other firm; that is, players' strategies are said to be strategic substitutes.[1] This assumption is reversed in Eaton and Grossman [1986] where that firm's own marginal revenue increases with an increase in the price of the other firm; that is, players' strategies are strategic complements. It is the difference in the assumption on firms' strategic behavior, which is reflected in the opposite tangencies of firms' reaction function in the output plane, that plays the central role in

[1]For strategic substitutes and strategic compliments, see Bulow, Geanakoplos and Klemperer (1985). Geometrically, players' strategies are said to be strategic substitutes (compliments), if their best response functions are downward (upward) sloping.

driving opposite policy implications. It is well known that this critical assumption relates to the model's stability conditions and the curvature of the inverse demand functions as well as the elasticity of the demand curve. (See for example, Brander and Spencer [1984a] and [1984b] and Jones [1987].) Focusing on these issues, Bandyopadhyay [1997] simplifies Brander and Spencer [1985] and shows that demand elasticities and cost asymmetry determine optimal trade policy direction - either export subsidy, export tax, or free trade - when one government unilaterally intervenes. Bandyopadhyay [1997] also demonstrates that, depending on demand elasticities, the Nash equilibrium in the policy game may be unstable when both governments bilaterally intervene. However, Bandyopadhyay [1997] does not explain the mechanism whereby the difference in marginal costs affects firms' rivalry in the good market in relation to demand elasticities and the governments' interventions.

In this paper, we extend Brander and Spencer [1985] by introducing a hyperbolic inverse demand curve which is the second familiar demand function in economics. By specifying a unit elastic demand, we examine how such a minor change alters government and firm behavior as well as the characteristics of the equilibria and clarify how the cost asymmetry relates to the unilateral trade policy. We show that: (i) the difference of constant marginal costs endogenously determines whether firms' strategies are strategic substitutes or complements when the inverse demand function is hyperbolic, and we clarify conditions for this; and (ii) trade induced by export subsidies of the home government is not necessarily beneficial for the home firm.

The study is set out as follows. In Section 2, we model a simple nonlinear international duopoly model, and derive some specific features of firms' rivalry in the market. There is no government intervention in the market. This is altered in the next Section. It is demonstrated that reaction functions are mound-shaped and that whether goods are strategic substitutes or strategic complements is endogenously defined. Section 3 considers strategic trade policies in a Stackelberg game, in which the home government moves first before the international Cournot-Nash duopolistic competition starts. We derive the Stackelberg equilibrium and the optimal level of the trade policy. Then we demonstrate that the home government's trade policy, for example a production subsidy, can provide a strategic advantage to the home firm if a choice variable of the foreign firm is a strategic substitute and a strategic disadvantage if a choice variable of the foreign firm is a strategic complement. Finally, concluding remarks are provided in Section 4.

2 International Duopoly with Hyperbolic Demand Function

2.1 Cournot-Nash Competition

We consider an international duopoly model, in which the home firm produces output x with constant marginal cost c_x, and the foreign firm producing output y with constant marginal cost c_y. Goods are homogeneous, and we assume that firms export their products only to the third country and compete in quantities in Cournot fashion with price p. For simplicity, we assume that neither firm faces any domestic demands in their domestic markets. We further assume the inverse demand function to be hyperbolic,[2]

$$p(x,y) = \frac{1}{x+y}. \tag{1}$$

The profits of the home firm and foreign firm are, therefore,

$$\begin{aligned} \Pi_h(x,y) &= p(x,y)x - c_x x, \\ \Pi_f(x,y) &= p(x,y)y - c_y y. \end{aligned} \tag{2}$$

Each firm maximizes its profit with respect to its own output given the rival's output. Equating the partial derivatives of profit functions to zero and arranging terms yields

$$\begin{aligned} r_h(y) &\equiv \sqrt{\frac{y}{c_x}} - y, \\ r_f(x) &\equiv \sqrt{\frac{x}{c_y}} - x, \end{aligned} \tag{3}$$

where $r_h(y)$ is a reaction function of the home firm, and $r_f(x)$ is a reaction function of the foreign firm. Each reaction curve is mound-shaped with its highest point for x and y such that $x = y$. Substituting $y = x$ into the first equation of (3) and solving for x gives the maximum value of the home firm's output, denoted by x_M, and the associated maximizer of y, denoted by y_M,

$$x_M = \frac{1}{4c_x} \text{ and } y_M = x_M. \tag{4}$$

[2] This function is a special case of $p(x,y) = \frac{1}{(x+y)^\lambda}$ where λ is a reciprocal of market demand elasticity. The general case is considered in Matsumoto and Serizawa [2003].

