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Abstract

This study proposes an equilibrium model of the term structures of bonds and equities, which has a similar descriptive ability to the reduced-form model proposed by Lettau and Wachter (LW) (J. Financial Economics, 2011), and yet offers economic implications about preferences and consumption dynamics. The ability is obtained by letting the parameters of recursive utility depend on state variables of the economy. The model is calibrated by matching it with the LW model, showing that it can produce the term structure of real interest rates with either a positive or a negative slope and the term structure of dividend risk premiums with a negative slope, both of which stand as challenges to current pricing models. It also shows that while the implied behavior of state-dependent time preference is reasonable, modifications of parameter values and cash-flow processes are necessary for state-dependent risk aversion to behave reasonably.

Keywords: Term structure, Interest rate, Dividend strip, Risk premium, Sharpe ratio, Recursive utility, State-dependent preference.

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1 Introduction

The pricing of cash flows at various points in time is one of the central issues in finance. The term structure of interest rates, which has long been studied, is based on fixed cash flows. Stochastic cash flows such as dividends lead to the term structure of dividend strips or zero-coupon equities, which is of relatively recent focus. This study builds an equilibrium model of the term structures of zero-coupon bonds and equities and discusses the preferences and consumption dynamics implicit in these term structures.

Essentially, any equilibrium model can price any asset. However, few models aim to explain both zero-coupon bonds and equities from a term-structure perspective. To model their term structures, at least two challenges are known. The first is the term structure of real interest rates, which is upward-sloping, flat, or downward-sloping. This indecisive shape in turn requires a model to be flexible. The second is the term structure of risk premiums of dividend strips, which is on average downward-sloping, as evidenced by various data sources such as index options (van Binsbergen, Brandt, and Koijen, 2012), index dividend futures (van Binsbergen et al., 2013; van Binsbergen and Koijen, 2017), and the cross-section of stocks (Weber, 2017).[1] However, as illustrated by van Binsbergen et al. (2012), the downward slope is difficult to explain by using well-established equilibrium models such as the external habit model of Campbell and Cochrane (1999), the long-run risks model of Bansal and Yaron (2004), and the disaster models of Barro (2009) and Gabaix (2012). These challenges motivated recent studies to improve equilibrium models, which are compared with ours below.

Without imposing equilibrium conditions, it may not be very difficult to model the term structures of real interest rates and dividend risk premiums consistently with the stylized facts. Indeed, Lettau and Wachter (2011) (hereafter LW) propose a reduced-form model that can explain these term structures. The key driver behind their success is the specification of the stochastic discount factor (SDF), which increases in response to a negative shock to realized dividend growth. This specification is effective for raising the risk of holding short-term dividend strips. Furthermore, LW assume that a negative shock to realized dividend growth is highly likely to raise expected dividend growth. This assumption makes long-term dividend strips less risky because they rise in value when the SDF is high. Consequently, the term structure of dividend risk premiums slopes downward. In addition, the LW model can generate an upward-sloping,

[1] See also Schulz (2016), who points out that the evidence of a downward slope is inconclusive when returns to short-term dividend claims are adjusted for taxes or liquidities.

flat, or downward-sloping term structure of real interest rates by simply controlling for the correlation between realized dividend growth and real risk-free rate.

These important mechanisms of the LW model are exogenous. Our goal is to endogenize them. The purpose of this study is to develop an equilibrium model that offers implications about the preferences and consumption dynamics and yet has a similar descriptive ability to the reduced-form model proposed by LW. For this purpose, we ask what kind of utility function supports LW's SDF. Our answer is to let the parameters of a recursive utility function of the representative agent depend on the state variables of the economy. Meanwhile, we model cash-flow processes as simply as in the original LW model, although we later consider a minimal extension of these processes.

The idea of state-dependent preferences itself is not new. In fact, Gordon and St-Amour (2000, 2004), Melino and Yang (2003), Chabi-Yo, Garcia, and Renault (2008), Berrada, Detemple, and Rindisbacher (2013), and Dew-Becker (2014) consider models in which preference parameters themselves change over time. A distinct feature of the current model is that both the risk-aversion and time-preference parameters are driven by many state variables such as (expected) consumption and inflation growth as well as financial variables such as the risk-free rate and price of risks, meaning that the agent can fine-tune her preferences by looking at the economy and asset markets.

A state-dependent risk aversion is beneficial for amplifying the variation in the SDF and hence capturing high equity risk premiums. Moreover, it leads to time-varying price of risks, which naturally explains time-varying risk premium (the product of the price of risks and an asset-specific quantity of risks) with the source of variation not limited to stochastic volatility, or stochastic intensity in the case of jumps, of cash-flow processes. Furthermore, it offers an additional channel of raising the slope of the term structure of nominal interest rates other than the standard channel of a negative correlation between consumption and inflation growth. A state-dependent time preference also has the advantage of generating various shapes in the term structure of real interest rates. Suppose, for example, there is a shock that raises the SDF. If this shock also affects agent's time preference in a way in which she more heavily discounts (utilities from) future cash flows, the prices of real bonds will fall, with the fall more significant for longer-term bonds because of the compound effect. Real bonds therefore cannot be hedging instruments against events that raise the SDF, and the real term structure will thus slope upward.

The parameters of the proposed model are calibrated by matching it with the LW model, which has two advantages. The first is to achieve a similar descriptive ability to the LW model. Indeed, the proposed model can closely replicate various term-structure shapes generated by the LW model. The second is to obtain an equilibrium foundation of the LW model. It is possible to imply the preferences and consumption dynamics from the LW model through the calibration of the proposed model. This calibration approach is similar in spirit to Backus, Boyarchenko, and Chernov (2018), who first establish the facts about the level and shape of the various term structures and then identify the features theoretical models (in both reduced and structural forms) should possess to be consistent with the facts. As the facts here, we use the term structures generated by the LW model. Unlike Backus et al. (2018) focusing on the extension of cash-flow processes while using the standard recursive utility, we extend the utility function with cash-flow processes kept simple.

The calibration results contain unrealistic implications about the preferences and/or consumption dynamics. Most notably, given consumption volatility of less than 4% per year, the mean and standard deviation of state-dependent risk aversion reach 150 and 128, respectively. Conversely, when mean risk aversion reduces to 30, then the implied consumption volatility reaches nearly 9%.

One possibility of these implications is that the parameter values originally calibrated by LW are unrealistic. Since the LW model is a reduced-form model, it allows for many combinations of the parameter values, any of which can explain the observed term structures. However, once some equilibrium conditions are imposed, few combinations of the parameter values are consistent with not only the observed term structures but also realistic preferences and consumption dynamics. Our model uncovers which combinations are more appropriate. Another possibility is that the dynamics of cash flows in the LW model are too simple. We then slightly deviate from the LW model by introducing jumps into consumption and dividend growth, which are interpreted as disasters. The change in parameter values and modification of cash-flow processes together are shown to be effective for making risk aversion and consumption growth economically plausible, while retaining the ability to explain the various term structures.

Related literature

Our model extends the recursive utility function of Epstein and Zin (1989, 1991) and Weil (1989) in a way in which the risk-aversion and time-preference parameters depend on the state variables of the economy. Melino and Yang (2003) consider the recursive utility function with state-dependent parameters more generally in that the elasticity of intertemporal substitution (EIS) is also state-dependent; however, they do not model how these parameters evolve over time. The law of motion of risk aversion is modeled by Gordon and St-Amour (2000), and Chabi-Yo et al. (2008) by using Markov-switching processes. Berrada et al. (2013) also use them for driving both risk aversion and time preference. Gordon and St-Amour (2004) and Dew-Becker (2014) model risk aversion as driven by autoregressive processes. Our model is similar to the last two studies regarding how to model time variation but different from them in that time preference is also state-dependent and that the preference parameters are driven by many state variables that drive the economy. Mehra and Sah (2002) study the impact of changes in a time-preference parameter or a risk-aversion parameter on the volatility of stock returns. We address the importance of time-varying preference parameters for describing the term structures.

Whether and how risk preferences vary has been examined using surveys and/or experiments. Andersen et al. (2008) perform field experiments using lottery choices with real monetary rewards and find that risk preferences are state dependent with respect to personal finances but not macroeconomic perspectives. Brunnermeier and Nagel (2008), Chiappori and Paiella (2011), and Liu, Yang, and Cai (2016) ask whether wealth drives risk preferences, proxied by a share of risk assets in wealth portfolio, reporting mixed results. Kuhnen and Knutson (2011), Cohn et al. (2015), and Guiso, Sapienza, and Zingales (2018) relate the change in risk preferences to emotions and find evidence that anxiety or fear can raise risk aversion by an economically significant magnitude.[2]

The possibility that time preference is varying over time and/or across individuals has also been considered; see Frederick, Loewenstein and O'Donoghue (2002) for a review and Halevy (2015) for recent experimental evidence of the variation in subjective discount rate. Becker and Mulligan (1997), and Stern (2006) model an endogenous subjective discount rate as a function of future-oriented capital, invested for increasing the propinquity of future utilities. Unlike their work, time preference in our model is driven by exogenous variables, which simplifies the consumption problem and keeps the model tractable for asset-pricing purposes. The exogenous subjective discount rate in this study is also different from the well-established, horizon-dependent

[2]In Cohn et al. (2015, pp. 863-4): "In standard theory, expectations typically do not affect preferences. If, however, price expectations affect fear levels, they may also directly affect risk preferences." We model preference parameters as a function of state variables including those affecting expected cash-flow growth.

time preference that discounts nearby cash flows more heavily than distant ones; see, for example, Thaler (1981). However, Harris and Laibson (2001) and Luttmer and Mariotti (2003) show that the horizon-dependent time preference leads to an effective subjective discount rate that depends on the state variables affecting endowment growth (unless the agent has log utility).

Recent studies develop equilibrium asset-pricing models to explain the stylized facts about dividend strips. We limit our attention to some of these studies that explicitly present the results for the entire term structures of risk premiums and return volatilities of dividend strips, which are summarized in Table 1; see van Binsbergen and Koijen (2017) for a broader review.

There are two main approaches to improving equilibrium models: one is to improve preferences and the other cash flows. This study belongs to the former. Recent studies taking the preference approach are as follows. Curatola (2015) considers heterogeneous agents who have loss-averse utility, in which the reference point between gain and loss is set at the external consumption habit, so that unlike many standard habit formation models, consumption is allowed to be below the habit. Because the loss-averse agents are willing to hold long-term dividend claims to hedge the risks of future consumption being below the habit, the term structure of dividend risk premiums slopes downward. Meanwhile, the term structure of real interest rates slopes upward because long-term real bonds cannot hedge increase in the habit and hence decrease in the surplus consumption. Doh and Wu (2016) impose a structure on the long-run risks model such that both the equilibrium wealth-consumption ratio and the price of a one-period dividend strip are quadratic functions of the state variables and then reverse-engineer the consumption and dividend processes consistent with the imposed structure. The resulting risk premiums of dividend strips are first decreasing with maturity and then increasing, which is not surprising as the premiums are also quadratic in the state variables. Our model is as flexible as the reduced-form model proposed by LW and can generate both a downward-sloping term structure of dividend risk premiums and either an upward-sloping or a downward-sloping term structure of real interest rates. This flexibility is owing to state-dependent preferences.

Recent studies that modify cash-flow processes propose various mechanisms that make short-run growth in dividends volatile and procyclical relative to long-run growth. Belo, Collin-Dufresne, and Goldstein (2015) consider as a mechanism a stationary financial leverage ratio. In their model, in response to a temporal increase (decrease) in corporate earnings measured by EBIT, a firm is assumed to increase (decrease) debt to keep the leverage ratio to a stationary level, which further increases (decreases) the cash distributed to shareholders as dividends.

Consequently, dividends change more intensely than earnings in the short run. In the long run, however, both EBIT and dividends are exposed to the same amount of risks because of the stationarity of the leverage ratio, which makes EBIT and dividends cointegrated. Favilukis and Lin (2016) consider as a mechanism wage rigidity in a production economy, where a negative transitory shock to technology, corresponding to poor economic states, reduces dividends more than wages that are settled infrequently. Lopez, Lopez-Salido and Vazquez-Grande (2015) consider a similar logic but instead use nominal rigidity that induces a countercyclical wage share of output and hence a procyclical dividend share. Marfè (2017) also uses the wage channel together with the limited participation of asset markets. Specifically, in his model, shareholders who receive and consume dividends provide wage insurance to workers, which is effective for the short run but not for the long run because both dividends and wages are cointegrated (i.e., they share the same long-run risks). Then, dividends, or equivalently shareholders' consumption in equilibrium, are more prone to transitory shocks than wages. Meanwhile, only shareholders can access asset markets. Consequently, in the eyes of pricing agents (i.e., shareholders), short-term dividend strips look riskier than long-term dividend strips.

Hasler and Marfè (2016) introduce recovery after disaster into cash-flow processes as well as the stochastic mean of cash-flow growth and stochastic intensity of disaster occurrence. While the latter two features alone may generate an upward-sloping term structure of dividend risk premiums as does the Wachter (2013) model, the fast recovery in dividend growth after a large negative shock reduces the risks of holding long-term dividend strips, more than offsetting the long-run risks associated with the stochastic mean growth and disaster intensity.

A novel approach taken by Croce, Lettau, and Ludvigson (2015) for generating a downward-sloping term structure of dividend risk premiums is that they do not change cash-flow processes from those originally specified by Bansal and Yaron (2004) but do change the way in which they are estimated. In their framework, the agent overestimates the impact of short-run shocks to consumption growth on dividend growth because she erroneously revises a long-run component of dividend growth that is irrelevant to short-run shocks to consumption growth. Consequently, she requires high premiums for holding short-term dividend strips. Conversely, long-run shocks to consumption growth, which are originally small, are difficult to infer from dividend growth because they are contaminated by large idiosyncratic shocks to dividend growth. Then, long-run consumption risks are not properly priced into long-term dividend strips, and they do not command high risk premiums.

Our model taking the preference approach complements the models taking the cash-flow approach. Indeed, it can easily be combined with more sophisticated cash flows, which further improves statistical adequacy and economic plausibility.

The rest of the manuscript is organized as follows. Section 2 presents the model. Section 3 explains how to calibrate the parameters of the proposed model with a brief introduction of the LW model. Section 4 verifies the performance of the proposed model and discusses the implied consumption dynamics and preferences. Section 5 introduces jumps into cash-flow processes to obtain more plausible economic implications. Section 6 concludes. The technical arguments are collected in the appendices.

2 Model

Our model is built on a simple exchange economy, in which the flow of endowments is exogenously provided and a rational, representative agent has recursive utility of Epstein and Zin (1989, 1991) and Weil (1989). Section 2.1 first specifies the utility function and then extends it in a way in which the risk-aversion and time-preference parameters depend on the state variables of the economy. Sections 2.2–2.4 specify the endowment process and state-dependent preferences such that the recursive equation for agent's continuation value is solved in closed form for a certain case, which is presented in Section 2.5. Section 2.6 derives an analytical approximation of the continuation value for a general case, guided by the results of Section 2.5. Finally, Section 2.7 provides the pricing formulas for zero-coupon bonds and equities. The derivations of the key equations are provided in Appendix A.

2.1 Preference

Let U_t denote the time-t utility of the representative agent, which is specified by the following recursive form:

$$U_t = \{(1-\beta)C_t^{\rho} + \beta E_t[U_{t+1}^{1-\gamma}]^{\rho/(1-\gamma)}\}^{1/\rho} , \tag{1}$$

where C_t is aggregate consumption at time t to be determined by the agent (the decision variable) and $E_t[\cdot]$ stands for expectation conditioned on time t. There are three parameters in U_t: β represents time preference or subjective discount factor (typically somewhat less than one), γ is a coefficient of risk aversion, and ρ is related to EIS as $1/(1-\rho)$.

To capture the average term structures of zero-coupon bonds and equities, we let β and

γ be state-dependent as in Melino and Yang (2003). Unlike their study, we set ρ to zero or equivalently the EIS to unity. This restriction has the advantage of keeping the model simple without reducing the goodness-of-fit to at least the average term structures. The unit EIS is considered by Piazzesi and Schneider (2006) to model the term structure of interest rates. Hansen et al. (2007) show that a model with the unit EIS can be used as a basis for approximating more general models.

By substituting (β_t, γ_t) for (β, γ) and $\rho = 0$ in (1),

$$U_t = C_t^{1-\beta_t} E_t[U_{t+1}^{1-\gamma_t}]^{\beta_t/(1-\gamma_t)} \ . \tag{2}$$

To solve the optimal consumption problem, we make the following assumptions: (i) β_t and γ_t are exogenous, and (ii) $0 < \beta_t < 1$ for all t. Assumption (i) is also considered by Gordon and St-Amour (2004), who model the risk-aversion coefficient directly as a stochastic process. The analogous assumption is made by Campbell and Cochrane (1999) in the form of the external habit. Since β_t and γ_t are not affected by the decision variable, the optimal consumption problem can be solved in the same way as in the case of constant preference parameters. Assumption (ii) guarantees that the period utility in (2) is concave with respect to the decision variable (see (83) in Appendix A) and that wealth is positive in equilibrium (derived just below).

Let $C_t^e > 0$ be the time-t endowment and W_t be time-t wealth, which in the endowment economy is the cum-dividend value of a claim to the flow of endowments. The gross rate of return to wealth, $R_{w,t+1}$, is defined by

$$R_{w,t+1} = \frac{W_{t+1}}{W_t - C_t^e} \ . \tag{3}$$

Then, the budget constraint for the agent is

$$R_{w,t+1}(W_t - C_t) = W_{t+1} \ . \tag{4}$$

Let V_t be the continuation value, which is the solution to the following problem:

$$V_t = \max_{C_t} U_t \qquad \text{subject to (4)} \ . \tag{5}$$

Because of the unit EIS, the optimal consumption, C_t^*, has a closed form irrespective of how β_t and γ_t are specified:

$$C_t^* = (1 - \beta_t) W_t \ . \tag{6}$$

Unlike a constant-parameter case, the wealth-consumption ratio, W_t/C_t^*, varies over time even for the unit EIS.

The equilibrium condition is that the agent consumes the given endowment, $C_t^* = C_t^e$. Wealth in equilibrium, W_t^*, is then solved as $W_t^* = \frac{1}{1-\beta_t} C_t^e$ from (6). By substituting W_t^* into (3), the equilibrium gross rate of return to wealth, $R_{w,t+1}^*$, is

$$R_{w,t+1}^* = \frac{1}{\beta_t} \frac{1-\beta_t}{1-\beta_{t+1}} \frac{C_{t+1}^e}{C_t^e} \ . \tag{7}$$

The continuation value in equilibrium, V_t^*, is the solution to the following recursive equation. Define $\nu_t = V_t^*/C_t^e$, and ν_t satisfies [3]

$$\nu_t = E_t \left[\left(\nu_{t+1} \frac{C_{t+1}^e}{C_t^e} \right)^{1-\gamma_t} \right]^{\beta_t/(1-\gamma_t)} \ . \tag{8}$$

The SDF, M_{t+1}, is obtained as

$$M_{t+1} = \beta_t \frac{1-\beta_{t+1}}{1-\beta_t} \left(\frac{\nu_{t+1}}{\nu_t^{1/\beta_t}} \right)^{1-\gamma_t} \left(\frac{C_{t+1}^e}{C_t^e} \right)^{-\gamma_t} \ . \tag{9}$$

In general, the recursive equation for ν_t cannot be solved in closed form, which makes the SDF unavailable in closed form. In the next subsections, we specify the endowment process and state-dependent preferences in a way in which ν_t is solved in closed form for a constant time preference and approximately for a state-dependent time preference, keeping accuracy in mind.