Solving $r_h(y) = 0$ for y gives two levels of outputs, the lower y_l and the higher y_h, that lead to zero optimal output of the home firm,

$$y_l = 0 \text{ and } y_h = \frac{1}{c_x}. \tag{5}$$

Since the foreign firm is exactly the same as the home firm, its reaction function has the same properties where x, y, c_x and the subscript, h, for the home firm are, respectively, replaced with y, x, c_y and the subscript, f, for the foreign firm. To avoid the notational confusion, we denote the maximum and the maximizer of the reaction curve of the foreign firm by y_m and x_m. It can be checked that

$$\frac{dr_f(x)}{dx} \gtreqless 0 \text{ according to } x \lesseqgtr x_m \tag{6}$$

where the maximum output of the foreign firm and the associated maximizer are

$$y_m = \frac{1}{4c_y} \text{ and } x_m = y_m, \tag{7}$$

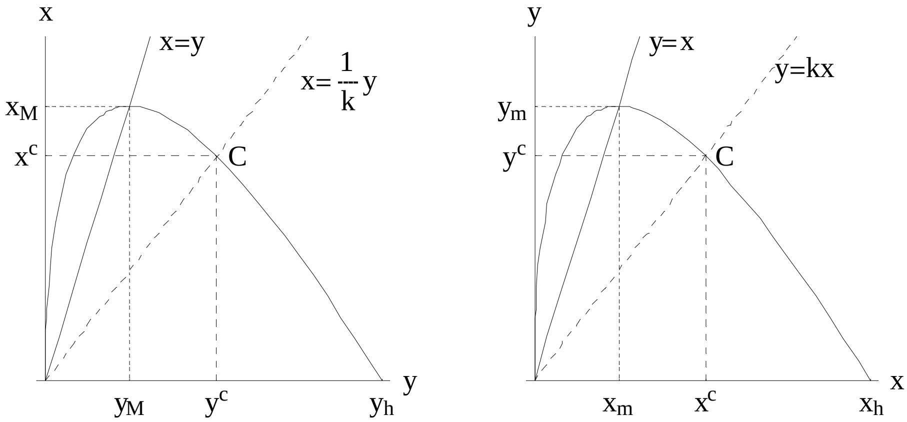
and the home firm's outputs that leads to zero optimal output of the foreign firm are

$$x_l = 0 \text{ and } x_h = \frac{1}{c_y}. \tag{8}$$

Above all, the mound-shaped reaction curves of the home and the foreign firms start at origin, which have the peaked points of (y_M, x_M) and (x_m, y_m) respectively, through which the critical line $y = x$ or $x = y$ passes, and then drops to zero at $y = y_h$ on the y-axes and $x = x_h$ on the x-axes as illustrated in Figure 1(A) and Figure 1(B).[3]

Proposition 1 *The reaction curves of duopolistic firms are mound-shaped when the inverse demand function is hyperbolic.*

[3] Note that the $x = y$ or $y = x$ locus is the diagonal. The ratio between the x-axis and the y-axis is appropriately adjusted. Please ignore the dashed lines and k for now.



(A) Home Firm with $k < 1$ (B) Foreign Firm with $k > 1$

Figure 1. Mound-shaped Reaction Curves

2.2 Strategic Substitutes and Complements

The mound-shaped reaction curve shows a sharp contrast to the traditional linear reaction curve, either downward-sloping or upward-sloping, and plays the central role in the following analysis. Before proceeding, we digress to detect a possible economic source of these curves. However, we focus only on the home firm because the same source is shared by the foreign firm which is symmetric with the home firm.

Since the cost function is assumed to be linear, a possible source of the nonlinearity should be found in the revenue function. Revenue is the price times the quantity sold, and the marginal revenue of the home firm has the well-known formula,

$$MR = p + x\frac{\partial p}{\partial x}. \tag{9}$$

(9) implies that a change of its own output generates two types of effects on revenue. The first term p implies changes of revenue caused by a unit change of output, keeping the price level constant, while the second term describes changes of revenue caused by a unit change of price, keeping the output level constant. An alternative expression of (9) is also familiar

$$MR = p(1 - \frac{1}{|\varepsilon_x|}), \tag{10}$$

where ε_x is the price elasticity of the home firm's demand given output of the foreign firm and can be expressed as

$$\left|\frac{d\log x}{d\log p}\right| = \frac{x+y}{x}. \tag{11}$$

(10) and (11) means that if y is positive, the elasticity $|\varepsilon_x|$ is greater than unity, and thus the marginal revenue of the home firm is positive.