2.2 Dynamics

Following LW, all the variables are assumed to be homoscedastic. Define $c_t = \ln C_t^e$ and $\Delta c_{t+1} = c_{t+1} - c_t$. The evolution of the rate of growth in endowment, which is equal to aggregate consumption in equilibrium, is specified as

$$\Delta c_{t+1} = \mu_c + b_c' x_t + \sigma_c' z_{t+1} \ , \tag{10}$$

where μ_c is the unconditional mean of the consumption-growth rate (given that the unconditional mean of x_t is zero), x_t is a d-dimensional vector of state variables, and z_{t+1} is a $(d+3)$-dimensional vector of *i.i.d.* normal random variables. The reason for the $(d+3)$ dimension will be clear soon.

To price nominal zero-coupon bonds, a general price index, Π_t, is introduced, which is assumed to be determined exogenously. Define $\pi_t = \ln \Pi_t$ and $\Delta\pi_{t+1} = \pi_{t+1} - \pi_t$. Then, the evolution of the rate of inflation growth is specified as

$$\Delta\pi_{t+1} = \mu_\pi + b_\pi' x_t + \sigma_\pi' z_{t+1} \ , \tag{11}$$

[3] We call ν_t the continuation value unless otherwise noted because V_t^* does not explicitly appear in the discussions to follow.

where the parameters and variables are interpreted similarly to the consumption process in (10).

To price dividend strips, the flow of aggregate dividends needs to be specified. Let D_t be the aggregate dividend paid at time t. Define $d_t = \ln D_t$ and $\Delta d_{t+1} = d_{t+1} - d_t$. Then, the evolution of the rate of dividend growth is specified as

$$\Delta d_{t+1} = \mu_d + b_d' x_t + \sigma_d' z_{t+1} \; . \tag{12}$$

The aggregate dividend can be regarded as levered consumption in an endowment economy. The most direct description of this relation is $D_t = (C_t^e)^a$ for some constant $a > 1$ (Abel, 1999; Campbell, 2003). In addition, it is often assumed that Δc_{t+1} and Δd_{t+1} are cointegrated (Bansal, Gallant, and Tauchen, 2007). We consider their link when calibrating the parameters in Section 3.2.

Finally, a d-dimensional state vector x_t is assumed to follow

$$x_{t+1} = \Phi_x' x_t + \sigma_x' z_{t+1} \; . \tag{13}$$

Notice that the unconditional mean of x_t is zero and that there are $(d + 3)$ variables in the economy.

For notational convenience, define $s_{ij} = \sigma_i' \sigma_j$. For example, the covariance between innovations in Δc_{t+1} and Δd_{t+1} is denoted as $s_{cd} = \sigma_c' \sigma_d$ (scaler). Likewise, the covariance between innovations in Δc_{t+1} and x_{t+1} is denoted as $s_{cx} = \sigma_x' \sigma_c$ ($d \times 1$ vector) and the variance of innovation in x_{t+1} as $s_{xx} = \sigma_x' \sigma_x$ ($d \times d$ matrix).

2.3 Risk aversion

We specify the coefficient of risk aversion as a linear function of the state vector:

$$\gamma_t = \mu_\gamma + b_\gamma' x_t \; . \tag{14}$$

The linear specification has the advantage of obtaining the SDF in closed form when time preference is constant. It is also useful for a state-dependent time preference, which is discussed in Section 2.6.

One caveat is that since x_t is Gaussian, γ_t becomes negative with a positive probability. This shortcoming is also seen in the previous work. Gordon and St-Amour (2004) and Dew-Becker (2014) specify γ_t as a part of the VAR(1) system and an AR(1) process, respectively; however, they do not theoretically impose the positivity of γ_t. The probability of $\gamma_t < 0$ in this study is addressed after calibrating the parameters in Section 4.1.

2.4 Time preference

We consider the following two specifications:

$$\text{(S1)}\quad \beta_t \;=\; \beta\;, \tag{15}$$

$$\text{(S2)}\quad \beta_t \;=\; 1-\exp\{\mu_\beta + b_\beta' x_t\} \qquad (\mu_\beta < 0)\;. \tag{16}$$

S1 allows us to solve the recursive equation (8) for ν_t in closed form. The resulting formula is exponentially linear in x_t, leading to the SDF of the affine class. The prices of zero-coupon bonds and equities are therefore available in closed form, which are also exponentially linear in x_t.

For any specification of β_t except S1, ν_t has no closed form. To retain tractability, we thus perform an analytical approximation of ν_t in a way in which the affine-pricing framework is available as for S1. S2 aims to retain the accuracy of the approximation rather than being based on economic reasoning or statistical adequacy. Specifically, once $\ln \nu_t$ is approximated as a linear function of x_t (this approximation is inevitable for any specification of β_t), the price of risks is derived as a linear function of x_t without further approximation. Intuitively, this is understood by noting that the SDF given in (9) has a term $1-\beta_{t+1}$, which in S2 is exponentially linear in x_{t+1}. Moreover, the wealth-consumption ratio given in (6) is log-linear in x_t.

One caveat of S2 is that β_t becomes negative with a positive probability, violating the lower bound constraint in Assumption (ii). The severity of this violation depends on the parameter values and therefore is addressed after the calibration in Section 4.1.

2.5 SDF for S1

We derive the continuation value and SDF for S1. Although our interest is in S2, the results for S1 are worth presenting for three reasons. First, they are an extension of the results presented by Hansen, Heaton and Li (2008). The extension is in the coefficient of risk aversion: this is constant in Hansen et al. (2008), whereas it is a linear function of the Gaussian state vector in this study. Second, the fact that the SDF for S1 derived here is exact while that for S2 is approximate clarifies the source of the approximation and provides the sense of the accuracy (the cost of S2). Third, through the comparison with S1, it is highlighted how the risk-free rate and price of risks are extended (the benefit of S2).

The recursive equation (8) for ν_t is solved as

$$\nu_t = \exp\{\mu_\nu + b_\nu' x_t\}\;, \tag{17}$$

where μ_ν and b_ν are the solutions to the following simultaneous equations:

$$\mu_\nu = \beta \left\{ \mu_\nu + \mu_c - \frac{1}{2} v_{c\nu} (\mu_\gamma - 1) \right\} , \tag{18}$$

$$b_\nu = \beta \left(b_c + \Phi_x b_\nu - \frac{1}{2} v_{c\nu} b_\gamma \right) , \tag{19}$$

where

$$v_{c\nu} = \mathrm{var}_t[\ln \nu_{t+1} + \Delta c_{t+1}] = b'_\nu s_{xx} b_\nu + 2 s'_{cx} b_\nu + s_{cc} . \tag{20}$$

(18) and (19) are quadratic equations because $v_{c\nu}$ is quadratic in b_ν. Appendix B provides the condition for μ_ν and b_ν to be real and addresses which real root to select. It is noted that setting $b_\gamma = 0$ in (19) (i.e., a constant risk-aversion coefficient) leads to the continuation value presented by Hansen et al. (2008).

Next, we drive the price-of-risk vector, denoted as λ_t. This is the (negative) loading on the innovation vector z_{t+1} in the SDF. By taking the log of (9) with β_t replaced by β and defining $m_{t+1} = \ln M_{t+1}$,

$$m_{t+1} = (1 - \gamma_t) \ln \nu_{t+1} - \gamma_t \Delta c_{t+1} + res^m_t , \tag{21}$$

where res^m_t collects the remaining terms observed at time t. By substituting (17) and then (10) and (13) into (21), we have $m_{t+1} - E_t[m_{t+1}] = -\lambda'_t z_{t+1}$, where

$$\lambda_t = (\sigma_x b_\nu + \sigma_c) \gamma_t - \sigma_x b_\nu . \tag{22}$$

Since γ_t is assumed to be linear in x_t, so is λ_t. Owing to γ_t, the risk premium of any asset is also time-varying even without time-varying volatility of cash flows. A potential drawback of λ_t in (22) is that it is driven by γ_t alone, which is a certain linear combination of the d-dimensional state vector as given in (14). This implies that the correlation between risk premiums of any pair of assets is one in absolute value. LW make a similar assumption that the price-of-risk vector is driven by one factor and point out the drawbacks of this assumption.

The one-period real risk-free rate, denoted as $r_{f,t+1}$, is the solution to the following Euler equation: $r_{f,t+1} = -\ln E_t[M_{t+1}]$. Then, it is also derived as a linear function of x_t:

$$r_{f,t+1} = A_f + B'_f x_t , \tag{23}$$

where

$$A_f = -\ln \beta + \mu_c - \frac{1}{2} s_{cc} - (s'_{cx} b_\nu + s_{cc})(\mu_\gamma - 1) , \tag{24}$$

$$B_f = b_c - (s'_{cx} b_\nu + s_{cc}) b_\gamma . \tag{25}$$

Finally, m_{t+1} can be rewritten in a conventional form as

$$m_{t+1} = -r_{f,t+1} - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t' z_{t+1} \ . \tag{26}$$

Since both $r_{f,t+1}$ and λ_t are linear in x_t, m_{t+1} falls into the affine class.

2.6 SDF for S2

The recursive equation (8) for ν_t cannot be solved in closed form for a state-dependent time preference, which is denoted here as $\beta(x_t)$ to clarify the dependence on x_t. To retain tractability, we therefore approximate ν_t as an exponentially linear function of x_t. First, $\beta(x_t)$ and $\beta(x_t)x_t$ are linearized around $x_t = 0$ (the unconditional mean):

$$\begin{aligned} \beta(x_t) &\approx \beta_0 + \beta_1' x_t \ , & (27) \\ \beta(x_t)x_t &\approx \beta_0 x_t \ , & (28) \end{aligned}$$

where $\beta_0 = \beta(0)$ and $\beta_1 = \frac{d\beta(x_t)}{dx_t}|_{x_t=0}$. For S2, these are, respectively, $\beta_0 = 1 - e^{\mu_\beta}$ and $\beta_1 = -e^{\mu_\beta} b_\beta$. Then, ν_t is approximated by an exponentially linear function of x_t as given in (17), where the coefficients satisfy the following simultaneous quadratic equations:

$$\begin{aligned} \mu_\nu &= \beta_0 \left\{ \mu_\nu + \mu_c - \frac{1}{2} v_{c\nu}(\mu_\gamma - 1) \right\} \ , & (29) \\ b_\nu &= \beta_0 \left(b_c + \Phi_x b_\nu - \frac{1}{2} v_{c\nu} b_\gamma \right) + \beta_1 \left\{ \mu_\nu + \mu_c - \frac{1}{2} v_{c\nu}(\mu_\gamma - 1) \right\} \ , & (30) \end{aligned}$$

where $v_{c\nu}$ is given in (20). Notice that the second term on the RHS of (30) is newly added by S2.

The accuracy of the approximation of ν_t is examined in Appendix C. In brief, it seems to be maintained for the parameter values determined by the calibration procedure in Section 3.2 and given specifically in Tables 2 and 4. Intuitively, the reason for the high accuracy is that $\beta(x_t)$ changes little, as will be addressed in Section 4.1 and Figure 1(b). Then, (27) and (28) are not bad approximations after all.

Once $\ln \nu_t$ is approximated as a linear function of x_t, the price-of-risk vector λ_t is derived as linear in x_t without further approximation, which is due to S2 together with a linear specification of γ_t. Specifically, from (9), the log SDF can be written as

$$m_{t+1} = \ln(1 - \beta_{t+1}) + (1 - \gamma_t) \ln \nu_{t+1} - \gamma_t \Delta c_{t+1} + res_t^m \ , \tag{31}$$

where res_t^m collects the remaining terms observed at time t. By substituting (16) and (17) and then (10) and (13) into (31), we have as before $m_{t+1} - E_t[m_{t+1}] = -\lambda_t' z_{t+1}$, where

$$\lambda_t = (\sigma_x b_\nu + \sigma_c)\gamma_t - \sigma_x(b_\beta + b_\nu) \; . \tag{32}$$

From the linear specification of γ_t given in (14), λ_t is also linear in x_t. Apart from b_ν that is different between S1 and S2, $-\sigma_x b_\beta$ is newly added by the extension to state-dependent time preference.

Finally, to obtain the one-period real risk-free rate $r_{f,t+1}$ as a linear function of x_t, we need to rely on another approximation, which is to linearize $\ln \beta(x_t)$ around $x_t = 0$. Specifically for S2,

$$\ln(1 - \exp\{\mu_\beta + b_\beta' x_t\}) \approx \ln(1 - e^{\mu_\beta}) - \frac{e^{\mu_\beta}}{1 - e^{\mu_\beta}} b_\beta' x_t \; . \tag{33}$$

Again, this approximation may not be a serious concern because of the small variation in $\beta(x_t)$ noted above. Then, $r_{f,t+1}$ is approximated as given in (23), where the coefficients are as follows:

$$A_f = -\ln \beta_0 + \mu_c - \frac{1}{2} v_{c\beta} - (\mu_\gamma - 1)(s_{cx}' b_\nu + s_{cc}) + (\mu_\gamma - 1)(s_{xx} b_\nu + s_{cx})' b_\beta \; , \tag{34}$$

$$B_f = b_c - (s_{cx}' b_\nu + s_{cc}) b_\gamma + \left\{ \frac{1}{\beta_0} I_{d\times d} - \Phi_x + b_\gamma (s_{xx} b_\nu + s_{cx})' \right\} b_\beta \; , \tag{35}$$

where $I_{d\times d}$ is a d-by-d identity matrix and

$$v_{c\beta} = \text{var}_t[\ln(1 - \beta_{t+1}) - \Delta c_{t+1}] = b_\beta' s_{xx} b_\beta - 2 s_{cx}' b_\beta + s_{cc} \; . \tag{36}$$

By setting $b_\beta = 0$, A_f and B_f in (34) and (35) reduce to those in (24) and (25), respectively.

2.7 Prices of zero-coupon bonds and equities

Both the risk-free rate and the price of risks are derived as linear functions of the Gaussian state vector exactly for S1 and approximately for S2. We now turn to the pricing of zero-coupon bonds and equities by utilizing the affine framework.

2.7.1 Real zero-coupon bonds

Let $P_{t,n}^R$ be the time-t real price of a zero-coupon bond maturing at time $t+n$ with the face value normalized to one unit of consumption. The Euler equation for $P_{t,n}^R$ is $P_{t,n}^R = E_t[M_{t+1} P_{t+1,n-1}^R]$ with the initial condition $P_{t,0}^R = 1$. The solution is of the form

$$P_{t,n}^R = \exp\{A_n^R + B_n^{R\prime} x_t\} \; , \tag{37}$$

where A_n^R and B_n^R are determined recursively, starting with $A_0^R = 0$ and $B_0^R = 0$. The recursive equations are provided in Appendix D.

2.7.2 Nominal zero-coupon bonds

Let $P_{t,n}^N$ be the time-t real price of a zero-coupon bond maturing at time $t+n$ with the face value normalized to one in nominal terms or equivalently $1/\Pi_{t+n}$ in real terms. The Euler equation for $P_{t,n}^N$ is $P_{t,n}^N = E_t[M_{t+1}P_{t+1,n-1}^N]$ with the initial condition $P_{t,0}^N\Pi_t = 1$. It follows that

$$P_{t,n}^N\Pi_t = E_t\left[M_{t+1}(P_{t+1,n-1}^N\Pi_{t+1})\frac{\Pi_t}{\Pi_{t+1}}\right] . \tag{38}$$

The solution to (38) is of the form

$$P_{t,n}^N\Pi_t = \exp\{A_n^N + B_n^{N\prime}x_t\} , \tag{39}$$

where A_n^N and B_n^N are determined recursively, starting with $A_0^N = 0$ and $B_0^N = 0$. The recursive equations are provided in Appendix D.

2.7.3 Zero-coupon equities or dividend strips

Let $P_{t,n}^D$ be the time-t real price of a zero-coupon equity that pays D_{t+n} at time $t+n$. The Euler equation for $P_{t,n}^D$ is $P_{t,n}^D = E_t[M_{t+1}P_{t+1,n-1}^D]$ with the initial condition $P_{t,0}^D/D_t = 1$. It follows that

$$\frac{P_{t,n}^D}{D_t} = E_t\left[M_{t+1}\frac{P_{t+1,n-1}^D}{D_{t+1}}\frac{D_{t+1}}{D_t}\right] . \tag{40}$$

The solution to (40) is of the form

$$P_{t,n}^D/D_t = \exp\{A_n^D + B_n^{D\prime}x_t\} , \tag{41}$$

where A_n^D and B_n^D are determined recursively, starting with $A_0^D = 0$ and $B_0^D = 0$. The recursive equations are provided in Appendix D.

3 Calibration

We calibrate the parameters of the proposed model by matching it with the LW model. Specifically, both the one-period real risk-free rate $r_{f,t+1}$ and the price-of-risk vector λ_t are matched between the two models, which means from equation (26) that the two models have the identical SDF and therefore that they agree with the price of any asset. This calibration approach has two advantages. First, it provides the proposed model with the opportunity to inherit a high descriptive ability of the LW model with respect to the average term structures of zero-coupon bonds and equities. Second, it provides the LW model with an equilibrium foundation, thereby uncovering the preferences and consumption dynamics implicit in this reduced-form model. Section 3.1 introduces the LW model and Section 3.2 explains the calibration procedure.

3.1 The LW model

The LW model has the following six variables (the notation is slightly different from the original one):

Δd_t : dividend growth rate

$\Delta \pi_t$: inflation growth rate

$x_{d,t}$: factor driving the expected dividend growth rate

$x_{\pi,t}$: factor driving the expected inflation growth rate

$x_{f,t}$: factor driving the real risk-free rate

$x_{\lambda,t}$: factor driving the price of risks

Note that consumption growth rate does not appear in the LW model as it is a reduced-form model. The last four variables are collected in a state vector, denoted as x_t^{LW}:

$$x_t^{LW} = (x_{d,t} \quad x_{\pi,t} \quad x_{f,t} - \mu_f \quad x_{\lambda,t} - \mu_\lambda)' \ , \tag{42}$$

where μ_f and μ_λ are the unconditional means of $x_{f,t}$ and $x_{\lambda,t}$, respectively (those of $x_{d,t}$ and $x_{\pi,t}$ are implicitly assumed to be zero). The dynamics of these variables are specified as

$$\Delta d_{t+1} = \mu_d + x_{d,t} + \sigma_d' z_{t+1} \ , \tag{43}$$

$$\Delta \pi_{t+1} = \mu_\pi + x_{\pi,t} + \sigma_\pi' z_{t+1} \ , \tag{44}$$

$$x_{t+1}^{LW} = \Phi_x^{LW\prime} x_t^{LW} + \sigma_x^{LW\prime} z_{t+1} \ . \tag{45}$$

The log SDF of the LW model, denoted as m_{t+1}^{LW}, is specified exogenously as

$$m_{t+1}^{LW} = -x_{f,t} - \frac{1}{2} s_{dd} x_{\lambda,t}^2 - x_{\lambda,t} \sigma_d' z_{t+1} \ , \tag{46}$$

where $s_{dd} = \sigma_d' \sigma_d$. A notable feature of m_t^{LW} is that it is driven by the same innovation term as driving dividend growth, $\sigma_d' z_{t+1}$. The conditional correlation between m_{t+1}^{LW} and Δd_{t+1} is then $-x_{\lambda,t}/|x_{\lambda,t}|$. Since the parameters calibrated by LW imply $\Pr\{x_{\lambda,t} > 0\} = 0.99$, these two variables can safely be regarded as perfectly negatively correlated. That is, a negative shock to dividend growth almost always raises the SDF. This mechanism is the key to generating high risk premiums of short-term dividend strips that are strongly affected by shocks to dividend growth. Risk premiums arising from shocks to the other variables are nonzero as long as these shocks have nonzero correlations with the dividend-growth shock.