We are now concerned with how the home firm's marginal revenue changes in response to changes in the foreign firm's choice of output. When output of the foreign firm increases, there are two sorts of effects according to the decomposition of the marginal revenue given in (9): one affects the first term p via shifting the demand curve and the other affects the second term through changing the slope of the demand curve. The first effect is to decrease the marginal revenue as the increase in output makes the price decrease in order to sell the larger quantity, so revenue of the home firm decreases, provided that the level of its output is constant. The second effect is to increase the marginal revenue by changing the slope of the demand curve of the home firm. The opposite direction of these two effects implies that the total change can be of either, depending on the relative magnitude between the increases and the decreases. To see the total effect under the hyperbolic inverse demand in (1), we differentiate MR with respect y to get

$$\frac{\partial MR}{\partial y} = \frac{x}{(x+y)^2}\left(2 - |\varepsilon_x|\right). \tag{12}$$

Thus the marginal revenue increases when y increases if the price elasticity of the home firm's demand is less than two in absolute value. Similarly, the marginal revenue decreases when y increases if the price elasticity is greater than two in absolute value. Returning to the profit function of the home firm and deriving its cross derivative we have

$$\frac{\partial}{\partial y}\left(\frac{\partial \Pi_h}{\partial x}\right) = \frac{\partial MR}{\partial y}. \tag{13}$$

Due to the definition given by Bulow, *et al* [1985], output of the home firm can be a *strategic substitute* or a *strategic complement* according to the marginal profit being negative or positive in response to a change in the foreign firm's output. Thus (12) and (13) implies the following.

Proposition 2 *Under the hyperbolic demand, the home firm regards its output as a strategic complement when demand elasticity is less than two in absolute value and as a strategic substitute when greater than two in absolute* *value. That is,*

$$\frac{\partial}{\partial y}\left(\frac{\partial \Pi_h}{\partial x}\right) \gtreqless 0 \textit{ according to } 2 \lesseqgtr |\varepsilon_x| \,.$$

The home firm's price elasticity in demand defined in (11) is the reciprocal of the market share of the home firm denoted by S_x

$$|\varepsilon_x| = \frac{x+y}{x} = \frac{1}{S_x}. \tag{14}$$

If its market share is quite large, the individual demand curve the home firm faces is almost identical with the market demand curve, so the demand elasticity is close to unity. The home firm behaves like a monopolist. If the home firm produces a very small part of the market, its market share is very small and the individual demand curve the home firm faces is effectively infinite. The home firm behaves like a competitive firm. Thus Proposition 2 can be put in another way: good x of the home firm is a strategic complement (respectively substitute) to good y of the foreign firm when S_x is greater (respectively less) than 0.5. The market share reflects the extent to which a firm dominates the market. The next question which we naturally raise is *under what economic circumstances the home firm can dominate the market*?

2.3 Cournot Equilibrium

The intersection of two reaction curves determines the Cournot equilibrium.[4] By solving simultaneously two equations of (3) for x and y, we have the equilibrium outputs of the home and foreign firms:

$$\begin{aligned} x^c &= \frac{c_y}{(c_x+c_y)^2}, \\ y^c &= \frac{c_x}{(c_x+c_y)^2}, \end{aligned} \tag{15}$$

and the equilibrium price

$$p^c = c_x + c_y, \tag{16}$$

[4] We should introduce some expectation formations, construct the dynamic process of Cournot output adjustment and then define the Cournot equilibrium as a fixed point of the dynamic process. Instead, we assume the Cournot equilibrium is stable and focus on the static analysis in this paper. Since Puu's complex duopoly model generates interesting dynamics involving chaos (See Puu [2000]), we discuss the case of unstable equilibrium in a different paper.

where superscript c is attached to the variables associated with the Cournot equilibrium.[5] To see what determines the relative magnitude between x^c and y^c, we divide the first equation by the second in (15) to get

$$\frac{y^c}{x^c} = k, \tag{17}$$

where k is the ratio of the marginal production cost of the home firm over the marginal production cost of the foreign firm,

$$k = \frac{c_x}{c_y}. \tag{18}$$

Thus we find that

$$k \lesseqgtr 1 \Longrightarrow x^c \gtreqless y^c \text{ and thus } S_x^c \gtreqless \frac{1}{2}, \tag{19}$$

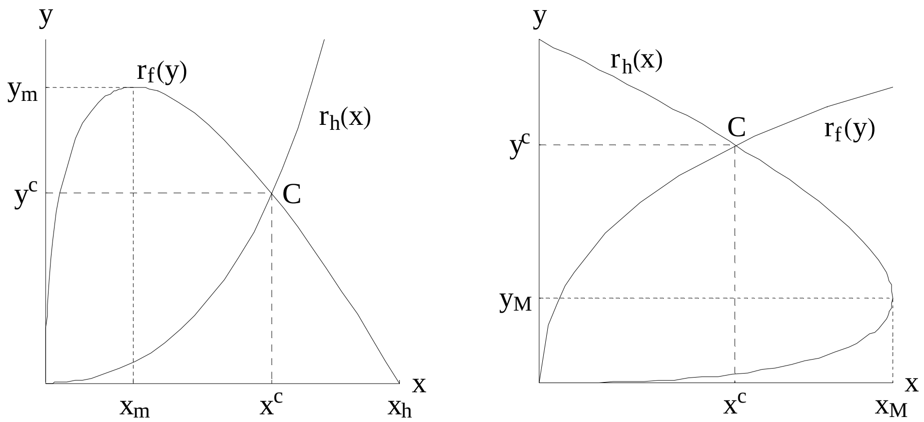
where S_x^c is the market share of the home firm at the Cournot equilibrium. From (17), (18) and Proposition 2, we can say that the home firm regards x^c as a strategic complement if $k < 1$ and a strategic substitute if $k > 1$.

This is confirmed geometrically in Figure 1(A). The Cournot equilibrium satisfies (15) and locates on the reaction curve. We see that the $x = y$ locus passes through the peaked point, (y_M, x_M), of the home firm's reaction curve. Thus if the $x^c = \frac{1}{k} y^c$ locus is flatter than the $x = y$ locus, then it crosses the downward sloping part of the reaction curve and then x^c becomes a strategic substitute to y^c because $y^c > x^c$ at the Cournot equilibrium, as illustrated by the dashed line in Figure 1(A). On the other hand if the locus is steeper, x^c becomes a strategic complement to y^c because $y^c < x^c$. Thus, the last result and (13) imply the following: output of the home firm is a strategic substitute for $1 < k$, i.e., $c_x > c_y$, and a strategic complement for $1 > k$, i.e., $c_x < c_y$. By the same principle, the foreign firm regards y as a strategic substitute for $1 > k$, i.e., $c_x < c_y$ and a strategic complement for $1 < k$, i.e., $c_x > c_y$. We call a firm with the lower marginal cost of production *relatively efficient* and a firm with the higher marginal cost *relatively inefficient.* Then we can summarize these results concerning sources of strategic substitutes and complements as follows.

Proposition 3 *A relatively efficient firm regards its own output as a strategic complement and a relatively inefficient firm regards its own output as a strategic substitute.*

[5] Mound-shaped reaction curves starting at the origin implies that curves cross each other twice at $(0, 0)$ and (x^c, y^c). The former is the trivial equilibrium point and the latter is the non-trivial equilibrium point. Our concern is with the non-trivial point and thus no further consideration is given to the trivial point.

Figure 2 visualizes the results of Proposition 3. In Figure 2(A) the home firm is relatively efficient and the foreign firm is relatively inefficient as the ratio of marginal costs is less than unity, $k = 0.25$. It can be seen that the reaction curve of the home firm is upward sloping (i.e., x is a strategic complement) and the reaction curve of the foreign firm is downward sloping (i.e., y is a strategic substitute) in the neighborhood of the Cournot Equilibrium. In Figure 2(B) where the relative efficiency is reversed as $k = 5$, we observe the opposite results to the one observed in Figure 2(A) that is x is a strategic substitute and y is a strategic complement in the neighborhood of the Cournot Equilibrium.



(A) $c_x = 1, c_y = 4$ (B) $c_x = 1,\ c_y = 0.2.$

Figure 2 Reaction Curves and Cournot Equilibrium

Next, we derive the firms' equilibrium profits calling them the *Cournot profits,* which can be calculated as well by substituting x^c and y^c of (15) into Π_h and Π_f of (2),

$$\Pi_h^c = \left(\frac{c_y}{c_x + c_y} \right)^2, \qquad \Pi_f^c = \left(\frac{c_x}{c_x + c_y} \right)^2. \tag{20}$$

From (20), we find that the ratio of the Cournot profits is the square of the Cournot output ratio,

$$\frac{\Pi_h^c}{\Pi_f^c} = \left(\frac{1}{k} \right)^2. \tag{21}$$

Therefore, from (19) we have

$$k \lesseqgtr 1 \Longrightarrow \Pi_h^c \gtreqless \Pi_f^c \tag{22}$$

(19) and (22)imply the following.

Proposition 4 *In a duopoly market with hyperbolic inverse demand and constant marginal costs, a relatively efficient firm produces larger output and makes a larger profit than a relatively inefficient firm at the Cournot equilibrium.*

Proposition 4 can be alternatively put. From (10), (14) and the profit maximization condition (i.e., $MR = MC$), we derive the well-known result that if the price elasticity of the market is constant, the price-cost margin is greater for the firm with larger market share

$$S_i = \frac{p - c_i}{p} \text{ for } i = x, y.$$

This implies that at a given price, a firm with lower marginal cost (i.e., a relatively efficient firm) produces more than the one with higher marginal cost, and if marginal costs are equal among firms, then firms produce the same level of output and thus have the same market share.