Table 2 summarizes the parameter values of the LW model, which are collected from tables 1–3 in LW (2011). The unconditional means, standard deviations, and autocorrelations are expressed in annual terms, except for the conditional first and second moments of $x_{\lambda,t}$ expressed in raw numbers. The annual numbers are transformed into quarterly raw numbers when substituted into the models.

Several notes on the parameter values are in order. First, the autoregressive matrix of x_t^{LW}, Φ_x^{LW}, is diagonal. The expected dividend-growth factor $x_{d,t}$ and risk-free rate factor $x_{f,t}$ are relatively persistent as the autoregressive coefficients are equal to and larger than 0.9, respectively. Second, the correlation between innovations in Δd_t and $x_{d,t}$ is -0.83, indicating that a negative shock to realized dividend growth is more likely to increase expected dividend growth. An important implication of the negative correlation is that long-term dividend strips are not as risky as short-term ones because a negative shock to realized dividend growth, which always raises the SDF, raises the level of future dividends and thus the price of long-term dividend strips. The negative correlation together with the innovation term of the SDF given in (46) are the key factors behind a downward-sloping term structure of dividend risk premiums. Third, the correlation between innovations in $\Delta\pi_t$ and $x_{\pi,t}$ is set to one, indicating that the realized and expected inflation growth rates move one for one. Fourth, the correlation between innovations in Δd_t and π_t is -0.3. Because π_t and $x_{\pi,t}$ are perfectly correlated, the correlation between innovations in Δd_t and $x_{\pi,t}$ is also -0.3. Then, a positive shock to realized and expected inflation growth is more likely to decrease realized dividend growth, which in turn raises the SDF. Meanwhile, the rise in realized and expected inflation growth lowers the payoffs of nominal bonds in real terms with both short and long maturities. Hence, nominal bonds cannot be hedging instruments against events that raise the SDF, leading to an upward-sloping term structure of nominal interest rates. Fifth, the correlation between innovations in Δd_t and $x_{f,t}$ is -0.3. This negative correlation contributes to generating an upward-sloping term structure of real interest rates. Specifically, following a negative shock to realized dividend growth, the SDF rises and the real risk-free rate tends to rise owing to the negative correlation. The rise in the real risk-free rate in turn lowers the prices of real bonds, indicating that real bonds cannot hedge the rise in the SDF.

In summary, all the variables, except $x_{\lambda,t}$, are correlated negatively with Δd_t and hence positively with m_t, which intuitively means that the agent dislikes increase in any of them. The factor risk premiums, which are computed as $-\text{cov}_t[m_{t+1}, \cdot\,] = s_{d\cdot}x_{\lambda,t}$, are then negative except those of Δd_t and $x_{\lambda,t}$. In the last row of Table 2, the factor risk premiums evaluated at $x_{\lambda,t} = \mu_\lambda$ are presented in annual percentage terms. First, by far the highest in absolute value is the factor risk premium of Δd_t, 17% per year. Then, an asset that has a positive exposure to Δd_t, such as short-term dividend strips, is supposed to command a positive risk premium. In fact, the risk premium of the one-quarter dividend strip is exactly 17%. Second, the factor risk premium of $x_{\lambda,t}$ is zero by the zero correlation between innovations in $x_{\lambda,t}$ and Δd_t. Then, although an asset has either a positive or a negative exposure to $x_{\lambda,t}$, this does not affect its risk premium. However, the exposure to $x_{\lambda,t}$ does affect the volatility and thus the Sharpe ratio of this asset. Third, the factor risk premiums of the rest of the variables are negative. Then, an asset that has a positive exposure to one of these variables commands a negative risk premium attributed to the variable.

3.2 Calibration procedure

The most straightforward approach for replicating the SDF of the LW model with that of the proposed model is to use the same variables. Specifically, we match $x_t = x_t^{LW}$ and inherit the dynamics of x_t^{LW} as well as those of Δd_t and $\Delta \pi_t$ into the proposed model. This means that the parameters associated with these dynamics are not calibrated in this study (i.e., we simply borrow them from LW). Then, the parameters to calibrate here are those associated with the consumption dynamics and state-dependent preferences that do not appear in the LW model.

We first re-specify the consumption process as

$$\Delta c_{t+1} = \mu_c + b_c x_{d,t} + \sigma_c' z_{t+1}\ , \tag{47}$$

which is similar to the dividend process given in (43). Precisely, the expected consumption growth is driven by the same state variable (scaled by b_c) as that driving the expected dividend growth. The parameters associated with the consumption process are as follows: $(\mu_c,\ b_c,\ s_{cc},\ s_{cd},\ s_{c\pi},\ s_{cx}')$. Among them, we fix $(\mu_c,\ b_c)$ to maintain a reasonable relationship between consumption and dividend growth. Specifically, we set $\mu_c = \mu_d$ and $b_c = 1/3$

following Bansal and Yaron (2004). Then, seven consumption parameters need to be calibrated.

Next, the parameters of the state-dependent preferences are $(\mu_\gamma,\ b'_\gamma)$ in the risk-aversion coefficient γ_t and $(\mu_\beta,\ b'_\beta)$ in the subjective discount factor β_t. Among these preference parameters, some elements of b_γ can be determined immediately. Specifically, the price-of-risk vector is driven by one factor, $x_{\lambda,t}$ in the LW model and γ_t in the proposed model, leading to

$$\gamma_t = \mu_\gamma + b_{\gamma 4}(x_{\lambda,t} - \mu_\lambda) \ . \tag{48}$$

Hence, $b_{\gamma i} = 0$ $(i = 1, 2, 3)$, resulting in seven preference parameters that need to be calibrated.

These unknown parameters are determined by numerically solving the following sets of constraint equations. The first set is obtained by matching the one-period real risk-free rate, which is given as a state variable in the LW model and derived as a linear function of the state variables in the proposed model. Specifically,

$$x_{f,t} = A_f + B_{f1}x_{d,t} + B_{f2}x_{\pi,t} + B_{f3}(x_{f,t} - \mu_f) + B_{f4}(x_{\lambda,t} - \mu_\lambda) \ . \tag{49}$$

Equation (49) holds for any x_t, leading to the five constraint equations:

$$B_{f1} = B_{f2} = B_{f4} = 0, \quad B_{f3} = 1, \quad A_f = \mu_f \ . \tag{50}$$

The second set of constraint equations is obtained by matching the factor risk premiums. In the LW model (augmented with the consumption process given in (47)):

$$-\mathrm{cov}_t[m^{LW}_{t+1},\ \Delta c_{t+1}] = s_{cd}x_{\lambda,t} \ , \tag{51}$$

$$-\mathrm{cov}_t[m^{LW}_{t+1},\ \Delta d_{t+1}] = s_{dd}x_{\lambda,t} \ , \tag{52}$$

$$-\mathrm{cov}_t[m^{LW}_{t+1},\ \Delta \pi_{t+1}] = s_{d\pi}x_{\lambda,t} \ , \tag{53}$$

$$-\mathrm{cov}_t[m^{LW}_{t+1},\ x_{t+1}] = s_{dx}x_{\lambda,t} \ . \tag{54}$$

Note that (54) is four dimensional. The corresponding factor risk premiums in the proposed model are

$$-\mathrm{cov}_t[m_{t+1},\ \Delta c_{t+1}] = (s'_{cx}b_\nu + s_{cc})\gamma_t - s'_{cx}(b_\beta + b_\nu) \ , \tag{55}$$

$$-\mathrm{cov}_t[m_{t+1},\ \Delta d_{t+1}] = (s'_{dx}b_\nu + s_{cd})\gamma_t - s'_{dx}(b_\beta + b_\nu) \ , \tag{56}$$

$$-\mathrm{cov}_t[m_{t+1},\ \Delta \pi_{t+1}] = (s'_{\pi x}b_\nu + s_{c\pi})\gamma_t - s'_{\pi x}(b_\beta + b_\nu) \ , \tag{57}$$

$$-\mathrm{cov}_t[m_{t+1},\ x_{t+1}] = (s'_{xx}b_\nu + s_{cx})\gamma_t - s'_{xx}(b_\beta + b_\nu) \ . \tag{58}$$

By substituting (48) into (55)–(58) and then matching the resulting equations with (51)–(54), we have the following fourteen equations:

$$\begin{aligned}
&\text{(Slope)} && \text{(Intersept)} \\
&b_{\gamma 4}(s'_{cx}b_\nu + s_{cc}) = s_{cd}\ , && (\mu_\gamma - b_{\gamma 4}\mu_\lambda)(s'_{cx}b_\nu + s_{cc}) - s'_{cx}(b_\beta + b_\nu) = 0\ , && (59)\\
&b_{\gamma 4}(s'_{dx}b_\nu + s_{cd}) = s_{dd}\ , && (\mu_\gamma - b_{\gamma 4}\mu_\lambda)(s'_{dx}b_\nu + s_{cd}) - s'_{dx}(b_\beta + b_\nu) = 0\ , && (60)\\
&b_{\gamma 4}(s'_{\pi x}b_\nu + s_{c\pi}) = s_{d\pi}\ , && (\mu_\gamma - b_{\gamma 4}\mu_\lambda)(s'_{\pi x}b_\nu + s_{c\pi}) - s'_{\pi x}(b_\beta + b_\nu) = 0\ , && (61)\\
&b_{\gamma 4}(s'_{xx}b_\nu + s_{cx}) = s_{dx}\ , && (\mu_\gamma - b_{\gamma 4}\mu_\lambda)(s'_{xx}b_\nu + s_{cx}) - s'_{xx}(b_\beta + b_\nu) = 0\ . && (62)
\end{aligned}$$

Taken together, there are fourteen unknown parameters: two in γ_t, five in β_t, and seven for consumption variance and covariances. Meanwhile, nineteen constraint equations are needed for perfect replication: five from the risk-free rate and fourteen from the factor risk premiums. Hence, perfect replication is impossible in the first place. This is so even if $(\mu_c,\ b_c)$ are free parameters. In this case, these drift parameters are used for matching the factor risk premiums rather than capturing expected consumption growth. Consequently, unrealistic values are returned, and this is why we fix $(\mu_c,\ b_c)$ for a realistic consumption process.

Among these equations, the five equations in (50) and the seven *slope* equations in (59)–(62) are selected. By this selection, there is no difference in the loadings of each asset on the state vector, B_n^i ($i = \{R, N, D\}$), between the LW and proposed models; see Appendix D for more details. Additionally, given that the factor risk premium of Δd_t is by far the highest, the *intercept* equation (60) is also selected. This means that the factor risk premium of Δd_t is exactly matched between the two models, and so is the risk premium of the one-quarter dividend strip (17% per year). Finally, one free parameter is reserved for keeping positive definite the extended correlation matrix, which includes consumption growth but excludes realized inflation growth because of the perfect correlation with expected inflation growth. Without this constraint, a negative definite correlation matrix is returned in exchange for a closer fit to the SDF of the LW model.

Since the rest of the intercept equations, (59) and (61)–(62), are not satisfied, the average term structures differ between the two models, as shown in Section 4.3. We prioritize the slope equations over the intercept equations for two reasons. First, the constant terms in the pricing formulas, A_n^i ($i = \{R, N, D\}$), which matter for the average term structures, are computed recursively and dependently on the loadings; see equations (145), (148), and (151) in Appendix D. Second, it is difficult to find solutions to the intercept equations that satisfy the following two conditions: (i) $(\mu_\nu,\ b_\nu)$ are real and (ii) the extended correlation matrix is positive definite.

4 Baseline results

This section addresses whether the proposed model can replicate the term structures of bonds and equities generated by the LW model and discusses the preferences and consumption dynamics implicit in these term structures. There are numerous solutions to the set of constraint equations presented in Section 3.2. In Section 4.1, we first select several solutions to discuss their pattern and then describe one of them in Section 4.2. For this particular solution, we generate the term structures of zero-coupon bonds and equities in Section 4.3. In Section 4.4, we show that the model can also generate a downward-sloping term structure of real interest rates without much affecting the other term structures. We further show in Section 4.5 that the model can change the slope of the term structure of nominal interest rates without changing the correlation between consumption and inflation growth.

4.1 Several solutions and their pattern

Table 3 presents several solutions in ascending order of mean risk aversion μ_γ. First, there is an inverse relationship between μ_γ and the volatility of innovation in consumption growth $\sqrt{s_{cc}}$. In addition, this relationship is nonlinear: the rate of decrease in $\sqrt{s_{cc}}$ is much slower than the rate of increase in μ_γ. Specifically, at $\mu_\gamma = 30$, $\sqrt{s_{cc}}$ is 8.84% per year, which is high relative to the historical estimates discussed below and the corresponding dividend volatility set at 10%. It becomes half at around $\mu_\gamma = 120$ and less than 4% at $\mu_\gamma = 150$. Further reductions in $\sqrt{s_{cc}}$ are limited: $(\sqrt{s_{cc}},\ \mu_\gamma) = (3.43\%,\ 300),\ (3.32\%,\ 500),\ (3.28\%,\ 1000)$.

Second, the unconditional standard deviation of risk aversion, $SD[\gamma_t]$, also increases with μ_γ. In fact, the ratio of mean to standard deviation is nearly constant at 1.2 for any solutions. Because the standard deviation is large relative to the mean, γ_t becomes negative with a non-negligible probability. Figure 1(a) depicts the unconditional distribution of γ_t at Solution (e) of Table 3 (i.e., $\mu_\gamma = 150$). The probability of $\gamma_t < 0$ reaches 12%.[4] If a negative risk aversion is

[4] In lottery choice experiments, it is often observed that a certain proportion of the subjects exhibit a risk-neutral or a risk-loving behavior. For instance, Holt and Laury (2002) and Dohmen et al. (2011) report that around 80% of the subjects are classified as risk-averse and the rest as risk-neutral or risk-loving. Harrison, Lau, and Rutström (2007) obtain a similar result. More specifically, their figure 1 showing the distribution of subjects' elicited relative risk-aversion coefficients has a close resemblance to Figure 1(a) of the current research, although

unacceptable, it can be avoided by specifying γ_t as a positive function of x_t. In Appendix C, we consider a quadratic specification and examine the accuracy of the approximation for the continuation value ν_t. Alternatively, it is also possible to virtually avoid negative values by reducing the volatility of $x_{\lambda,t}$. This is originally set at 4 (see Table 2). In Section 5, we change the values of some of the parameters originally set by LW to see whether reasonable economic implications are obtained without reducing the descriptive ability for the average term structures presented in this section.

Third, the unconditional mean of the subjective discount factor, $E[\beta_t]$, expressed in quarterly terms, increases with μ_γ. At $\mu_\gamma = 30$, it is 0.969, which appears to be smaller than usually considered. At $\mu_\gamma = 90$, it increases to a reasonable value of 0.985. Conversely, the unconditional standard deviation of the subjective discount factor, $SD[\beta_t]$, decreases with μ_γ. The inverse relationship between the mean and variance of β_t is a natural consequence of the specification given in (16), which has the upper bound of one. Figure 1(b) depicts the unconditional distribution of β_t at Solution (e) of Table 3 (i.e., $\mu_\gamma = 150$). Obviously, β_t does not vary largely. Consequently, the unconditional probability of $\beta_t < 0$ is negligibly low, indicating that the specification of β_t given in (16) is virtually consistent with Assumption (ii) (i.e., $0 < \beta_t < 1$). Also of note is that the small variation in β_t is beneficial for the accuracy of the approximation of ν_t. As shown in Section 2.6, the source of the approximation lies in (27) and (28), which implies that the smaller the variation in β_t, the more accurate is the approximation: in the limit where β_t is constant, no approximation is necessary, as discussed in Section 2.5.

In summary, we face either large risk aversion or high consumption volatility, or both. This is a typical trade-off in the literature of equity premium puzzles. However, here it results from many equity risk premiums having the term structure with a sharply downward slope. This trade-off implies that the specification of the SDF and/or the calibration of the parameters provided by LW are not in fact realistic from an equilibrium point of view. While this problem is masked in the reduced-form model, it emerges once economic structures are imposed. In Section 5, we slightly change the specification of cash-flow processes as well as the values of some of the parameters to resolve this trade-off and recover realistic economic implications.

the magnitude of the coefficients differs (ours are much larger in absolute value). Curatola (2015) assumes in his model to explain a negative slope of dividend risk premiums that risk-loving agents represent a certain proportion of the population.

4.2 Implied consumption dynamics and preferences at a particular solution

Facing the trade-off between risk aversion and consumption volatility, we choose a reasonable level of consumption volatility, while giving up a reasonable level of risk aversion. Specifically, we focus here on Solution (e) of Table 3, characterized by $\mu_\gamma = 150$ and $\sqrt{s_{cc}} = 3.91\%$. The consumption volatility of 4% seems reasonable based on U.S. historical data. For example, in Barro (2006, table III), a sample standard deviation of real per capita GDP growth over 1890–2004 is 4.5%. In Mehra and Prescott (1985, table 1), originally from Grossman and Shiller (1981), a sample standard deviation of real per capita consumption growth over 1889–1978 is 3.6%.

We report all the calibrated parameters with particular attention to the following two points. The first is the mechanism of implying a large μ_γ and the second is the key parameters for determining the shape of the average term structure of real interest rates. The parameters of the consumption dynamics are first addressed, followed by those of the state-dependent preferences.

4.2.1 Consumption growth

Panel A of Table 4 presents implied correlations between innovations in consumption growth Δc_t and the other six variables in addition to the volatility of innovation in Δc_t. First, the correlation between Δc_t and dividend growth Δd_t, denoted as ρ_{cd}, is 0.88. Although this implied value is larger than typical estimates from time-series data on aggregate consumption and dividend growth, it may be rationalized by an economic theory of limited asset-market participation. Specifically, as assumed by Marfè (2017), only shareholders who earn dividends and consume them can have access to the markets, so that the SDF is that of shareholders. This assumption then supports not only the SDF of the LW model but also a high correlation between consumption and dividend growth.

Second, the correlation between Δc_t and expected dividend-growth factor $x_{d,t}$, denoted as ρ_{cx1}, is -0.76, which is close to the correlation between Δd_t and $x_{d,t}$ originally fixed at -0.83. These negative correlations contribute to making longer-term dividend strips less risky than shorter-term ones.

Third, the correlation between Δc_t and expected inflation-growth factor $x_{\pi,t}$, denoted as ρ_{cx2}, is -0.09, which is somewhat lower in absolute value than the correlation between Δd_t and $x_{\pi,t}$ originally fixed at -0.3. Note that the correlation between Δc_t and realized inflation growth $\Delta \pi_t$ is the same as ρ_{cx2} because of the perfect correlation between $\Delta \pi_t$ and $x_{\pi,t}$. The negative correlations of consumption growth with realized and expected inflation growth are the key channel through which equilibrium models generate a positive slope of the term structure of nominal interest rates (e.g., Piazzesi and Schneider, 2006; Wachter, 2006). Empirically, however, it may be difficult to find decisive evidence for a negative correlation. It is then beneficial to create alternative channels through which longer-term nominal bonds are made riskier. Indeed, the proposed model has such a channel, which is addressed in Section 4.4.

Fourth, the correlation between Δc_t and risk-free rate factor $x_{f,t}$, denoted as ρ_{cx3}, is -0.05. This is consistent in sign with, but smaller in magnitude than, the correlation between Δd_t and $x_{f,t}$ originally fixed at -0.3.