3 Strategic Trade Policy

The Cournot equilibrium is the best choice for both firms in the absence of government intervention as neither firm has an incentive to change its output decision. As shown in Brander and Spencer [1985] and Eaton and Grossman [1986], the domestic government can raise domestic welfare by shifting oligopoly profits from the foreign to the domestic firm. These results are strictly related to whether the goods are strategic substitutes or strategic complements. In the former, goods are assumed to be strategic substitutes and the optimal trade policy is to give a home firm an export subsidy. In the latter, strategic complements are assumed and so the optimal trade policy is an export tax. The natural question which we should raise is *whether active government intervention is still beneficial in the nonlinear duopoly setting.*

In this section, we analyze how intervention by the home government affects the market. A familiar two-stage game is set up: in the first stage, the home government decides the optimal subsidy level, and in the second stage, firms compete in quantities as in Section 2 taking the subsidy level as given. As the home government behaves as if a Stackelberg-leader and the firms Stackelberg-followers we call this game a Stackelberg competition. The model is solved by backward induction. That is, we first solve for the Cournot equilibrium output of the two firms in the second stage, and then we solve for the optimal level of subsidy in the first stage. The solution concept is subgame perfect Nash equilibrium (henceforth, SPNE).

3.1 Stackelberg-leader government and subsidy

First, we look into the second stage and we solve the Cournot equilibrium output when the home government intervenes in the market. Given the level of subsidy introduced by the home government, firms engage in a similar Cournot-Nash duopolistic competition as in the previous section, except that the profit of the home firm is affected by the subsidy. Suppose that the home government puts a subsidy s per unit of output produced by the home firm which reduces the marginal production cost of the home firm. Though the profit maximization problem of the foreign firm defined in (2) is not affected by the home government intervention, the profit maximization problem of the home firm becomes

$$Max\ p(x,y)x - (c_x - s)x,$$

where $c_x - s > 0$ is assumed. From the first order conditions of each firms' maximization problem, the home and the foreign firms' reaction functions are

$$\begin{aligned} x &= \sqrt{\frac{y}{c_x - s}} - y, \\ y &= \sqrt{\frac{x}{c_y}} - x. \end{aligned} \tag{23}$$

We can see that the characteristics of the home firm's mound-shaped reaction curve are not affected by s, which is the same as that obtained in the previous section. It starts at the origin. However, a simple calculation yields that its intercept on y-axes shifts upward by $\frac{1}{c_x - s}$ from $\frac{1}{c_x}$ obtained in the case without subsidy. Moreover, the shifted critical values (x_M^s, y_M^s), at which the reaction curve has the peak, are derived by the substitution of $y = x$

$$x_M^s = y_M^s = \frac{1}{4(c_x - s)}, \tag{24}$$

where superscript s means the case with a subsidy. Since the reaction function of the foreign firm is the same as that derived in Section 2, the critical values of the peaked point (x_m^s, y_m^s), are the same as

$$x_m^s = y_m^s = \frac{1}{4c_y} \tag{25}$$

By solving two equations of (23) simultaneously for x and y, we have the

Cournot-Nash equilibrium outputs under a subsidy:

$$x^{cs} = \frac{c_y}{(c_x - s + c_y)^2},$$
$$y^{cs} = \frac{c_x - s}{(c_x - s + c_y)^2}, \tag{26}$$

where superscript cs means the Cournot equilibrium under a subsidy. The ratio of Cournot outputs under a subsidy is

$$\frac{y^{cs}}{x^{cs}} = \frac{c_x - s}{c^y},$$

and the price that is determined by the Cournot outputs is similarly derived as

$$p^{cs} = c_x - s + c_y.$$

3.2 Subsidy Effect on the Market

We analyze the impact of a subsidy on the market in the nonlinear setting. To do this we differentiate each equation of (26) with respect to s to get

$$\frac{\partial x^{cs}}{\partial s} = \frac{2c_y}{(c_x - s + c_y)^3},$$
$$\frac{\partial y^{cs}}{\partial s} = \frac{c_x - s - c_y}{(c_x - s + c_y)^3}. \tag{27}$$

First we observe that the subsidy has a positive effect on the Cournot output of the home firm,

$$\frac{\partial x^{cs}}{\partial s} > 0 \ . \tag{28}$$

That is, if the home government increases (decreases) the subsidy, then the reaction curve of the home firm shifts to the right (left).

Second, we can see that an increase of the export subsidy increases the total volume of the exports to the third country regardless of the foreign firm's reaction,

$$\frac{\partial x^{cs}}{\partial s} + \frac{\partial y^{cs}}{\partial s} = \frac{1}{(c_x - s + c_y)^2} > 0. \tag{29}$$

Third, we see that the effect on the Cournot output of the foreign firm is in general ambiguous as the sign of $(c_x - s) - c_y$ in $\frac{\partial y^{cs}}{\partial s}$ can be either positive or negative. In particular, depending on the relative magnitude of those two terms, we have

$$\frac{\partial y^{cs}}{\partial s} < 0 \text{ if } k_s < 1, \qquad \frac{\partial y^{cs}}{\partial s} > 0 \text{ if } k_s > 1, \tag{30}$$

where k_s is a ratio of firms' marginal costs after the government intervention which is defined as

$$k_s = \frac{c_x - s}{c_y}. \tag{31}$$

As the characteristics of the reaction curves of the home firm and the foreign firm are the same as those obtained in the case without subsidy, Proposition 1 and Proposition 2 are applicable to the case with a subsidy. With the same reasoning, the foreign firm regards its output as

strategic substitute if $k_s < 1$,

strategic complement if $k_s > 1$.

Therefore it can be said that the subsidy s positively or negatively affects output of the foreign firm according to whether y^{cs} is the strategic complement or the strategic substitute.

The effect on home output is positive. This result is the same as that in the linear duopoly setting. However, contrary to the negative effect in the linear setting, the effect on foreign output can also be positive.[6] We summarize these results.

Proposition 5 *The subsidy of the home government has a positive effect on output of the home firm while it has either a negative effect on the foreign firm's output if the foreign output is a strategic substitute or a positive effect if the foreign output is a strategic complement. Further, regardless of the characterization of the related goods of the foreign firm, it has a positive effect on the total volume of exports to the third country.*

[6] In the Cournot-Nash duopoly competition with the linear inverse demand function, the equilibrium output of the home firm increases with subsidy by sacrificing the foreign firm's market share.

3.3 Subsidy Effect on Profits

We can also analyze the impact of the subsidy on Cournot profits. Inserting the Cournot equilibrium output (26) into firms' profit functions yields the Cournot equilibrium profits under the subsidy:

$$\begin{aligned} \Pi_h^{cs} &= \left(\frac{c_y}{c_x - s + c_y} \right)^2, \\ \Pi_f^{cs} &= \left(\frac{c_x - s}{c_x - s + c_y} \right)^2. \end{aligned} \tag{32}$$

The usual differentiating procedure produces changes in the equilibrium profits caused by the government strategy,

$$\begin{aligned} \frac{\partial \Pi_h^{cs}}{\partial s} &= \frac{2c_y^2}{(c_x - s + c_y)^3} > 0, \\ \frac{\partial \Pi_f^{cs}}{\partial s} &= -\frac{2c_y(c_x - s)}{(c_x - s + c_y)^3} < 0. \end{aligned} \tag{33}$$

The subsidy has a positive effect on the home firm's profit. However, the home government subsidy has a negative effect on the foreign firm's profit.

3.4 Optimal Subsidy

In this subsection we look into the first stage. Given the equilibrium output in the second stage, the home government now has to decide the optimal level of subsidy. This is why the home government is called a Stackelberg-leader. As the home government is assumed to maximize the home country's welfare, we define the welfare of the home country as the total sum of the profit of the home firm less total subsidy:

$$W = \{R(x^{cs}, y^{cs}) - (c_x - s)x^{cs}\} - sx^{cs}, \tag{34}$$

where R denotes the revenue. Substituting (26) for (34) and differentiating it with respect to s yields

$$\frac{dW}{ds} = \left(\frac{\partial R}{\partial x} - c_x \right) \frac{\partial x^{cs}}{\partial s} + \frac{\partial R}{\partial y}\frac{\partial y^{cs}}{\partial s} \tag{35}$$

Taking account of the first-order condition for the profit maximization of the home firm,

$$\frac{\partial R}{\partial x} - (c_x - s) = 0,$$

and the first-order condition for the government's maximization problem, $\frac{dW}{ds} = 0$, yields

$$s = \frac{\partial R}{\partial y} \frac{\partial y^{cs}/\partial s}{\partial x^{cs}/\partial s}, \tag{36}$$

where

$$\begin{aligned} \frac{\partial R}{\partial y} &= \frac{-x^{cs}}{(x^{cs} + y^{cs})^2}, \\ \frac{\partial y^c/\partial s}{\partial x^c/\partial s} &= \frac{c_x - s - c_y}{2c_y}. \end{aligned}$$

Inserting (26) into (36) and solving it for s gives the optimal level of subsidy,

$$s^* = c_y - c_x. \tag{37}$$

So we have $s^* \gtreqless 0 \Longrightarrow k \gtreqless 1$, which is equal to Proposition 1 in Bandyopadhyay (1997). Depending on the firm's relative competitiveness, the optimal trade policy becomes either an export subsidy, laissez-faire, or an export tax. Before government intervention, if the home firm is more efficient than the foreign firm, $c_y > c_x$, the home firm regards x as strategic complements and the foreign firm regards y as strategic substitutes. As a result, the optimal policy is an export subsidy, $s^* > 0$. By the same token, if the home firm is less efficient than the foreign firm, the home firm regards x as strategic substitutes and the foreign firm regards y as strategic complements so that an export tax is optimal, $s^* < 0$. Only when the two firms have the same marginal cost, no intervention by the home government is optimal, $s^* = 0$.