Finally, the correlation between Δc_t and price-of-risk factor $x_{\lambda,t}$, denoted as ρ_{cx4}, is 0.34, which is higher than the correlation between Δd_t and $x_{\lambda,t}$ originally fixed at zero. The positive ρ_{cx4} is from the positive covariance s_{cx4}, which is needed to match the factor risk premium of $x_{\lambda,t}$ between the LW and proposed models. Specifically, recall that the fourth row of the slope equation (62) is

$$b_{\gamma 4}(s'_{xx4}b_{\nu} + s_{cx4}) = s_{dx4} \quad (= 0) \ , \tag{63}$$

where s_{cx4} and s_{dx4} are, respectively, the fourth row of s_{cx} and s_{dx} (the vectors consisting of the covariances of innovation in x_t with innovations in Δc_t and Δd_t) and s_{xx4} is the fourth column of s_{xx} (the variance matrix of innovation in x_t). Note that s_{dx4} is originally set to zero by LW. On the LHS of (63), $s'_{xx4}b_{\nu} = \text{cov}_t[x_{\lambda,t+1}, \ \ln \nu_{t+1}]$ turns out to be negative, which is intuitive because a positive shock to the price of risks tends to lower the continuation value. It then follows that $s_{cx4} = -s'_{xx4}b_{\nu} > 0$.

The positive correlation between Δc_t and $x_{\lambda,t}$ appears counterintuitive, as a positive shock to consumption growth tends to raise the price of risks. Furthermore, since a positive shock to consumption growth lowers the SDF, it may be concerned that the SDF falls in response to a rise in $x_{\lambda,t}$. However, we show below that the SDF is in fact correlated *positively* with $x_{\lambda,t}$.

Taken together, the implied parameters of the consumption process are basically consistent with the predetermined parameters of the dividend process. Although the correlation between innovations in consumption growth and price of risks becomes positive, this is inevitable based on the calibration procedure.

4.2.2 Preferences

To ease our explanation, we call $1 - \beta_t$ the subjective discount *rate*. Panel B of Table 4 presents the implied parameters of risk aversion γ_t and log subjective discount rate $\ln(1 - \beta_t)$, together with the calculated parameters of the log continuation value $\ln \nu_t$.

First, the loadings on the expected inflation-growth factor $x_{\pi,t}$ are zero for all these functions. That γ_t does not depend on $x_{\pi,t}$ is noted in Section 3.2. $x_{\pi,t}$ does not affect $\ln(1 - \beta_t)$ because it does not enter into the risk-free rate owing to the constraint $B_{f2} = 0$ given in (50). Since $x_{\pi,t}$ has no influence on γ_t, β_t, or Δc_t, it has no channel of affecting ν_t.

Risk aversion

As reported in Table 3, the standard deviation of γ_t is 128. Behind such a large value, $b_{\gamma 4}$ (the coefficient of $x_{\lambda,t}$) is implied to be 8.94. A large $b_{\gamma 4}$ together with a large μ_γ are a consequence of explaining the high factor risk premium of Δd_t set by LW. Specifically, recall that the slope equation for matching the factor risk premium of Δd_t is

$$b_{\gamma 4}(s'_{dx} b_\nu + s_{cd}) = s_{dd} \ . \tag{64}$$

On the LHS, $s'_{dx} b_\nu = \text{cov}_t[\Delta d_{t+1}, \ \ln \nu_{t+1}]$ turns out to be negative, indicating that a positive shock to dividend growth tends to decrease the continuation value. This relation can intuitively be understood by recalling that from $\rho_{dx1} = -0.83$ a positive shock to Δd_t is more likely to reduce $x_{d,t}$ and hence $\ln \nu_t$ that has a positive coefficient of $x_{d,t}$, $b_{\nu 1} = 8.86$. Meanwhile, both $b_{\gamma 4}$ and s_{dd} in (64) are positive. It follows that $s_{cd} > -s'_{dx} b_\nu > 0$. However, because $s_{cd} = \sqrt{s_{cc} s_{dd}} \rho_{cd}$ is much smaller than s_{dd} given a reasonable $\sqrt{s_{cc}}$ (around 4% per year) and $\rho_{cd} \leq 1$, $b_{\gamma 4}$ must be large to equate the LHS with s_{dd}.

This logic can also be used to explain why a large $\sqrt{s_{cc}}$ is required to reduce $b_{\gamma 4}$. For a small $b_{\gamma 4}$, s_{cd} must be large to satisfy (64). However, because s_{dd} is fixed at 10% per year and ρ_{cd} has an upper limit of one, $\sqrt{s_{cc}}$ must increase.

A large $b_{\gamma 4}$ leads to a large μ_γ, which is explained by using the intercept equation for matching the factor risk premium of Δd_t:

$$(\mu_\gamma - b_{\gamma 4} \mu_\lambda)(s'_{dx} b_\nu + s_{cd}) = s'_{dx}(b_\beta + b_\nu) \ . \tag{65}$$

The RHS of (65) is

$$s'_{dx}(b_\beta + b_\nu) = \text{cov}_t[\Delta d_{t+1}, \ \ln \nu_{t+1}] + \text{cov}_t[\Delta d_{t+1}, \ \ln(1 - \beta_{t+1})] \ . \tag{66}$$

As noted above, the first term on the RHS of (66) is negative. The second term on the RHS turns out to be positive but is dominated by the first term. Then, the RHS of (65) is negative but not very large in absolute value. Meanwhile, $b_{\gamma 4}$ on the LHS of (65) is large to satisfy (64) and μ_λ is originally fixed at 17. To offset a large $b_{\gamma 4}\mu_\lambda$, μ_γ must also become large.

These explanations clarify why we face the trade-off between the reasonable values of risk aversion and consumption volatility. To reduce μ_γ, $b_{\gamma 4}$ must also be reduced. However, this is possible only by increasing the consumption volatility.

Because of $b_{\gamma 4} = 8.94$, the risk aversion increases with the price-of-risk factor. Since the correlation between Δc_t and $x_{\lambda,t}$ is positive (i.e., $\rho_{cx4} = 0.34$), so is the correlation between Δc_t and γ_t, which may be counterintuitive. However, this relationship does not violate the inverse relationship between $\ln \nu_t$ and γ_t because $\ln \nu_t$ has a negative coefficient of $x_{\lambda,t}$, $b_{\nu 4} = -0.003$. In fact, the covariance between $\ln \nu_t$ and γ_t is negative, $\text{cov}_t[\ln \nu_{t+1},\ \gamma_{t+1}] = -0.24$. This negative covariance in turn implies a positive covariance between the log SDF m_t and γ_t, which is fundamental for many models to explain high equity premiums. This relation is further addressed after reporting the parameters of the subjective discount rate.

Subjective discount rate

First, the signs of the coefficients in $\ln(1-\beta_t)$ are all opposite to those in $\ln \nu_t$, indicating that the subjective discount rate moves inversely with the continuation value. This movement makes sense by recalling that the wealth-consumption ratio is solved in equilibrium as $\ln(W_t^*/C_t^e) = -\ln(1-\beta_t)$. Hence, the inverse relationship between $\ln(1-\beta_t)$ and $\ln \nu_t$ is consistent with the parallel movement between the continuation value and wealth.

More precisely, $\ln(1-\beta_t)$ increases with $x_{f,t}$ and $x_{\lambda,t}$ because of $b_{\beta 3} = 30.2$ and $b_{\beta 4} = 0.014$, respectively. The positive relationship between the subjective discount rate and risk-free rate is reasonable. The positive $b_{\beta 4}$ implies that the agent raises her discount rate and hence becomes less patient when she becomes more risk averse. Conversely, the increase in $x_{d,t}$ lowers $\ln(1-\beta_t)$ because of $b_{\beta 1} = -8.66$. This implies that the agent lowers her discount rate and hence becomes more patient when she has brighter prospects for future consumption.

Finally, we discuss the implied conditional covariances between m_t and state-dependent preferences:

$$\text{cov}_t[m_{t+1},\ \gamma_{t+1}] = b'_\gamma\{s_{xx}(b_\beta + b_\nu) - (s_{xx}b_\nu + s_{cx})\gamma_t\} = 1.53\ , \tag{67}$$

$$\text{cov}_t[m_{t+1},\ \ln(1-\beta_{t+1})] = b'_\beta\{s_{xx}(b_\beta + b_\nu) - (s_{xx}b_\nu + s_{cx})\gamma_t\} = 10^{-5}(292 - 1.6\gamma_t)\ . \tag{68}$$

The conditional covariance between m_{t+1} and γ_{t+1} is positive, which is consistent with many equilibrium models in that the SDF increases with increasing risk aversion. It is constant because only the fourth element of b_γ is nonzero and the fourth element of $(s_{xx}b_\nu + s_{cx})$ is zero from the slope equation (62) for matching the factor risk premium of x_t. Meanwhile, the conditional covariance between m_{t+1} and $\ln(1-\beta_{t+1})$ depends on γ_t. It is positive at $\mu_\gamma = 150$ and remains so for $\gamma_t < 180$, suggesting that the agent is more likely to raise her discount rate when the SDF is high. Because future cash flows are discounted to a larger extent by the compound effect, real bonds are devalued, with the devaluation more significant for longer-term bonds. They therefore command high risk premiums and the resulting term structure of real interest rates slopes upward.

In summary, when consumption volatility is reasonable, the implied parameters in γ_t are large. They are mostly determined by the factor risk premium of dividend growth. On the other hands, the parameters in β_t are more closely related to (real) bonds and calibrated to explain the term structure of (real) interest rates. In Section 4.3, we generate the average term structures of zero-coupon bonds and equities at Solution (e) of Table 3.

4.3 Term structure shapes

All solutions, some of which are presented in Table 3, lead to similar average term structures because the factor loadings B_n^i $(i = \{R, N, D\})$ are the same among the solutions, which are in fact identical to those in the LW model by construction of the calibration. The constant terms A_n^i $(i = \{R, N, D\})$ differ among the solutions because not all of the intercept equations are satisfied and the resulting errors have different patterns. Still, A_n^is differ little because they are computed dependently on the (identical) loadings.

4.3.1 Factor loadings

We first show the term structure of factor loadings for the log-price of zero-coupon bonds and equities, B_n^i $(i = \{R, N, D\})$. The sign and shape of B_n^i give us a clue about the term structure of risk premiums. Since we know the sign and magnitude of the factor risk premiums, which are presented in Panel C of Table 4, we understand the risk premium of an asset if we know an asset-specific quantity of risks, which is associated with the loadings on the factors. Additionally, the shape of the term structure of excess-return volatilities can roughly be captured by examining the loadings.

Figure 2 plots B_n^i against n (quarters), which are normalized by multiplying the unconditional standard deviation of each factor, and hence interpreted as the loadings on the factors with unit volatility. Panel (c) shows the (normalized) loadings on the risk-free rate factor $x_{f,t}$, which is a common factor for all assets, generating the baseline risk. They are identical for all assets, negative for all $n \geq 1$, and decrease with n. Since the factor risk premium of $x_{f,t}$ is negative (-0.11% per year), the negative loadings on $x_{f,t}$ lead to positive risk premiums attributed to $x_{f,t}$ for all assets. Furthermore, the risk premiums are expected to increase with n as the loadings decrease with n. Since, for real bonds, the loadings on the other factors are zero as seen in the other panels of Figure 2, the term structure of real interest rates is expected to be upward-sloping.

Panel (b) shows that only nominal bonds have nonzero loadings on the expected inflation-growth factor $x_{\pi,t}$, which are negative for all $n \geq 1$ and decrease with n. Since the factor risk premium of $x_{\pi,t}$ is negative (-0.18% per year), the risk premiums attributed to $x_{\pi,t}$ become positive for all $n \geq 1$ and increase with n. These premiums are added to those attributed to $x_{f,t}$, and the resulting term structure of nominal interest rates will be above that of real interest rates with the difference between the two curves widening with n.

Panel (a) shows that only dividend strips have nonzero loadings on the expected dividend-growth factor $x_{d,t}$, which are positive for all $n \geq 1$ and increase with n. Since the factor risk premium of $x_{d,t}$ is negative (-0.46% per year), the risk premiums attributed to $x_{d,t}$ are negative for all $n \geq 1$ and more so for longer n. This means that a dividend strip has two opposing components of the risk premium: one is a positive risk premium attributed to $x_{f,t}$ and the other is a negative risk premium attributed to $x_{d,t}$. The latter dominates the former. Specifically, while the magnitude of the normalized loadings is similar in absolute value between $x_{d,t}$ and $x_{f,t}$ as seen in Figures 2(a) and 2(c), the normalized factor risk premium of $x_{d,t}$ is about three times larger in absolute value than that of $x_{f,t}$. Consequently, the term structure of dividend risk premiums will have a downward slope.

Finally, in Panel (d), the loadings on the price-of-risk factor $x_{\lambda,t}$ are presented, which are negative for all assets. They differ in shape, however. They decrease monotonically with n for real and nominal bonds whereas they are inversely hump-shaped for dividend strips. These shapes do not matter with the risk premium of any assets in the original LW model, however, because the factor risk premium of $x_{\lambda,t}$ is zero. They do matter with the volatility and hence the Sharpe ratio. Specifically, the negative hump around n =35–40 is expected to increase the volatility of returns to dividend strips with these maturities. In contrast to the LW model, the negative loadings on $x_{\lambda,t}$ also matter with any assets' risk premiums in the proposed model because the factor risk premium of $x_{\lambda,t}$ is negative, as presented in Panel C of Table 4. Hence, the negative loadings on $x_{\lambda,t}$ generate additional risk premiums. Consequently, the term structures of risk premiums in the proposed model will be above those in the LW model, which is indeed the case as seen below.

4.3.2 Level and volatility of real and nominal interest rates

Let $Y_{t,n}^{i}$ $(i = \{R, N\})$ be the yield to maturity of a zero-coupon bond maturing in n periods: $Y_{t,n}^{i} = -\frac{1}{n} \ln P_{t,n}^{i}$. By substitution of (37) and (39),

$$Y_{t,n}^{i} = -\frac{1}{n}(A_{n}^{i} + B_{n}^{i\prime} x_{t}) \qquad (i = \{R, N\}) \ . \tag{69}$$

Then, the unconditional mean and variance of $Y_{t,n}^{i}$ are $-A_{n}^{i}/n$ and $B_{n}^{i\prime}\text{var}[x_{t}]B_{n}^{i}/n^{2}$, respectively. These moments, expressed in quarterly terms, are annualized by multiplying by four.

Figure 3(a) plots the annualized unconditional mean of real interest rates, $4E[Y_{t,n}^{R}]$, against n (quarters) produced by the LW (dotted line) and proposed (solid line) models. The two plots are upward-sloping. By construction of the calibration in which the real risk-free rate is matched exactly between the two models, these plots start from the same point at $n = 1$. By increasing n, they deviate gradually. At $n = 160$ (40 years), the mean real interest rate for the proposed model is higher by 1% than that for the LW model in line with the argument in Section 4.3.1.

Figure 3(b) plots the annualized unconditional mean of nominal interest rates, $4E[Y_{t,n}^{N}]$. Again, the two plots are upward-sloping and above those for real interest rates because of the additional risk premiums attributed to $\Delta\pi_{t}$ and $x_{\pi,t}$. These plots start from almost the same point at $n = 1$ and deviate gradually, with the proposed model producing higher nominal rates. The deviation reaches 1.65% at $n = 160$.

Figures 3(c) and 3(d) plot the annualized unconditional standard deviation of real and nominal interest rates, $\sqrt{4\text{var}[Y_{t,n}^{i}]}$ $(i = \{R, N\})$. Both plots are downward-sloping. There is no discrepancy between the LW and proposed models because the loadings B_{n}^{i} $(i = \{R, N\})$ are the same for both models for any n.

Figures 4(a) and 4(b) plot the term structures of real and nominal interest rates generated by the proposed model when the risk-free rate factor $x_{f,t}$ is above (+2SD) or below (−2SD) two standard deviations from the mean, while the other factors are fixed at the mean. Consistent with our intuition, when $x_{f,t}$ is high (low), both real and nominal curves shift upward (downward) with the shift more significant at the short end.

Analogous plots are shown in Figures 4(c) and 4(d), where the price-of-risk factor $x_{\lambda,t}$ is above or below two standard deviations from the mean with the other factors fixed at the mean. This is equivalent to changing the risk-aversion coefficient γ_t by plus or minus two standard deviations from the mean by the specification of γ_t given in (48). Also consistent with our intuition, when the agent becomes more risk averse (i.e., γ_t is high), both curves shift upward. When she is less risk averse (i.e., γ_t is low), both real and nominal interest rates first decrease up to n =12–16 (three to four years) and then increase because of the constant term, $-A_n^i/n$, which is increasing in n as shown in Figures 3(a) and 3(b).

The term structures of interest rates in Figures 4(e) and 4(f) are drawn when the economy is "good" and "bad," respectively. We arguably define a good (bad) state of the economy as a state with low (high) $x_{\lambda,t}$ and high (low) $(x_{d,t},\ x_{\pi,t},\ x_{f,t})$. The high (low) value corresponds to two standard deviations above (below) the mean. When the economy is good, both real and nominal interest rates start from high levels, decrease with increasing n up to around $n = 40$ (10 years), and then turn slightly increasing. Overall, both real and nominal curves can be regarded as downward-sloping or flat. By contrast, when the economy is bad, these curves are sharply upward-sloping, starting from low levels. The nominal interest rate at $n = 1$ is negative as the model consists of the Gaussian state vector. These plots do not seem to deviate largely from real observations of the economy, although some level adjustments may be necessary.

4.3.3 Risk premium, volatility, and Sharpe ratio of dividend strips

Let $r^D_{t+1,n-1}$ be the log return to a dividend strip, defined and developed as

$$\begin{aligned} r^D_{t+1,n-1} &= \ln\left(\frac{P^D_{t+1,n-1}}{P^D_{t,n}}\right) \\ &= \ln\left(\frac{P^D_{t+1,n-1}/D_{t+1}}{P^D_{t,n}/D_t}\right) + \ln\left(\frac{D_{t+1}}{D_t}\right) \\ &= (\sigma_x B^D_{n-1} + \sigma_d)' z_{t+1} + res^D_t\ , \end{aligned} \tag{70}$$

where res^D_t collects the remaining terms observed at time t. We define the risk premium of a dividend strip, denoted as $RP^D_{t,n-1}$, based on the excess log return adjusted for convexity or Jensen's inequality term:

$$RP^D_{t,n-1} \;=\; E_t[r^D_{t+1,n-1}] - r_{f,t+1} + \frac{1}{2}\mathrm{var}_t[r^D_{t+1,n-1}]$$

$$= \quad -\text{cov}_t[m_{t+1},\ r^D_{t+1,n-1}]\ , \tag{71}$$

where the second equality follows from the Euler equation, $E_t[e^{m_{t+1}+r^D_{t+1,n-1}}] = 1$. By developing the conditional covariance in (71),

$$RP^D_{t,n-1} = A^{RPD}_{n-1} + B^{RPD}_{n-1}\gamma_t\ , \tag{72}$$

where

$$A^{RPD}_{n-1} \quad = \quad -(b_\beta + b_\nu)'(s_{xx}B^D_{n-1} + s_{dx})\ , \tag{73}$$

$$B^{RPD}_{n-1} \quad = \quad s'_{dx}b_\nu + s_{cd} + (s_{xx}b_\nu + s_{cx})'B^D_{n-1}\ . \tag{74}$$

The unconditional risk premium is then obtained as $E[RP^D_{t,n-1}] = A^{RPD}_{n-1} + B^{RPD}_{n-1}\mu_\gamma$.

Figure 5(a) plots the annualized unconditional mean of risk premiums, $4E[RP^D_{t,n-1}]$, against n (quarters) implied by the LW (dotted line) and proposed (solid line) models. Both plots are downward-sloping. They start from the same point, 17%, which is remarkably high. By increasing n, they deviate gradually, with the proposed model again producing higher risk premiums. At $n = 160$, the risk premiums implied by the LW and proposed models are, respectively, 4.4% and 5.8%, and the two curves are almost flat.