3.5 Stackelberg Equilibrium

Now that we have the optimal value of the home government's trade policy, the SPNE outcomes in the Stackelberg competition can be derived by inserting the optimal level of subsidy s^* for the equilibrium output in the second stage. Then, by inserting (37) into (26), (32), and (34), we have the SPNE output of each firm as well as other SPNE outcomes: the output of each firm is

$$x^S = \frac{c_y}{4c_x^2} \text{ and } y^S = \frac{2c_x - c_y}{4c_x^2}, \tag{38}$$

associated profits of each firm and the optimal welfare of the home country are

$$\Pi_h^S = \left(\frac{c_y}{2c_x}\right)^2, \; \Pi_f^S = \left(\frac{2c_x - c_y}{2c_x}\right)^2 \text{ and } W^S = \frac{c_y}{4c_x}, \tag{39}$$

where superscript S attached to the variables implies the Stackelberg equilibrium. Note that when the foreign firm is less competitive at the Stackelberg equilibrium to such an extent that $2c_x - c_y \leq 0$ holds, then the foreign firm does not export and thus the home firm monopolizes the third country market. To avoid such an interesting case, we assume $2c_x - c_y > 0$ hereafter.

If we substitute the optimal subsidy s^* into (31), the ratio of the marginal costs in the case with a subsidy is

$$k_s^* = \frac{2c_x - c_y}{c_y} \gtreqless 1 \Leftrightarrow k \gtreqless 1. \tag{40}$$

provided $2c_x - c_y > 0$. Thus the economic condition to define the strategic related goods in the case with a subsidy is the same as that in the case without subsidy. From (38) and (39), the ratios of Stackelberg outputs and Stackelberg profits are given by

$$\frac{y^S}{x^S} = k_s^* \text{ and } \frac{\Pi_f^S}{\Pi_h^S} = \left(\frac{y^S}{x^S}\right)^2. \tag{41}$$

According to (40), we can have a similar result to the Stackelberg equilibrium in the case with an optimal subsidy with the one at the Cournot equilibrium obtained in the case with no subsidy summarized in Proposition 4,

$$\text{if } \; k \gtreqless 1, \text{ then } x^S \lesseqgtr y^S \text{ and } \Pi_h^S \lesseqgtr \Pi_f^S. \tag{42}$$

For a graphical understanding of the optimal trade policy, we illustrate the reaction curves before and after the government intervention and the isoprofit curves of the home firm. The Cournot-Nash equilibrium is denoted by C and the Stackelberg equilibrium by S in Figures 3 and 4 where the pictures in the right figure are enlargements of those in the left figure. Reaction curves with no subsidy are depicted with solid lines while the reaction curves with the optimal subsidy are depicted with bold dotted lines in both figures. Two isoprofit curves of the home firms are illustrated with normal dotted lines in the right figure. There, one locus is horizontal at the Cournot equilibrium and the other touches the reaction curve of the foreign firm at the Stackelberg equilibrium. It can be observed that the isoprofit curve passing through the Cournot equilibrium is located above the isoprofit curve passing through the Stackelberg equilibrium, which implies the profit of the home firm after the government intervention is higher than the profit before. Alternatively put, the Stackelberg profit of the home firm is higher than the Cournot profit.

In Figure 3, we depict the case in which the home firm is relatively efficient to the foreign firm, i.e., $k < 1$. According to Proposition 3, x is a strategic complement and y is a strategic substitute.[7] Subsidizing the home firm shifts the reaction curve of the home firm rightward. Since $\dfrac{\partial x^{cs}}{\partial s} > 0$ and $\dfrac{\partial y^{cs}}{\partial s} < 0$ by (33), its effect is to encourage more production of x and less production of y. In consequence, the welfare of the home country is increased at the Stackelberg equilibrium as compared to the Cournot equilibrium as illustrated in the right figure in which the isoprofit curve is shifted downward.