Next, we compute the unconditional variance of excess return to a dividend strip, $\text{var}[r^D_{t+1,n-1} - r_{f,t+1}]$. First, it can be decomposed as

$$\text{var}[r^D_{t+1,n-1} - r_{f,t+1}] = \text{var}[E_t[r^D_{t+1,n-1} - r_{f,t+1}]] + E[\text{var}_t[r^D_{t+1,n-1} - r_{f,t+1}]]\ . \tag{75}$$

The first term on the RHS of (75) is developed as

$$\text{var}[E_t[r^D_{t+1,n-1} - r_{f,t+1}]] = \text{var}[RP^D_{t,n-1}] = B^{RP\prime}_{n-1}\text{var}[\gamma_t]B^{RP}_{n-1}\ . \tag{76}$$

The first equality in (76) follows from the definition of the risk premium given in (71), where $\text{var}_t[r^D_{t+1,n-1}]$ is actually constant. The second equality follows by substituting (72). From (14), $\text{var}[\gamma_t] = b'_\gamma\text{var}[x_t]b_\gamma$. The second term on the RHS of (75) is developed as

$$E[\text{var}_t[r^D_{t+1,n-1} - r_{f,t+1}]] = \text{var}_t[r^D_{t+1,n-1}] = B^{D\prime}_{n-1}s_{xx}B^D_{n-1} + 2s'_{dx}B^D_{n-1} + s_{dd}\ , \tag{77}$$

where the first equality follows because $r_{f,t+1}$ is observed at time t and $\text{var}_t[r^D_{t+1,n-1}]$ is constant. The second equality follows by substituting (70).

Figure 5(b) plots the annualized volatility, $\sqrt{4\text{var}[r^D_{t+1,n-1} - r_{f,t+1}]}$, implied by the LW and proposed models. Both plots are the same because B^D_n and B^{RPD}_n are the same for any $n \geq 1$.

The reason for the same B_n^D is that all the slope equations (59)–(62) are satisfied. In addition, B_n^{RPD} is the same because the intercept equation (60) associated with the factor risk premium of dividend growth is also satisfied. The volatility curve is hump-shaped with the peak around n =35–40, which corresponds to the trough of the term structure of loadings on $x_{\lambda,t}$ shown in Figure 2(d).

Finally, we compute the unconditional Sharpe ratio of dividend strips as a ratio of the unconditional mean of risk premiums to the unconditional volatility of excess returns. Figure 5(c) plots the annualized ratio, $4E[RP_{t,n-1}^D]/\sqrt{4\text{var}[r_{t+1,n-1}^D - r_{f,t+1}]}$. Both plots are sharply downward-sloping with the curve for the proposed model less steep. As seen in Figures 5(a) and 5(b), the risk premiums are high while the volatilities are low at the short end. This combination produces high Sharpe ratios. In the medium maturity range, the risk premiums decrease while the volatilities increase, leading to a sharp decrease in the Sharpe ratio. At the long end, since both risk-premium and volatility curves are almost flat, so is the curve of the Sharpe ratio.

Figure 6 presents the term structures of dividend risk premiums and Sharpe ratios generated by the proposed model when the price-of-risk factor $x_{\lambda,t}$ is above (+2SD) or below (−2SD) two standard deviations from the mean. It is noted that since the risk premiums are driven by γ_t, which is a linear function of $x_{\lambda,t}$ alone, changing the values of the other factors does not alter the plots. In addition, since the excess-return volatilities are constant for all maturities, the volatility curve does not change by changing the factor values. Accordingly, it is not surprising that only a parallel shift is observed in the risk-premium and Sharpe-ratio curves. Consistent with our intuition, the curves shift upward (downward) when $x_{\lambda,t}$ and thus γ_t is large (small).

To see a more flexible shift in the risk-premium curve such that the curve slopes upward in times of a good economy, it may be necessary to incorporate stochastic volatility into cash-flow processes.

In summary, the proposed model can produce the average term structures of zero-coupon bonds and equities that are close to those produced by the LW model. In the next subsection, we further examine whether the proposed model can also generate the term structure of real interest rates that is flat or downward-sloping without much affecting the shapes of the other term structures.

4.4 Changing the shape of the real term structure

The LW model can easily change the shape of the average term structure of real interest rates. This ability is rooted in the SDF driven by the same innovation term as that driving dividend growth, which makes it easy to change the correlations between the SDF and factors affecting the real term structure. Since the proposed model does not have such a simple mechanism, it cannot change the shape as easily as the LW model can. However, as demonstrated in Sections 4.1–4.3, the proposed model can replicate the LW model, which motivates us to take the following two steps to generate a downward-sloping term structure of real interest rates: (i) generate it by using the LW model, and (ii) replicate the LW model by using the proposed model.

In the first step, we change the correlation between innovations in dividend growth Δd_t and risk-free rate factor $x_{f,t}$, denoted as ρ_{dx3}, from -0.3 to 0.1 while keeping the other parameters unchanged in the LW model. When $\rho_{dx3} = 0.1$, the factor risk premium of $x_{f,t}$ is now positive because a positive shock to $x_{f,t}$ is more likely to increase Δd_t and hence decrease the SDF. Meanwhile, regardless of the value of the correlation parameter, the (log) price of real bonds is negatively exposed to $x_{f,t}$ and the negative exposure is increasing with maturity. Hence, the combination of a positive factor risk premium and increasingly negative exposures associated with $x_{f,t}$ results in increasingly negative risk premiums of real bonds. The term structure of real interest rates then has a negative slope. To lower the slope further, we simply increase the value of ρ_{dx3} (up to one). However, since the loadings on $x_{f,t}$ are the same for all the assets as shown in Figure 2(c), a positive slope of the term structure of nominal interest rates becomes less steep and a negative slope of the term structure of dividend risk premiums becomes steeper for a larger ρ_{dx3}.

In the second step, this exogenous mechanism through the correlation parameters is again endogenized by the proposed model using the parameters of the consumption dynamics and state-dependent preferences. These parameters need to be recalibrated entirely even though only a single parameter is changed in the original LW model. As in the baseline calibration, there are numerous solutions to the set of constraint equations presented in Section 3.2. We thus select a solution with $\mu_\gamma = 150$ to ease comparison with the baseline calibration.

In Figure 7, the average term structures generated by the LW (dotted line) and proposed (solid line) models are presented. (Those of volatilities are not shown to save space.) Each term structure starts from the same point as that in the baseline calibration presented in Figures 3 and 5. First, Figure 7(a) shows that the real term structure is indeed slightly downward-sloping.

The proposed model generates higher rates with the deviation reaching 0.55% at $n = 160$ (40 years). Second, Figure 7(b) shows that the term structure of nominal interest rates is flattened for both models, reflecting a downward-sloping real term structure. The nominal term structure is still positively sloped because of the positive risk premiums attributed to $x_{\pi,t}$. It is possible to further steepen this slope by increasing these risk premiums. In the LW model, this is achieved by making the correlation between dividend growth and realized/expected inflation growth more negative (up to minus one). In addition to this correlation channel, the proposed model has an alternative channel through the state-dependent preferences, which may be beneficial when empirical evidence of the correlation channel is weak. The benefit of this alternative channel is discussed in Section 4.5. Third, in Figures 7(c) and 7(d), the average term structures of dividend risk premiums and Sharpe ratios remain sharply downward-sloping. In fact, the downward slope is reinforced by $\rho_{dx3} = 0.1$ as expected.

Table 5 presents the calibrated parameters for $\rho_{dx3} = 0.1$. Panel A shows two major changes in the consumption process. First, the consumption volatility $\sqrt{s_{cc}}$ increases from 3.91% to 4.96%, which can also be explained by using the slope equation (64) for matching the factor risk premium of Δd_t. In this equation, $s'_{dx} b_\nu = \mathrm{cov}_t[\Delta d_{t+1},\ \ln \nu_{t+1}]$ becomes more negative than in the baseline calibration because increase in $x_{f,t+1}$, which decreases the log continuation value $\ln \nu_{t+1}$ in the previous and current calibrations because of $b_{\nu 3} < 0$, now tends to increase, rather than decrease, Δd_{t+1} by changing ρ_{dx3} from -0.3 to 0.1. Second, the correlation between innovations in Δc_t and $x_{f,t}$ (i.e., ρ_{cx3}) increases from -0.046 to 0.085 consistently with the change in ρ_{dx3} from -0.3 to 0.1.

Panel B of Table 5 presents the preference parameters. Overall, the signs of the parameters do not change from those in the baseline calibration shown in Table 4. Since we originally selected a particular solution with $\mu_\gamma = 150$, the implied value of $b_{\gamma 4}$ (the coefficient of $x_{\lambda,t}$ in γ_t) changes little from that in the baseline calibration. Instead, the implied parameters in $\ln(1 - \beta_t)$ exhibit some changes. Specifically, the unconditional mean of β_t increases from 0.987 to 0.998 mainly because of the decrease in μ_β from -4.39 to -6.34. Furthermore, the implied values of b_β change toward reducing the conditional covariance between $\ln(1 - \beta_t)$ and m_t. Specifically, $\mathrm{cov}_t[m_{t+1},\ \ln(1 - \beta_{t+1})] = 10^{-5}(171 - 11.5\gamma_t)$, which is now negative at $\mu_\gamma = 150$, indicating that the agent tends to lower her discount rate when the SDF is high. The decrease in this covariance is mainly attributed to the decrease in $b_{\beta 1}$ from -8.66 to -12. Then, a positive shock to $x_{d,t}$, which increases m_t as reflected in the negative factor risk premium of

$x_{d,t}$, decreases $\ln(1-\beta_t)$ more than previously. Furthermore, the increase in $b_{\beta 3}$ from 30 to 45 makes an additional contribution to decreasing this covariance. Specifically, a positive shock to $x_{f,t}$ increases $\ln(1-\beta_t)$ more than previously, while it is more likely to decrease m_t because of $\rho_{dx3} = 0.1$.

In summary, the proposed model is shown to be as flexible as the LW model in terms of generating the average term structure of real interest rates. This flexibility is crucial as the shape of the real term structure is indecisive. The state-dependent time preference contributes to this flexibility.

4.5 Raising the slope of the nominal term structure

As seen in Section 4.4, the real term structure slopes downward by setting $\rho_{dx3} = 0.1$; however, at the same time, the nominal term structure is flattened. We attempt to steepen the nominal slope while keeping the real slope negative by changing some of the preference parameters rather than the correlation parameters.

There are two approaches for this purpose. The first is a direct approach, which is to increase $b_{\gamma 2}$ (the coefficient of $x_{\pi,t}$ in γ_t), originally set at zero. Specifically, we set $b_{\gamma 2} = 90$. Then, the agent dislikes increase in $x_{\pi,t}$ more than previously as it increases the risk-aversion coefficient γ_t and thus the log SDF m_t. Consequently, she requires higher risk premiums for holding nominal bonds, and the term structure of nominal interest rates will be more positively sloped. It is noted that after changing the value of $b_{\gamma 2}$, we need to solve the recursive equation (8) for the continuation value ν_t. Then, the solution would not exist if $b_{\gamma 2}$ were changed largely from zero (the original value) because the rest of the parameters regarding the consumption dynamics and state-dependent preferences remain unchanged.

Figures 8(a) and 8(b) plot the average term structures of real and nominal interest rates for $b_{\gamma 2} = 90$ together with those for $b_{\gamma 2} = 0$ (the same plots shown in Figures 7(a) and 7(b)). The nominal term structure shifts upward, with the shift more significant at the long end. Consequently, the nominal spread between $n = 160$ and $n = 1$ increases from 1.44% for $b_{\gamma 2} = 0$ to 1.79% for $b_{\gamma 2} = 90$.

The second is an indirect approach, which is to increase $b_{\beta 2}$ (the coefficient of $x_{\pi,t}$ in $\ln(1 - \beta_t)$), originally set at zero. Specifically, we set $b_{\beta 2} = 2$. The reason that a positive $b_{\beta 2}$ steepens the slope of the nominal term structure is less obvious but will be understood by recalling the inverse relationship between $\ln(1-\beta_t)$ and $\ln \nu_t$ (see Tables 4 and 5). By setting $b_{\beta 2} > 0$, therefore, it is not surprising that the value of $b_{\nu 2}$, which is the coefficient of $x_{\pi,t}$ in $\ln \nu_t$ and obtained as a solution to the recursive equation (8), is negative. For $b_{\nu 2} < 0$, the agent dislikes increase in $x_{\pi,t}$ more than before (i.e., $b_{\nu 2} = 0$) as this reduces the continuation value. As is the case for $b_{\gamma 2}$, if $b_{\beta 2}$ were changed largely from zero (the original value), there would be no solution for ν_t.

Figures 8(c) and 8(d) plot the average term structures of real and nominal interest rates for $b_{\beta 2} = 2$ together with those for $b_{\beta 2} = 0$ (the same plots shown in Figures 7(a) and 7(b)). Again, the nominal term structure is more upward-sloping with the spread between long-term and short-term rates increased to 2.14%.

In summary, the proposed model can steepen the slope of the term structure of nominal interest rates when the term structure of real interest rates is downward-sloping and the correlation between consumption and inflation growth is moderate. The key factor is again the state-dependent preference parameters, which control agent's aversion to inflation risks.

5 Alternative parameter values and cash-flow dynamics

While the proposed equilibrium model can generate the term structures of zero-coupon bonds and equities as flexibly as the reduced-form LW model, it obtains some counterfactual implications about the consumption dynamics and/or preferences. The purpose of this section is therefore to make the proposed model plausible from an economic point of view through two steps. First, in Section 5.1, we change some of the parameter values originally calibrated by LW and recalibrate the parameters of the proposed model by adopting the same procedure as explained in Section 3.2. This change aims to reduce the mean and standard deviation of state-dependent risk aversion without much increasing the volatility of consumption growth. Second, in Section 5.2, we slightly deviate from the LW model and incorporate jumps into cash flows to further reduce the consumption volatility.

5.1 Changing some of the parameter values of the LW model and recalibrating the proposed model

The values of the following three parameters are changed, while the other parameters are kept fixed at the values presented in Table 2:

$$\begin{aligned}
&\sqrt{\text{var}_t[x_{\lambda,t+1}]}: && 4 && \rightarrow && 0.2\ , \\
&\sqrt{\text{var}_t[\Delta d_{t+1}]}: && 10\% && \rightarrow && 18\%\ \ \text{(per year)}\ , \\
&E[x_{\lambda,t}]: && 17 && \rightarrow && 2.62\ \ (= 0.085/0.18^2)\ ,
\end{aligned}$$

where $x_{\lambda,t}$ is the price-of-risk factor and Δd_t is realized dividend growth. The reduction in the volatility of $x_{\lambda,t}$ from 4 to 0.2 aims to reduce the variance of state-dependent risk aversion specified as $\gamma_t = \mu_\gamma + b_{\gamma 4}(x_{\lambda,t} - \mu_\lambda)$. This change, however, also reduces the volatility of returns to dividend strips, shifting downward the volatility term structure with the shift more significant at the short end. To offset this downward shift, the volatility of innovation in Δd_t, simply denoted as $\sqrt{s_{dd}}$, is increased from 10% to 18% per year, which seems to remain within an acceptable range. In the LW model, the increase in s_{dd} directly increases the factor risk premium of Δd_t, or equivalently the risk premium of the one-period dividend strip, as it is given by $E[-\text{cov}_t[m_{t+1}^{LW},\ \Delta d_{t+1}]] = s_{dd}\mu_\lambda$. To keep this in a reasonable range as well, $\mu_\lambda (= E[x_{\lambda,t}])$ is decreased from 17 to 2.62. Through this change, the factor risk premium of Δd_t is halved to 8.5% per year. Nonetheless, a level of 8.5% still seems to stand as a challenge to existing equilibrium models (see Table 1). In short, these changes shift part of risks from the price-of-risk factor to realized dividend growth.

After changing the values of the three parameters in the LW model as above, the parameters of the proposed model are calibrated by the same procedure as explained in Section 3.2. Again, among the numerous solutions to the set of constraint equations, we focus on the solution with $E[\gamma_t] = \mu_\gamma = 30$, aiming to highlight the degree to which the volatility of consumption growth decreases. While a level of 30 may still be high, this is within the range of values considered or estimated by the previous work: 21 (Bansal and Shaliastovich, 2013), 50 (Doh and Wu, 2016), 66 (van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez, 2012), and 75 (Rudebusch and Swanson, 2012).

The "No JUMP" row of Table 6 provides the results of the calibration. Total volatility (Total vol.) is computed as $\sqrt{\text{var}_t[\Delta c_{t+1}]}$ for consumption growth and $\sqrt{\text{var}_t[\Delta d_{t+1}]}$ for dividend growth. These are the same as $\sqrt{s_{cc}}$ $(= \sqrt{\sigma_c'\sigma_c})$ and $\sqrt{s_{dd}}$ $(= \sqrt{\sigma_d'\sigma_d})$, respectively, in the current model. In the extended model in Section 5.2, which introduces jumps into cash flows, $\sqrt{s_{cc}}$ and $\sqrt{s_{dd}}$ correspond to the Gaussian components of the total volatility.

The implied $\sqrt{s_{cc}}$ is 5.05% per year, which is lower than 8.84% in the baseline calibration presented at Solution (a) of Table 3 but still seems to be high relative to the historical estimates. An intuitive explanation of why $\sqrt{s_{cc}}$ is lower even though $\sqrt{s_{dd}}$ is raised from 10% to 18% is as follows. By decreasing the volatility of $x_{\lambda,t}$, $b_{\gamma 4}$ (the coefficient of $x_{\lambda,t}$ in γ_t) increases more rapidly than $\sqrt{s_{dd}}$ for γ_t to retain a sufficient variation. It then follows from the slope equation for the factor risk premium of Δd_t, given by $b_{\gamma 4}(s'_{dx}b_\nu + s_{cd}) = s_{dd}$, that the consumption volatility, which appears in $s_{cd} = \sqrt{s_{cc}s_{dd}}\rho_{cd}$, does not need to be as high as that in the baseline calibration.

The unconditional standard deviation of γ_t, denoted as $SD[\gamma_t]$, is implied to be 8.6, which is below 25 in Table 3. Panel (a) of Figure 9 depicts the unconditional distribution of γ_t. Although still nonzero under the normal distribution, the probability of $\gamma_t < 0$ is negligibly low, showing that one of the shortcomings of the proposed model is resolved.

The unconditional mean and standard deviation of subjective discount factor, denoted as $E[\beta_t]$ and $SD[\beta_t]$, are 0.996 and 0.00085, respectively. Compared with the corresponding values at Solution (a) of Table 3, the mean is more reasonable and the standard deviation is much lower because of the lower volatility of $x_{\lambda,t}$. As noted in Sections 2.6 and 4.1, the lower volatility of β_t improves the accuracy of the approximation of the continuation value ν_t, which is beneficial for the model. Panel (b) of Figure 9 depicts the unconditional distribution of β_t, showing that the peak is near the upper bound of one and that the left tail is not long.

Figure 10 presents the average term structures of real and nominal interest rates, where the focus is now placed on the plots labeled "No Jump." Panels (a) and (b) show that both real and nominal yield curves slope upward. The slopes, however, are less steep than those in the baseline calibration in Figure 3 because of the smaller mean of $x_{\lambda,t}$. Panels (c) and (d) show that both real and nominal volatility curves slope downward. The negative slopes are more pronounced than those in the baseline calibration because of the lower volatility of $x_{\lambda,t}$.

Figure 11 presents the term structures of risk premiums, excess-return volatilities, and Sharpe ratios of dividend strips, where we address the plots labeled "No Jump." Panel (a) shows that the proposed model can still generate a downward-sloping term structure of dividend risk premiums. The risk premiums at $n = 1$ (one quarter) and $n = 160$ (40 years) are 8.50% and 5.77% per year, respectively. Although the range is narrower than in the baseline calibration, it is still comparable to those for the previous models listed in Table 1. Panel (b) shows that the volatility curve is first decreasing up to around $n = 40$ (10 years) and then turns slightly increasing. The volatilities at $n = 1, 40$, and 160 are 18.0%, 15.2%, and 16.3% per year, respectively. Compared with the baseline calibration in Figure 5, the volatilities at the short end are higher, reflecting the increase in the volatility of Δd_t. In the medium maturity range, they are lower because of the lower volatility of $x_{\lambda,t}$. At the long end, the volatilities in the baseline and alternative calibrations converge to a similar level. Panel (c) shows that the model can also generate a downward-sloping term structure of Sharpe ratios, ranging from 0.47 ($n = 1$) to 0.35 ($n = 160$). This range seems to be comparable to those in the previous models although it is narrower than in the baseline calibration.

In summary, the proposed model can still generate the term structures that stand as challenges to equilibrium models. However, changing the parameter values alone may be insufficient because the volatility of Δc_t still seems to be high. In Section 5.2, we modify the dynamics of cash flows to overcome this problem. This modification is minimal as our fundamental interest is in the extension of preferences rather than cash flows. Backus et al. (2018) also propose a simple modeling of jumps, so that returns to contingent claims written on cash flows at distant points in time are not excessively correlated.

5.2 Introducing jumps into cash flows

To further reduce the volatility of consumption growth, we introduce jumps into the consumption and dividend processes. The (negative) jumps can be interpreted as disastrous events in line with Reitz (1988), Barro (2009), Gabaix (2012), and Wachter (2013). The agent dislikes jump shocks (infrequent but large negative shocks) to consumption growth more than Gaussian shocks (small but frequent shocks) if these two types of shocks have the same volatility in a statistical sense. Put it another way, for a given level of agent's measure of consumption risks, it is possible to reduce statistical measures of consumption risks by introducing jumps.

5.2.1 Cash-flow dynamics and the derived SDF

Our introduction of jumps is simple. We assume that realized consumption and dividend growth alone can jump, whereas neither the inflation growth nor the state vector can. Additionally, we assume that both the jump intensity and the jump size are constant. Then, we respecify the consumption and dividend processes as

$$\Delta c_{t+1} = \mu_c + b_c x_{d,t} + \sigma_c' z_{t+1} + (\ln \xi) N_{t+1} , \quad (78)$$

$$\Delta d_{t+1} = \mu_d + x_{d,t} + \sigma_d' z_{t+1} + k(\ln \xi) N_{t+1} , \quad (79)$$

where N_t follows an i.i.d. Poisson distribution with intensity parameter $l > 0$ and is independent of the Gaussian shock z_t. A jump size in consumption growth is captured by ξ $(0 < \xi < 1)$. When a single jump occurs at time $t+1$ (i.e., $N_{t+1} = 1$), $C_{t+1}^e = \xi C_t^e$, ignoring the other components. Multiple jumps at a point in time are also possible, which can be interpreted as representing the severity of the disaster. Specifically, when $N_{t+1} = n$, $C_{t+1}^e = \xi^n C_t^e$. However, this interpretation makes it difficult to identify l and ξ separately. Then, we fix l at 1/40, which roughly corresponds to the frequency at which a jump occurs once in every ten years on average. It is noted that our purpose of implying a reasonable behavior of risk aversion together with a realistic level of consumption volatility can be achieved for other values of l.

The same N_{t+1} is used for capturing jumps in dividend growth, which means that the jump event occurs to both consumption and dividend simultaneously. However, the jump size for dividend growth is amplified by $k > 1$ because $D_{t+1} = \xi^{nk} D_t$ for $N_{t+1} = n$, ignoring the other components.

A number of extensions are possible and accordingly the results below may further be improved. First, the jump size can be stochastic. A conventional probability distribution such as an exponential, gamma, or normal distribution does not violate model's tractability. Second, the jump intensity can be stochastic. Gabaix (2012) and Watcher (2013) demonstrate the importance of time-varying jump intensity for capturing high equity premiums. Third, the disaster can be followed by recovery. Hasler and Marfè (2016) model consumption and dividend processes that mean-revert after a large fall, with the rate of mean reversion differing between the two processes, and successfully explain a downward-sloping term structure of dividend risk premiums.

For notational simplicity, we denote the consumption process by the sum of the Gaussian and jump components as $\Delta c_{t+1} = \Delta c_{t+1}^G + \Delta c_{t+1}^J$, where $\Delta c_{t+1}^J = (\ln \xi) N_{t+1}$ and Δc_{t+1}^G is the remaining Gaussian component. Likewise, the dividend process is denoted as $\Delta d_{t+1} = \Delta d_{t+1}^G + \Delta d_{t+1}^J$.

The log continuation value $\ln \nu_t$ is also approximated by a linear function of the state vector x_t as $\ln \nu_t = \mu_\nu + b_\nu' x_t$, where $(\mu_\nu,\ b_\nu')$ are the solution to the simultaneous quadratic equations, which are slightly modified by the jump component. Appendix A provides these equations and Appendix C examines the accuracy of the approximation of ν_t.

The log SDF m_{t+1} is derived as

$$m_{t+1} = -r_{f,t+1} - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t' z_{t+1} - \gamma_t(\ln \xi)N_{t+1} - l(\xi^{-\gamma_t} - 1) \ . \tag{80}$$

The real risk-free rate $r_{f,t+1}$ is also derived as a linear function of x_t: $r_{f,t+1} = A_f + B_f' x_t$, where $(A_f,\ B_f')$ are also adjusted for the jump component, presented in Appendix A. The price-of-risk vector λ_t associated with z_{t+1} is of the same form as that given in (32), which now depends indirectly on the jump parameters through $(\mu_\nu,\ b_\nu')$.

The log prices of zero-coupon bonds and equities are approximated as linear in x_t: $\ln P^i_{t,n} = A^i_n + B^{i\,\prime}_n x_t$ $(i = \{R, N, D\})$. For real and nominal bonds, the recursive equations for $(A^i_n,\ B^{i\,\prime}_n)$ $(i = \{R, N\})$ are of the same form as those without the jump component, although they contain the jump parameters implicitly through $(\mu_\nu,\ b_\nu',\ A_f,\ B_f')$. By contrast, for dividend strips, the recursive equations for $(A^D_n,\ B^{D\,\prime}_n)$ have additional terms related to the jump component because their payoffs depend directly on future dividends that are exposed to jump shocks. The price of a dividend strip when jumps are included is derived in Appendix D.

5.2.2 Calibration

The parameters of the extended model are calibrated by taking the following conditions into account: $E[\gamma_t] = 30$; $E[\Delta c_t] = E[\Delta d_t] = 1.29\%$ per year in equations (78) and (79) (the same level as in the baseline calibration); $\text{var}_t[\Delta c_{t+1}] = 4\%$ per year in equation (78); $\text{var}_t[\Delta d_{t+1}] = 18\%$ per year in equation (79); $l = 1/40$; and the average term structures of interest rates and dividend risk premiums are similar to those in Section 5.1.

The "JUMP" row of Table 6 provides the results of the calibration. The continuation value and hence the SDF exist that satisfy the conditions listed above. Indeed, we can successfully reduce the volatility of consumption growth while keeping reasonable the behavior of state-dependent preferences. Of the total volatility of Δc_t set at 4%, the Gaussian component $\sqrt{s_{cc}}$ reaches 3.98%. Once a jump event occurs with the intensity set at $l = 1/40$, the current consumption falls by 1.26% (computed by $\xi - 1$) from the previous quarter. These results imply that the role of jumps is not crucial for consumption growth. It is, however, for dividend growth. Of the total volatility of Δd_t set at 18%, the Gaussian component $\sqrt{s_{dd}}$ is 16.52%, and upon the occurrence of a single jump, the dividend falls by more than 20% (computed by $\xi^k - 1$) from the previous quarter.

The standard deviation of state-dependent risk aversion decreases from 8.6 to 8.1 when jumps are included. The difference, however, does not seem to be economically large, based on the unconditional distribution of γ_t in Figure 9(a). In both cases, the probability of $\gamma_t < 0$ is negligibly low.

On the contrary, the inclusion of jumps can change the mean and standard deviation of subjective discount factor β_t. The unconditional mean $E[\beta_t]$ decreases to 0.986, which remains within a reasonable range. The unconditional standard deviation $SD[\beta_t]$ increases to 0.00232. Consequently, the accuracy of the approximation of the continuation value ν_t is maintained even by the introduction of jumps (see Appendix C). Figure 9(b) plots the unconditional distribution of β_t. It shifts leftward and has a longer left tail, which is explained intuitively as follows. We do not change the level of the one-period risk-free rate between with and without jumps in the calibration procedure. With jumps, however, there is a downward pressure on the one-period risk-free rate because the agent is more willing to hold real bonds to hedge jump risks. To offset this downward pressure, the subjective discount rate $1 - \beta_t$ must instead rise and hence β_t falls. Furthermore, since β_t shifts away from the upper bound of one, there is more room for β_t to fluctuate.

5.2.3 Term structures

Figure 10 plots the term structures of interest rates and their volatilities. By construction of the calibration procedure explained in Section 5.2.2, each plot is similar with and without jumps. The same is true for the term structure of dividend risk premiums in Figure 11(a). On the contrary, a difference appears in Figure 11(b), which shows the term structure of excess-return volatilities of dividend strips. By including jumps, the volatility curve in the medium to long maturity range shifts upward while at $n = 1$, the volatility is similar with and without jumps because of the constraint that the volatility of dividend growth is the same, set at 18%. At $n = 160$, the excess-return volatility with jumps is 17.0%, higher than that without jumps, 16.3%. The higher volatility arises from a higher covariance between realized and expected dividend growth. The covariance is higher (less negative) because jumps are introduced only into realized dividend growth while keeping its total volatility fixed. Since the excess-return volatility is slightly higher with jumps than without, the Sharpe ratios with jumps decrease slightly faster, as shown in Figure 11(c).

In summary, the change in some of the parameter values and the inclusion of jumps into cash flows together are helpful for improving the proposed model, which now offers economically reasonable implications about the consumption dynamics and preferences while keeping the ability to generate the various term structures.

6 Concluding remarks

This study proposes an equilibrium asset-pricing model that jointly produces the term structures of zero-coupon bonds and equities. For this purpose, we extend a recursive utility function in a way in which the parameters capturing risk aversion and time preference are driven by state variables of the economy and asset markets. The parameters of the proposed model are calibrated by matching the stochastic discount factor of the proposed model with that exogenously specified by Lettau and Wachter (2011; LW). This calibration approach allows the proposed model to have a similar descriptive ability to the LW model, and the LW model to have an equilibrium foundation. With the help of the LW model, the proposed model can produce a downward-sloping term structure of dividend risk premiums when the term structure of real interest rates slopes either upward or downward. Explaining these term structures is considered to be a challenge for equilibrium models.

In terms of an equilibrium foundation, we find that the values of the parameters originally calibrated by LW may be unrealistic when more economic structures are imposed. Most notably, it is implied that the mean and variance of state-dependent risk aversion is too high under a reasonable level of consumption-growth volatility. We then change some of the parameter values of the LW model, which shifts part of risks from a price-of-risk factor to realized dividend growth. We further introduce jumps into the consumption and dividend processes, which can be interpreted as disasters. The modified version of the model then implies an economically plausible behavior of the preferences and consumption growth without losing the descriptive ability for the average term structures.

While this study shows that the state-dependent preferences are useful for endogenizing the key mechanism of the LW model, it does not address which state variables are effective from actual data on cash flows and market prices. A further challenge is to present micro evidence on the state-dependent preferences. Even though we accept that the preferences are changeable, it is unclear whether they change as predicted by the model. These important questions are left for future research.

Appendix A: Derivation of key equations

The optimal consumption given in equation (6)

Substitute V_t into (2) with C_t replaced by C_t^* (the optimal consumption for the agent):

$$V_t = C_t^{*1-\beta_t} E_t[V_{t+1}^{1-\gamma_t}]^{\beta_t/(1-\gamma_t)} \ . \tag{81}$$

Assume that V_t is of the form $V_t = \phi_t W_t$, where ϕ_t is a state-dependent variable identified below. Substitute first this form and then the budget constraint given in (4) into the RHS of (81):

$$V_t = C_t^{*1-\beta_t}(W_t - C_t^*)^{\beta_t} E_t[(\phi_{t+1} R_{w,t+1})^{1-\gamma_t}]^{\beta_t/(1-\gamma_t)} \ . \tag{82}$$

(82) satisfies the first order condition (FOC): $\partial V_t/\partial C_t^* = 0$. Given that β_t and γ_t are exogenous by Assumption (i), solving the FOC yields C_t^* given in (6). By Assumption (ii), the second order condition is met:

$$\frac{\partial^2 V_t}{\partial C_t^{*2}} = -\beta_t(1-\beta_t)V_t \left\{ \frac{W_t}{C_t^*(W_t - C_t^*)} \right\}^2 < 0 \ . \tag{83}$$

Finally, by substituting (6) into (82), V_t is confirmed to be of the assumed form, where

$$\phi_t = (1-\beta_t) \left\{ \frac{\beta_t}{1-\beta_t} E_t[(\phi_{t+1} R_{w,t+1})^{1-\gamma_t}]^{1/(1-\gamma_t)} \right\}^{\beta_t} \ . \tag{84}$$

Recursive equation (8) for the value function

Replace first $R_{w,t+1}$ with $R_{w,t+1}^*$ on the RHS of (84) and then substitute (7):

$$\frac{\phi_t^*}{1-\beta_t} = E_t \left[\left(\frac{\phi_{t+1}^*}{1-\beta_{t+1}} \frac{C_{t+1}^e}{C_t^e} \right)^{1-\gamma_t} \right]^{\beta_t/(1-\gamma_t)} \ , \tag{85}$$

where ϕ_t^* is used in place of ϕ_t to emphasize the equilibrium. Meanwhile, the continuation value in equilibrium is $V_t^* = \phi_t^* W_t^* = \frac{\phi_t^*}{1-\beta_t} C_t^e$. Define $\nu_t = V_t^*/C_t^e = \frac{\phi_t^*}{1-\beta_t}$. Substituting this into (85) yields (8).

SDF given in equation (9)

A simple way of deriving the SDF is to use the Euler equation for wealth (a claim to the flow of endowments). Rearrange (8):

$$1 = E_t \left[\left(\frac{\nu_{t+1}}{\nu_t^{1/\beta_t}} \right)^{1-\gamma_t} \left(\frac{C_{t+1}^e}{C_t^e} \right)^{-\gamma_t} \frac{C_{t+1}^e}{C_t^e} \right] \ . \tag{86}$$

By (7),

$$\frac{C^e_{t+1}}{C^e_t} = \beta_t \frac{1-\beta_{t+1}}{1-\beta_t} R^*_{w,t+1} \ . \tag{87}$$

Substitute (87) into (86) and rearrange the terms, we have $1 = E_t[M_{t+1} R^*_{w,t+1}]$, where M_{t+1} is given in (9).

Value function given in equations (17)–(19)

Assume that the solution to the recursive equation (8) is of the following form: $\nu_t = \exp\{\mu_\nu + b'_\nu x_t\}$. Substitute this form into (8) together with $\beta_t = \beta$:

$$\begin{aligned} \exp\{\mu_\nu + b'_\nu x_t\} &= E_t[\exp\{(1-\gamma_t)(\mu_\nu + b'_\nu x_{t+1} + \Delta c_{t+1})\}]^{\beta/(1-\gamma_t)} \\ &= \exp\left\{\beta\left(E_t[\chi_{t+1}] - \frac{1}{2}(\gamma_t - 1)\mathrm{var}_t[\chi_{t+1}]\right)\right\} \ , \end{aligned} \tag{88}$$

where

$$\chi_t = \mu_\nu + b'_\nu x_t + \Delta c_t \ . \tag{89}$$

Note that

$$E_t[\chi_{t+1}] = \mu_\nu + \mu_c + (b_c + \Phi_x b_\nu)' x_t \ , \tag{90}$$

$$\mathrm{var}_t[\chi_{t+1}] = b'_\nu s_{xx} b_\nu + 2 s'_{cx} b_\nu + s_{cc} \ \ (= v_{c\nu}) \ . \tag{91}$$

Substitute these conditional moments and $\gamma_t = \mu_\gamma + b'_\gamma x_t$ into the RHS of (88), and then take the log of both sides:

$$\mu_\nu + b'_\nu x_t = \beta\left\{\mu_\nu + \mu_c + (b_c + \Phi_x b_\nu)' x_t - \frac{1}{2} v_{c\nu}(\mu_\gamma + b'_\gamma x_t - 1)\right\} \ . \tag{92}$$

For the assumed form of ν_t to be true, (92) must hold for any x_t, leading to the simultaneous equations for $(\mu_\nu, \ b'_\nu)$ given in (18) and (19).

Risk-free rate given in equations (23)–(25)

By substituting (17) together with $\beta_t = \beta$ into (9), the SDF can be rewritten using χ_{t+1} defined in (89) as

$$M_{t+1} = \beta \nu_t^{-(1-\gamma_t)/\beta} \exp\{(1-\gamma_t)\chi_{t+1} - \Delta c_{t+1}\} \ . \tag{93}$$

Take the conditional expectation of both sides of (93):

$$E_t[M_{t+1}] = \beta \nu_t^{-(1-\gamma_t)/\beta} E_t[\exp\{(1-\gamma_t)\chi_{t+1}\}] \ E_t[e^{-\Delta c_{t+1}}] \exp\{(\gamma_t - 1)\mathrm{cov}_t[\chi_{t+1}, \ \Delta c_{t+1}]\} \ . \tag{94}$$

Meanwhile, the recursive equation (8) can be rewritten using χ_t as

$$\nu_t^{(1-\gamma_t)/\beta} = E_t[\exp\{(1-\gamma_t)\chi_{t+1}\}] \ . \tag{95}$$

Substitute (95) into the RHS of (94), develop the conditional moments, and rearrange the terms:

$$\begin{aligned} r_{f,t+1} &= -\ln E_t[M_{t+1}] \\ &= -\ln\beta + \mu_c - \frac{1}{2}s_{cc} - (s'_{cx}b_\nu + s_{cc})(\mu_\gamma - 1) + \{b_c - (s'_{cx}b_\nu + s_{cc})b_\gamma\}' x_t \ . \end{aligned} \tag{96}$$

Collecting the intercept and slope terms of (96) into A_f and B_f, respectively, yields (24) and (25).

Approximation of the value function given in equations (17) and (29)–(30)

Substitute $\nu_t = \exp\{\mu_\nu + b'_\nu x_t\}$ into the RHS of (8), develop the conditional expectation, and rearrange the terms:

$$\ln\ (\text{RHS of (8)}) = \beta(x_t)\left\{\mu_\nu + \mu_c + (b_c + \Phi_x b_\nu)' x_t - \frac{1}{2}v_{c\nu}(\mu_\gamma + b'_\gamma x_t - 1)\right\} \ , \tag{97}$$

which is basically the same as the RHS of (92) except that β is replaced by $\beta(x_t)$. Since (97) is not equal to $\mu_\nu + b'_\nu x_t$ (the log of the LHS of (8)) for any x_t, it is approximated as linear in x_t. Specifically, nonlinear terms associated with $\beta(x_t)$ and $\beta(x_t)x_t$ are linearized around $x_t = 0$ (the unconditional mean) as given in (27) and (28), respectively. Then, matching the intercept and slope terms yields (29) and (30).