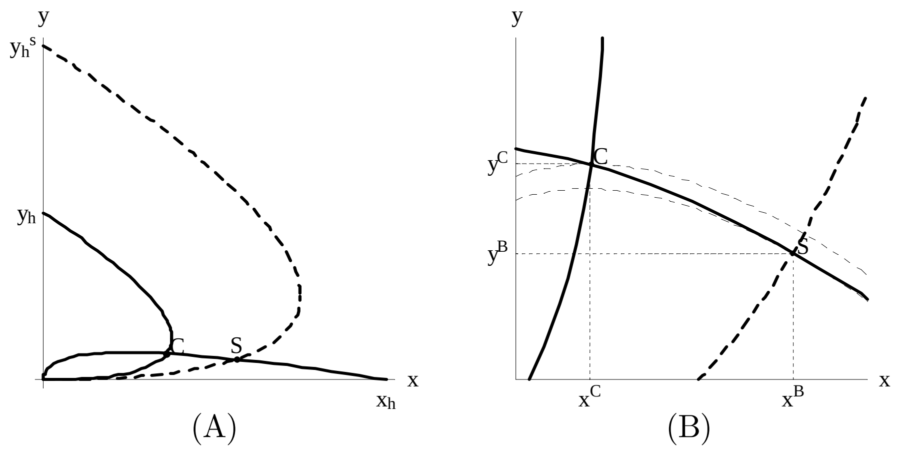


Figure 3. Positive Subsidy for (SC,SS) where $c_x = 2$ and $c_y = 3$.

In Figure 4, we depict the case in which the home firm is relatively inefficient as compared to the foreign firm, i.e., $k > 1$. According to Proposition 3 again, x is a strategic substitute and y a strategic complement. Since the positive subsidy shifts the reaction curve of the home firm rightward so that the home firm's isoprofit curve crosses at a higher point than C, which reduces the home country's welfare. Thus, it is optimal for the home government to restrict the production of the inefficient home firm by levying it an export tax. The export tax shifts the reaction curve of the home firm leftward, so that the Stackelberg equilibrium S is placed at the lower position than C.

[7] (SC, SS) in Figure 3 means that the home firm regards its output as a strategic complement, and the foreign firm's a strategic substitute. (SS, SC) in Figure 4 below is defined in the same manner.

As shown in (33), we have $\dfrac{\partial x^{cs}}{\partial s} > 0$ and $\dfrac{\partial y^{cs}}{\partial s} > 0$.

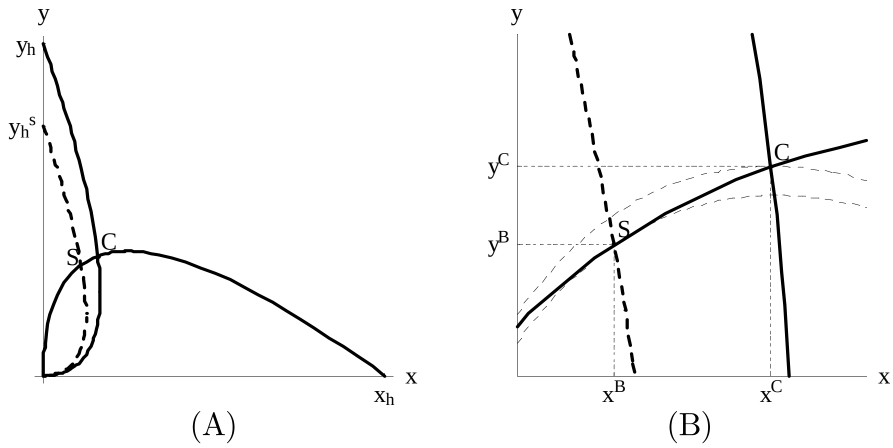


Figure 4. Export tax for (SS,SC) where $c_x = 2$ and $c_y = 1$.

However, it is true that a production subsidy can generate an increase in exports that possibly reduces welfare of the home country. The general conclusion from the graphical analysis is that a positive subsidy benefits the home firm when the foreign firm regards its output as a strategic substitute and is harmful to the home firm when the foregin firm regards its output as a strategic complement.

4 Concluding Remarks

In the international duopoly models, Brander and Spencer [1985] and Eaton and Grossman [1986] derive opposite policy implications for an export subsidy and an export tax, respectively. This is because in the former, products are exogenously assumed to be strategic substitutes and, for the latter, strategic complements. In this paper, we introduced a hyperbolic function and explained that whether products are strategic substitutes or strategic complements depends on demand elasticity. Under a unit elastic demand curve, we showed that a firm's strategic behavior is endogenously determined by a firm's relative competitiveness, which relates to a firm's mound-shaped reaction curves. We also derive the conditions where the optimal trade policy will be an export subsidy or an export tax for the home firm.

As mentioned in footnote 4, the stability assumption for our model is critical to our results and we deal with unstable equilibria in a subsequent paper. Furthermore, a bilateral policy game before the duopolistic competition in the good market must also be investigated.

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