Approximation of the risk-free rate given in equations (23) and (34)–(35)

By substituting (17) into (9), the SDF can be rewritten as

$$M_{t+1} = \frac{\beta_t}{1-\beta_t}\nu_t^{-(1-\gamma_t)/\beta_t}\exp\{(1-\gamma_t)\chi_{t+1} + \psi_{t+1}\} \ , \tag{98}$$

where χ_{t+1} is defined in (89) and

$$\psi_t = \mu_\beta + b'_\beta x_t - \Delta c_t \ . \tag{99}$$

Take the conditional expectation of both sides of (98):

$$\begin{aligned} E_t[M_{t+1}] = & \frac{\beta_t}{1-\beta_t}\nu_t^{-(1-\gamma_t)/\beta_t} \\ & \times E_t[\exp\{(1-\gamma_t)\chi_{t+1}\}]E_t[e^{\psi_{t+1}}]\exp\{(1-\gamma_t)\text{cov}_t[\chi_{t+1},\ \psi_{t+1}]\} \ . \end{aligned} \tag{100}$$

Note that the recursive equation (8) can also be rewritten as $\nu_t^{(1-\gamma_t)/\beta_t} = E_t[\exp\{(1-\gamma_t)\chi_{t+1}\}]$. Substitute this into the RHS of (100) and develop the conditional expectation:

$$E_t[M_{t+1}] = \frac{\beta_t}{1-\beta_t} \exp\left\{E_t[\psi_{t+1}] + \frac{1}{2}\text{var}_t[\psi_{t+1}] + (1-\gamma_t)\text{cov}_t[\chi_{t+1},\ \psi_{t+1}]\right\} \ . \tag{101}$$

Note that

$$E_t[\psi_{t+1}] = \mu_\beta - \mu_c + (\Phi_x b_\beta - b_c)' x_t \ , \tag{102}$$

$$\text{var}_t[\psi_{t+1}] = b_\beta' s_{xx} b_\beta - 2 s_{cx}' b_\beta + s_{cc} \ \ (= v_{c\beta}) \ , \tag{103}$$

$$\text{cov}_t[\chi_{t+1},\ \psi_{t+1}] = (s_{xx} b_\nu + s_{cx})' b_\beta - (s_{cx}' b_\nu + s_{cc}) \ . \tag{104}$$

Substitute these conditional moments together with (14) and (16) into the RHS of (101), and rearrange the terms:

$$\begin{aligned} r_{f,t+1} = & -\ln\beta_t + \mu_c - \frac{1}{2} v_{c\beta} + \{(s_{xx} b_\nu + s_{cx})' b_\beta - (s_{cx}' b_\nu + s_{cc})\}(\mu_\gamma - 1) \\ & + \left[b_c + (I_{d\times d} - \Phi_x) b_\beta + \{(s_{xx} b_\nu + s_{cx})' b_\beta - (s_{cx}' b_\nu + s_{cc})\} b_\gamma\right]' x_t \ . \end{aligned} \tag{105}$$

The leading term on the RHS of (105), $-\ln\beta_t$, is nonlinear in x_t. It is then linearized around $x_t = 0$ (the unconditional mean) as given in (33). Then, collecting the intercept and slope terms into A_f and B_f, respectively, yields (34) and (35).

Risk premium of a dividend strip given in equations (72)–(74)

Note that

$$m_{t+1} - E_t[m_{t+1}] = -\lambda_t' z_{t+1} \ , \tag{106}$$

$$r^D_{t+1,n-1} - E_t[r^D_{t+1,n-1}] = (\sigma_x B^D_{n-1} + \sigma_d)' z_{t+1} \ , \tag{107}$$

where $\lambda_t = (\sigma_x b_\nu + \sigma_c)\gamma_t - \sigma_x(b_\beta + b_\nu)$. Substitute (106) and (107) into the RHS of (71),

$$\begin{aligned} RP^D_{t,n-1} &= \{(\sigma_x b_\nu + \sigma_c)\gamma_t - \sigma_x(b_\beta + b_\nu)\}'(\sigma_x B^D_{n-1} + \sigma_d) \\ &= -(b_\beta + b_\nu)'(s_{xx} B^D_{n-1} + s_{dx}) + \{s_{dx}' b_\nu + s_{cd} + (s_{xx} b_\nu + s_{cx})' B^D_{n-1}\}\gamma_t \ . \end{aligned} \tag{108}$$

Approximation of the value function with jumps in consumption growth

By $\Delta c_t = \ln(C_t^e / C_{t-1}^e) = \Delta c_t^G + \Delta c_t^J$, the recursive equation for the log continuation value can be written as

$$\ln\nu_t = \frac{\beta_t}{1-\gamma_t} \ \ln E_t[\exp\{(1-\gamma_t)(\ln\nu_{t+1} + \Delta c^G_{t+1} + \Delta c^J_{t+1})\}] \ . \tag{109}$$

Because Δc_t^G and Δc_t^J are mutually independent and because ν_t is a function of the state vector that has no jump component, the RHS of (109) can be factorized as follows:

$$\ln \nu_t = \frac{\beta_t}{1-\gamma_t} \left(\ln E_t[\exp\{(1-\gamma_t)(\ln \nu_{t+1} + \Delta c_{t+1}^G)\}] + \ln E_t[\exp\{(1-\gamma_t)\Delta c_{t+1}^J\}] \right) . \quad (110)$$

For $\Delta c_{t+1}^J = (\ln \xi) N_{t+1}$, the second term on the RHS of (110) is developed as

$$\frac{\beta_t}{1-\gamma_t} \ln E_t[\exp\{(1-\gamma_t)(\ln \xi) N_{t+1}\}] = \frac{l\,\beta_t(\xi^{1-\gamma_t}-1)}{1-\gamma_t} . \quad (111)$$

To approximate $\ln \nu_t$ as $\ln \nu_t = \mu_\nu + b_\nu' x_t$, it is necessary to linearize the RHS of (111) as,

$$\frac{l\,\beta_t(\xi^{1-\gamma_t}-1)}{1-\gamma_t} \approx -(1-e^{\mu_\beta}) l k_0 + l k_1' x_t , \quad (112)$$

where

$$\begin{aligned} k_0 &= \frac{\xi^{1-\mu_\gamma}-1}{\mu_\gamma - 1} , & (113) \\ k_1 &= e^{\mu_\beta} k_0 b_\beta + \frac{1-e^{\mu_\beta}}{\mu_\gamma - 1} \{k_0 + \xi^{1-\mu_\gamma} \ln \xi\} b_\gamma . & (114) \end{aligned}$$

It is noted that this approximation is unavoidable even for $\beta_t = \beta$ (constant). Then, it is concerned that the approximation of ν_t is less accurate, which is addressed in Appendix C.

By the additional approximation given in (112), $(\mu_\nu,\ b_\nu')$ satisfy the following equations:

$$\begin{aligned} \mu_\nu &= (1-e^{\mu_\beta}) \left\{ \mu_\nu + \mu_c - \frac{1}{2} v_{c\nu}(\mu_\gamma - 1) - l k_0 \right\} , & (115) \\ b_\nu &= (1-e^{\mu_\beta}) \left(b_c + \Phi_x b_\nu - \frac{1}{2} v_{c\nu} b_\gamma \right) - e^{\mu_\beta} \left\{ \mu_\nu + \mu_c - \frac{1}{2} v_{c\nu}(\mu_\gamma - 1) \right\} b_\beta + l k_1 . & (116) \end{aligned}$$

Risk-free rate with jumps in consumption growth

We approximate the risk-free rate $r_{f,t+1}$ as $r_{f,t+1} = A_f + B_f' x_t$. First, the SDF given in (9) is rewritten as $M_{t+1} = M_{t+1}^G M_{t+1}^J$, where

$$\begin{aligned} M_{t+1}^G &= \beta_t \frac{1-\beta_{t+1}}{1-\beta_t} \left(\frac{\nu_{t+1}}{\nu_t^{1/\beta_t}} \right)^{1-\gamma_t} e^{-\gamma_t \Delta c_{t+1}^G} , & (117) \\ M_{t+1}^J &= e^{-\gamma_t (\ln \xi) N_{t+1}} . & (118) \end{aligned}$$

Then, by the Euler equation, $r_{f,t+1} = -\ln E_t[M_{t+1}^G] - \ln E_t[M_{t+1}^J]$. The conditional expectation of the Gaussian part is the same as before and that of the Jump part is developed as

$$-\ln E_t[e^{-\gamma_t (\ln \xi) N_{t+1}}] = -l(\xi^{-\gamma_t} - 1) . \quad (119)$$

This term is added to the previous equation for $r_{f,t+1}$ without jumps. To approximate $r_{f,t+1}$ as linear in x_t, the RHS of (119) is linearized as,

$$-l(\xi^{-\gamma_t}-1) \approx -l(\xi^{-\mu_\gamma}-1) + l\xi^{-\mu_\gamma}(\ln\xi) b'_\gamma x_t \ . \tag{120}$$

Then, $r_{f,t+1} = A_f + B'_f x_t$, where

$$\begin{aligned} A_f &= -\ln(1-e^{\mu_\beta}) + \mu_c - \frac{1}{2} v_{c\beta} + (\mu_\gamma - 1)\{(s_{xx}b_\nu + s_{cx})' b_\beta - (s'_{cx}b_\nu + s_{cc})\} \\ &\quad - l(\xi^{-\mu_\gamma}-1) \ , && (121) \\ B_f &= b_c + \{l\xi^{-\mu_\gamma}\ln\xi - (s'_{cx}b_\nu + s_{cc})\} b_\gamma + \left\{ \frac{1}{1-e^{\mu_\beta}} I_{d\times d} - \Phi_x + b_\gamma (s_{xx}b_\nu + s_{cx})' \right\} b_\beta && (122) \end{aligned}$$

Appendix B: Condition for the continuation value to be real

For a state-dependent subjective discount factor, $\beta(x_t)$, the continuation value is approximated as $\nu_t = \exp\{\mu_\nu + b'_\nu x_t\}$, where μ_ν and b_ν are the solution to the simultaneous quadratic equations given in (29) and (30). This appendix presents the condition on which μ_ν and b_ν are real. Also, it addresses which real root to select.

Recall that (29) and (30) are, respectively,

$$\begin{aligned} \mu_\nu &= \beta_0 \left\{ \mu_\nu + \mu_c - \frac{1}{2} v_{c\nu}(\mu_\gamma - 1) \right\} \ , \\ b_\nu &= \beta_0 \left(b_c + \Phi_x b_\nu - \frac{1}{2} v_{c\nu} b_\gamma \right) + \beta_1 \left\{ \mu_\nu + \mu_c - \frac{1}{2} v_{c\nu}(\mu_\gamma - 1) \right\} \ . \end{aligned}$$

By (29), $v_{c\nu}$ can be expressed as a linear function of μ_ν. Then, substitute this into (30) and rearrange the terms:

$$b_\nu = c_0 + c_1 \mu_\nu \ , \tag{123}$$

where

$$\begin{aligned} c_0 &= (I_{d\times d} - \beta_0 \Phi_x)^{-1} \beta_0 \left(b_c - \frac{\mu_c}{\mu_\gamma - 1} b_\gamma \right) \ , && (124) \\ c_1 &= (I_{d\times d} - \beta_0 \Phi_x)^{-1} \left(\frac{\beta_1}{\beta_0} + \frac{1-\beta_0}{\mu_\gamma - 1} b_\gamma \right) \ . && (125) \end{aligned}$$

Substituting (123) back into (29), where $v_{c\nu} = b'_\nu s_{xx} b_\nu + 2s'_{cx} b_\nu + s_{cc}$, yields a quadratic equation with respect to μ_ν as

$$\alpha_2 \mu_\nu^2 + 2\alpha_1 \mu_\nu + \alpha_0 = 0 \ , \tag{126}$$

where

$$\alpha_2 = c_1' s_{xx} c_1 \ , \tag{127}$$

$$\alpha_1 = c_0' s_{xx} c_1 + s_{cx}' c_1 + \frac{1-\beta_0}{\beta_0(\mu_\gamma - 1)} \ , \tag{128}$$

$$\alpha_0 = c_0' s_{xx} c_0 + 2 s_{cx}' c_0 + s_{cc} - \frac{2\mu_c}{\mu_\gamma - 1} \ . \tag{129}$$

Then, the condition for real μ_ν is that the determinant of (126) is non-negative:

$$\alpha_1^2 - \alpha_2 \alpha_0 \geq 0 \ . \tag{130}$$

By (123), this is also the condition for real b_ν.

Which root to select

Given that (126) has two real roots, we always select a larger root in order to avoid a negative value of μ_ν if a smaller root is negative. It is likely that at least one root is positive for typical sets of parameter values, which is explained as follows. (29) can be rewritten as

$$\mu_\nu = \frac{\beta_0}{1-\beta_0} \left\{ \mu_c - \frac{1}{2} v_{c\nu} (\mu_\gamma - 1) \right\} \ , \tag{131}$$

where $\beta_0 = 1 - e^{\mu_\beta}$. By $\mu_\beta < 0$, it holds that $0 < \beta_0 < 1$ and hence that $\beta_0/(1-\beta_0) > 0$. Furthermore, it is typically the case that the mean term ($\mu_c = E[\Delta c_t]$) dominates the (scaled) variance term ($v_{c\nu} = \mathrm{var}_t[\ln \nu_{t+1} + \Delta c_{t+1}]$) even if μ_γ is large.

Appendix C: Accuracy of approximation of the value function

We express here an approximation of the value function as ν_t^{AP}, which is derived as $\nu_t^{AP} = \exp\{\mu_\nu + b_\nu' x_t\}$. The approximation error is defined by

$$e_t = \nu_t - \nu_t^{AP} = E_t[(\nu_{t+1} e^{\Delta c_{t+1}})^{1-\gamma_t}]^{\beta_t/(1-\gamma_t)} - \exp\{\mu_\nu + b_\nu' x_t\} \ . \tag{132}$$

Since the true form of ν_t is unknown, it is difficult to evaluate the conditional expectation on the RHS of (132) and hence the error e_t. We therefore compute a pseudo approximation error. First, we decompose e_t as $e_t = e_{1,t} + e_{2,t}$, where

$$e_{1,t} = E_t[(\nu_{t+1} e^{\Delta c_{t+1}})^{1-\gamma_t}]^{\beta_t/(1-\gamma_t)} - E_t[(\nu_{t+1}^{AP} e^{\Delta c_{t+1}})^{1-\gamma_t}]^{\beta_t/(1-\gamma_t)} \ , \tag{133}$$

$$e_{2,t} = E_t[(\nu_{t+1}^{AP} e^{\Delta c_{t+1}})^{1-\gamma_t}]^{\beta_t/(1-\gamma_t)} - \exp\{\mu_\nu + b_\nu' x_t\} \ . \tag{134}$$

Then, $e_{2,t}$ is a pseudo approximation error, where the log of $E_t[(\nu_{t+1}^{AP} e^{\Delta c_{t+1}})^{1-\gamma_t}]^{\beta_t/(1-\gamma_t)}$ is equal to (97). Unless $e_{1,t}$ and $e_{2,t}$ offset each other, $e_{2,t}$ undervalues the approximation error, which requires caution for interpreting the results obtained in the following three cases.

C1. Linear risk aversion γ_t with $E[\gamma_t] = 150$

Panel A of Table A1 presents $e_{2,t}/\nu_t^{AP}$ in percentage terms when γ_t is linear with the parameter values given in Table 4. The errors are evaluated when the factors are above or below k $(= 1, 2, 3)$ standard deviations from the mean (i.e., zero). By construction of the approximation, $e_{2,t} = 0$ at $x_t = 0$. The label "All factors" indicates that all factors change simultaneously, whereas the label "Individual factors" indicates that only a factor in each row changes with the other factors fixed at the mean. All the errors in the $x_{\pi,t}$-row are zero as ν_t does not depend on $x_{\pi,t}$.

First, by changing all elements of x_t proportionally, the pseud errors are at most -0.12%. Second, by changing only the expected dividend-growth factor $x_{d,t}$ or the risk-free rate factor $x_{f,t}$, the pseud errors are negligibly small. Third, since the price-of-risk factor $x_{\lambda,t}$ varies more intensively than the other factors, it is expected to have a larger impact on the accuracy of the approximation. While this is indeed the case, the pseud errors are at most 0.1%.

C2. Quadratic γ_t with $E[\gamma_t] = 150$

In Section 4.3, we refer to a quadratic specification of γ_t to avoid negative values of γ_t. We specify $\gamma_t = q_0 + q_1 x_{\lambda,t}^2$ with $q_0,\ q_1 > 0$. In order to derive $\ln \nu_t$ as a linear function of x_t, γ_t needs to be linearized around $x_{\lambda,t} = \mu_\lambda$:

$$\gamma_t \approx q_0 + q_1 \mu_\lambda^2 + 2 q_1 \mu_\lambda (x_{\lambda,t} - \mu_\lambda) \ . \tag{135}$$

By matching the intercept and slope terms between (48) and (135), we have $q_1 = b_{\gamma 4}/(2\mu_\lambda)$ and $q_0 = \mu_\gamma - q_1 \mu_\lambda^2$. By this matching, we do not re-calibrate the parameters of the model but simply use those presented in Tables 2 and 4.

Panel B of Table A1 presents $e_{2,t}/\nu_t^{AP}$ in percentage terms for a quadratic γ_t. As expected, the pseud errors are larger than those for a linear γ_t due to the additional approximation given in (135). Still, they are less than 1% in absolute value.

C3. Linear γ_t and jumps in cash-flow processes

To obtain $\ln \nu_t^{AP} = \mu_\nu + b_\nu' x_t$ in the case of jumps, we need to rely on a further approximation presented in (112), which is avoided if γ_t is constant. Then, it follows that the smaller the variation in γ_t, the more accurate is the (additional) approximation. Note that in Section 5.1, we reduce the volatility of $x_{\lambda,t}$ and hence the volatility of $\gamma_t = \mu_\gamma + b_{\gamma 4}(x_{\lambda,t} - \mu_\lambda)$, which works for reducing the approximation error.

Panel C of Table A1 presents $e_{2,t}/\nu_t^{AP}$ in percentage terms, which are computed at the parameter values given (partially) in Table 6. As expected, the additional approximation is not a serious concern. The pseud errors are at most -0.2% when all elements of x_t are above three standard deviations from the mean. When the value of each factor is changed, there is no case in which the pseud error exceeds 0.1% in absolute value.

Appendix D: Term structure formulas

Risk-neutral drift

To simply express the recursive equations for the prices of zero-coupon bonds and equities, we bundle model parameters into those in the risk-neutral probability measure. Specifically, we first describe the risk-neutral dynamics as

$$\Delta d_{t+1} = \mu_d^Q + b_d^{Q\prime} x_t + \sigma_d' z_{t+1}^Q \ , \tag{136}$$

$$\Delta \pi_{t+1} = \mu_\pi^Q + b_\pi^{Q\prime} x_t + \sigma_\pi' z_{t+1}^Q \ , \tag{137}$$

$$x_{t+1} = \mu_x^Q + \Phi_x^{Q\prime} x_t + \sigma_x' z_{t+1}^Q \ , \tag{138}$$

where z_{t+1}^Q is an *i.i.d.* normal random vector in the risk-neutral probability measure. The risk-neutral drift of Δd_{t+1} satisfies for any x_t

$$\mu_d^Q + b_d^{Q\prime} x_t = E_t[\Delta d_{t+1}] + \mathrm{cov}_t[m_{t+1},\ \Delta d_{t+1}] \ . \tag{139}$$

The conditional covariance on the RHS of (139) is

$$\mathrm{cov}_t[\Delta d_{t+1},\ m_{t+1}] = -\sigma_d' \lambda_t = s_{dx}'(b_\beta + b_\nu) - (s_{dx}' b_\nu + s_{cd})\mu_\gamma - (s_{dx}' b_\nu + s_{cd}) b_\gamma' x_t \ . \tag{140}$$

Then, μ_d^Q and b_d^Q on the LHS of (139) are identified as

$$\mu_d^Q = \mu_d + s_{dx}'(b_\beta + b_\nu) - \mu_\gamma (s_{dx}' b_\nu + s_{cd}) \ , \quad b_d^Q = b_d - b_\gamma (s_{dx}' b_\nu + s_{cd}) \ . \tag{141}$$

Likewise,

$$\mu_\pi^Q = \mu_\pi + s'_{\pi x}(b_\beta + b_\nu) - \mu_\gamma(s'_{\pi x} b_\nu + s_{c\pi}) , \qquad b_\pi^Q = b_\pi - b_\gamma(s'_{\pi x} b_\nu + s_{c\pi}) , \tag{142}$$

$$\mu_x^Q = s_{xx}(b_\beta + b_\nu) - \mu_\gamma(s_{xx} b_\nu + s_{cx}) , \qquad \Phi_x^Q = \Phi_x - b_\gamma(s_{xx} b_\nu + s_{cx})' . \tag{143}$$

By the calibration, the slope terms of the factor risk premiums are matched exactly between the LW and proposed models. This is equivalent to matching the slope terms of the risk-neutral drift (i.e., b_d^Q, b_π^Q, and Φ_x^Q) between the two models.

Real zero-coupon bonds

The price of a real zero-coupon bond maturing in n periods, $P_{t,n}^R$, satisfies the following Euler equation:

$$P_{t,n}^R = E_t[M_{t+1} P_{t+1,n-1}^R] = e^{-r_{f,t+1}} E_t^Q[P_{t+1,n-1}^R] , \tag{144}$$

where the second equality is by the change from the physical to risk-neutral probability measures and $E_t^Q[\cdot]$ stands for the conditional expectation under the risk-neutral probability measure. The initial condition is $P_{t,0}^R = 1$. By substituting $P_{t,n}^R = \exp\{A_n^R + B_n^{R\prime} x_t\}$ into the RHS of (144), developing the conditional expectation under the risk-neutral probability measure, and matching the intercept and slope terms on both sides, we obtain the following recursive equations for A_n^R and B_n^R:

$$A_n^R = A_{n-1}^R - A_f + \mu_x^{Q\prime} B_{n-1}^R + \frac{1}{2} B_{n-1}^{R\prime} s_{xx} B_{n-1}^R , \tag{145}$$

$$B_n^R = \Phi_x^Q B_{n-1}^R - B_f , \tag{146}$$

with the initial condition $A_0^R = 0$ and $B_0^R = 0$.

Φ_x^Q in (146) is the same between the LW and proposed models as documented above. Also, B_f, the loading on the state vector in the real risk-free rate $r_{f,t+1}$, is the same between the two models as (50) holds by the calibration procedure. Consequently, B_n^R is the same between the two models for any n.

Nominal zero-coupon bonds

Rewrite the Euler equation (38) for the real price of a nominal zero-coupon bond as

$$P_{t,n}^N \Pi_t = E_t\left[M_{t+1}(P_{t+1,n-1}^N \Pi_{t+1}) \frac{\Pi_t}{\Pi_{t+1}}\right] = e^{-r_{f,t+1}} E_t^Q\left[(P_{t+1,n-1}^N \Pi_{t+1}) \frac{\Pi_t}{\Pi_{t+1}}\right] , \tag{147}$$

with the initial condition $P^N_{t,0}\Pi_t = 1$. By substituting $P^N_{t,n}\Pi_t = \exp\{A^N_n + B^{N\prime}_n x_t\}$ into (147), developing the conditional expectation under the risk-neutral probability measure, and matching the intercept and slope terms on both sides, we obtain the following recursive equations for A^N_n and B^N_n:

$$A^N_n = A^N_{n-1} - A_f - \mu^Q_\pi + \frac{1}{2}s_{\pi\pi} + (\mu^Q_x - s_{\pi x})' B^N_{n-1} + \frac{1}{2}B^{N\prime}_{n-1} s_{xx} B^N_{n-1} \ , \tag{148}$$

$$B^N_n = \Phi^Q_x B^N_{n-1} - B_f - b^Q_\pi \ , \tag{149}$$

with the initial condition $A^N_0 = 0$ and $B^N_0 = 0$. Notice that B^N_n is the same between the LW and proposed models for any n because Φ^Q_x, B_f, and b^Q_π are the same between the two models by the calibration.

Zero-coupon equities

Rewrite the Euler equation (40) for the price of a dividend strip as

$$\frac{P^D_{t,n}}{D_t} = E_t\left[M_{t+1}\frac{P^D_{t+1,n-1}}{D_{t+1}}\frac{D_{t+1}}{D_t}\right] = e^{-r_{f,t+1}}E^Q_t\left[\frac{P^D_{t+1,n-1}}{D_{t+1}}e^{\Delta d_{t+1}}\right] \ , \tag{150}$$

with the initial condition $P^D_{t,0}/D_t = 1$. By substituting $P^D_{t,n}/D_t = \exp\{A^D_n + B^D_n{}' x_t\}$ into (150), developing the conditional expectation under the risk-neutral probability measure, and matching the intercept and slope terms on both sides, we obtain the following recursive equations for A^D_n and B^D_n:

$$A^D_n = A^D_{n-1} - A_f + \mu^Q_d + \frac{1}{2}s_{dd} + (\mu^Q_x + s_{dx})' B^D_{n-1} + \frac{1}{2}B^{D\prime}_{n-1} s_{xx} B^D_{n-1} \ , \tag{151}$$

$$B^D_n = \Phi^Q_x B^D_{n-1} - B_f + b^Q_d \ , \tag{152}$$

with $A^D_0 = 0$ and $B^D_0 = 0$. For the same reason as above, B^D_n is the same between the LW and proposed models for any n.

Zero-coupon equities with jumps in consumption and dividend growth

The SDE M_{t+1} is decomposed into the Gaussian and jump components, which are mutually independent, as

$$M_{t+1} = M^G_{t+1}M^J_{t+1} = e^{-r_{f,t+1}}\frac{M^G_{t+1}}{E_t[M^G_{t+1}]}\frac{M^J_{t+1}}{E_t[M^J_{t+1}]} \ .$$

Also, $\Delta d_{t+1} = \Delta d^G_{t+1} + \Delta d^J_{t+1}$. Meanwhile, $P^D_{t+1,n-1}/D_{t+1}$ is a function of x_{t+1} that has no jump component. Then, the recursive equation (150) is developed as

$$\frac{P^D_{t,n}}{D_t} = e^{-r_{f,t+1}}E_t\left[\frac{M^G_{t+1}}{E_t[M^G_{t+1}]}\frac{P^D_{t+1,n-1}}{D_{t+1}}e^{\Delta d^G_{t+1}}\right]E_t\left[\frac{M^J_{t+1}}{E_t[M^J_{t+1}]}e^{\Delta d^J_{t+1}}\right] \ . \tag{153}$$

The first conditional expectation (multiplied by $e^{-r_{f,t+1}}$) on the RHS of (153) is the same as that developed without jumps. The second conditional expectation is developed by substituting $M_{t+1}^J = e^{-\gamma_t(\ln\xi)N_{t+1}}$ and $\Delta d_{t+1}^J = k(\ln\xi)N_{t+1}$ as

$$E_t[\exp\{(k-\gamma_t)(\ln\xi)N_{t+1} - l(\xi^{-\gamma_t}-1)\}] = \exp\{l(\xi^k-1)\xi^{-\gamma_t}\}\ . \tag{154}$$

To derive $\ln(P_{t,n}^D/D_t)$ as a linear function of x_t, $\xi^{-\gamma_t}$ is linearized as

$$\xi^{-\gamma_t} \approx \xi^{-\mu_\gamma}\{1-(\ln\xi)b_\gamma' x_t\}\ . \tag{155}$$

Then, $\ln(P_{t,n}^D/D_t) = A_n^D + B_n^{D\prime} x_t$, where A_n^D and B_n^D are determined recursively as

$$A_n^D = A_{n-1}^D - A_f + \mu_d^Q + \frac{1}{2}s_{dd} + (\mu_x^Q + s_{dx})' B_{n-1}^D + \frac{1}{2}B_{n-1}^{D\prime} s_{xx} B_{n-1}^D + l(\xi^k-1)\xi^{-\mu_\gamma}, \tag{156}$$

$$B_n^D = \Phi_x^Q B_{n-1}^D - B_f + b_d^Q - l(\xi^k-1)\xi^{-\mu_\gamma}(\ln\xi)b_\gamma\ , \tag{157}$$

with $A_0^D = 0$ and $B_0^D = 0$.

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Paper	Classification	Term structure of dividend strips' risk premiums	volatilities	Real yield curve	RRA	EIS	Subjective dis. rate
Curatola (2015)	Preference; loss aversion	from 7% to 4% at $n = 20$ (years)	from 33% to 20% at $n = 20$ (years)	Upward	4.69	NA	3.00%
Doh and Wu (2016)	Preference; quadratic wealth	from 0% to -18% at $n = 5$ to 100% at $n = 50$	NA	Upward	50	1.4	0.36%
Belo et al. (2015)	Cash-flow; financial leverage	from 10% to 4% at $n = 20$	from 30% to 10% at $n = 20$	NA	10	1.5	1.32%
Lopez et al. (2015)	Cash-flow; nominal rigidities	from 12% to 2.3% at $n = 2$ to 8% at $n = 30$	from 33% to 6% at $n = 2$ to 21% at $n = 30$	Upward	2	0.5	1.08%
Favilukis and Lin (2016)	Cash-flow; wage rigidities	from 6% to 0.6% at $n = 25$	from 45% to 2.5% at $n = 25$ to 7% at $n = 40$	NA	6.5	2.0	2.41%
Marfè (2017)	Cash-flow; wage insurance	from 6.9% to 4.8% at $n = 50$	from 19.2% to 12.8% at $n = 50$	Upward	10	1.5	3.56%
Hasler and Marfè (2016)	Cash-flow; disaster+revcovery	from 7.7% to 6.1% at $n = 20$	from 17.3% to 6.1% at $n = 20$	Upward	5.5	0.8	3.77%
Croce et al. (2015)	Learning	from 6.1% to 4.6% at $n = 25$	from 16.2% to 15.5% at $n = 20$	NA	10	1.5	1.19%

Table 1: Previous models of the term structure of zero-coupon equities
Selected studies are presented which report the entire term structures of risk premiums and return volatilities of dividend strips. RRA and EIS stand for relative risk aversion and elasticity of intertemporal substitution, respectively. Subjective discount rate is continuously compounded and annualized.

	Δd	$\Delta\pi$	x_d	x_π	x_f	x_λ
Unconditional means						
$\mu_{\cdot}$	1.29%	3.68%	–	–	0.96%	17.0
Standard deviations of innovation terms						
$\sqrt{s_{\cdot\cdot}}$	10.0%	1.18%	0.32%	0.35%	0.19%	4.00
Autocorrelations						
diag(Φ_x)	–	–	0.90	0.78	0.92	0.85
Correlations between innovation terms						
Δd	1.00	−0.30	−0.83	−0.30	−0.30	0.00
$\Delta\pi$		1.00	0.00	1.00	0.00	0.00
x_d			1.00	0.00	0.00	0.35
x_π				1.00	0.00	0.00
x_f					1.00	0.00
Unconditional factor risk premiums						
	17.00%	−0.60%	−0.45%	−0.18%	−0.10%	0.00

Table 2: Parameter values of the LW model

These values are collected from tables 1–3 in LW (2011). Unconditional means, standard deviations, and autocorrelations are annualized, except for the unconditional mean of $x_{\lambda,t}$ and the standard deviation of innovation in $x_{\lambda,t}$ expressed in raw numbers. The last row presents annualized, unconditional factor risk premiums.

Solution	Consumption $\sqrt{s_{cc}}$ (%, year)	Risk aversion $E[\gamma_t]$	$SD[\gamma_t]$	Subjective discount factor $E[\beta_t]$	$SD[\beta_t]$ ($\times 10^2$)
(a)	8.84	30	25	0.969	1.097
(b)	5.89	60	51	0.981	0.532
(c)	4.77	90	77	0.985	0.392
(d)	4.22	120	102	0.986	0.331
(e)	3.91	150	128	0.987	0.298

Table 3: Moments for consumption and preferences at selected solutions
Table 3 presents the annualized volatility of innovation in consumption growth ($\sqrt{s_{cc}}$), and the unconditional mean ($E[\cdot]$) and standard deviation ($SD[\cdot]$) of state-dependent preferences at selected solutions to the set of constraint equations given in Section 3.2. The solutions are in ascending order of $E[\gamma_t]$.

Panel A: Consumption volatility and correlations						
	$\sqrt{s_{cc}}$	ρ_{cd}	ρ_{cx1}	ρ_{cx2}	ρ_{cx3}	ρ_{cx4}
	3.91%	0.877	−0.755	−0.086	−0.046	0.341

Panel B: State-dependent preferences and continuation value					
	constant	x_d	x_π	x_f	x_λ
γ_t	150	0.00	0.00	0.000	8.9357
$\ln(1-\beta_t)$	−4.392	−8.66	0.00	30.170	0.0136
$\ln \nu_t$	0.071	8.86	0.00	−0.817	−0.0029

Panel C: Unconditional factor risk premiums						
	Δd	$\Delta \pi$	x_d	x_π	x_f	x_λ
	17.00%	−0.60%	−0.46%	−0.18%	−0.11%	−0.172

Table 4: Implied parameters of consumption dynamics and preferences
Table 4 presents the calibrated parameters at Solution (e) of Table 3. Panel A presents the annualized volatility of innovation in consumption growth ($\sqrt{s_{cc}}$) and the correlations between innovations in consumption growth and the rest of the variables. The correlation with *realized* inflation growth is not shown because it is the same as the correlation with *expected* inflation growth (ρ_{cx2}). Panel B presents the parameters in risk aversion γ_t, log subjective discount rate $\ln(1-\beta_t)$, and log continuation value $\ln \nu_t$. All of these functions are linear in $x_t' = (x_{d,t},\ x_{\pi,t},\ x_{f,t}-\mu_f,\ x_{\lambda,t}-\mu_\lambda)$. Panel C presents annualized, unconditional factor risk premiums, except for x_λ expressed in row numbers.

Panel A: Consumption volatility and correlations

$\sqrt{s_{cc}}$	ρ_{cd}	ρ_{cx1}	ρ_{cx2}	ρ_{cx3}	ρ_{cx4}
4.96%	0.898	−0.792	−0.069	0.085	0.281

Panel B: State-dependent preferences and continuation value

	constant	x_d	x_π	x_f	x_λ
γ_t	150	0.00	0.00	0.000	8.7817
$\ln(1-\beta_t)$	−6.337	−12.00	0.00	44.639	0.0025
$\ln \nu_t$	0.458	12.36	0.00	−1.619	−0.0035

Panel C: Unconditional factor risk premiums

Δd	$\Delta\pi$	x_d	x_π	x_f	x_λ
17.00%	−0.60%	−0.45%	−0.18%	0.02%	0.015

Table 5: Implied parameters for $\rho_{dx3} = 0.1$

ρ_{dx3} stands for the correlation between innovations in dividend growth and risk-free-rate factor. It is first changed from −0.3 to 0.1 in the LW model, and then the parameters of the proposed model are re-calibrated in the same procedure as explained in Section 3.2. The same legend as in Table 4 follows.

	Consumption			Dividend			Risk aversion		Subjectiv dis. factor	
	Total vol. (%, year)	$\sqrt{s_{cc}}$ (%, year)	Jump size (%, quarter)	Total vol. (%, year)	$\sqrt{s_{dd}}$ (%, year)	Jump size (%, quarter)	$E[\gamma_t]$	$SD[\gamma_t]$	$E[\beta_t]$	$SD[\beta_t]$ ($\times 10^2$)
No JUMP	5.05	5.05	-	18.00	18.00	-	30	8.6	0.996	0.085
JUMP	4.00	3.98	-1.26	18.00	16.52	-20.23	30	8.1	0.986	0.232

Table 6: Moments for cash flows and preferences with and without jumps when the mean and volatility of $x_{\lambda,t}$ are reduced
The values of the following three parameters originally calibrated by LW (2011) are first changed (the previous values are in parenthesis): $\sqrt{\text{var}_t[\Delta d_{t+1}]} = 18\%$ (10%) per year; $\sqrt{\text{var}_t[x_{\lambda,t+1}]} = 0.2$ (4); $E[x_{\lambda,t+1}] = 0.085/0.18^2$ ($0.17/0.1^2$). Then, the parameters of the proposed model without jumps are calibrated by the same procedure as explained in Section 3.2, and the results are presented in the "No JUMP" row. The "JUMP" row presents the results for the proposed model augmented with jumps in consumption and dividend processes. The parameters are calibrated under the following conditions: The term structures of interest rates and dividend risk premiums are similar to those without jumps; the total volatility (Total vol.), which is decomposed into the Gaussian ($\sqrt{s_{cc}}$) and jump parts, is set at 4% per year for consumption growth and 18% per year for dividend growth. By construction, the total volatility and the Gaussian component are the same for No JUMP. The jump size represents the degree to which time-t consumption or dividend falls from the previous quarter when a single jump occurs at time t, the frequency of which is set at once in every ten years on average.

	-3S.D.	-2S.D.	-1S.D.	$+1$S.D.	$+2$S.D	$+3$S.D.
Panel A: Linear γ_t						
All factors	-0.060	-0.030	-0.008	-0.011	-0.048	-0.124
Individual factors						
x_d	0.046	0.020	0.005	0.004	0.017	0.038
x_π	0.000	0.000	0.000	0.000	0.000	0.000
x_f	-0.002	-0.001	0.000	0.000	-0.001	-0.002
x_λ	0.056	0.028	0.008	0.009	0.042	0.105
Panel B: Quadratic γ_t						
All factors	-0.812	-0.364	-0.092	-0.094	-0.378	-0.858
Individual factors						
x_d	0.046	0.020	0.005	0.004	0.017	0.038
x_π	0.000	0.000	0.000	0.000	0.000	0.000
x_f	-0.002	-0.001	0.000	0.000	-0.001	-0.002
x_λ	-0.695	-0.307	-0.076	-0.074	-0.289	-0.635
Panel C: Linear γ_t with jumps in consumption and dividend growth						
All factors	-0.099	-0.047	-0.013	-0.015	-0.064	-0.155
Individual factors						
x_d	0.047	0.020	0.005	0.005	0.018	0.039
x_π	0.000	0.000	0.000	0.000	0.000	0.000
x_f	-0.004	-0.002	0.000	0.000	-0.001	-0.003
x_λ	0.000	0.000	0.000	0.000	0.000	0.001

Table A1: Approximation errors of ν_t^{AP}
The value function is approximated as $\nu_t^{AP} = \exp\{\mu_\nu + b_\nu' x_t\}$. The table reports $e_{2,t}/\nu_t^{AP}$ in percentage terms, where $e_{2,t}$ stands for a pseudo approximation error defined as $e_{2,t} = E_t[(\nu_{t+1}^{AP} e^{\Delta c_{t+1}})^{1-\gamma_t}]^{\beta/(1-\gamma_t)} - \nu_t^{AP}$. The values of $e_{2,t}$ are computed at the values of the factors above or below k $(= 1, 2, 3)$ standard deviations (S.D.) from the mean (i.e., zero). In each panel, the label "All factors" indicates that all factors change proportionally, whereas the label "Individual factors" indicates that only a factor in each row changes with the other factors fixed at the mean. By construction of the approximation, $e_{2,t} = 0$ at $x_t = 0$ (mean). Panels A and B are for a linear risk-aversion and a quadratic risk-aversion, respectively, evaluated at the parameter values given in Tables 2 and 4. Panel C is for a linear risk-aversion with jumps in consumption and dividend processes, evaluated at the parameter values given (partially) in Table 6.

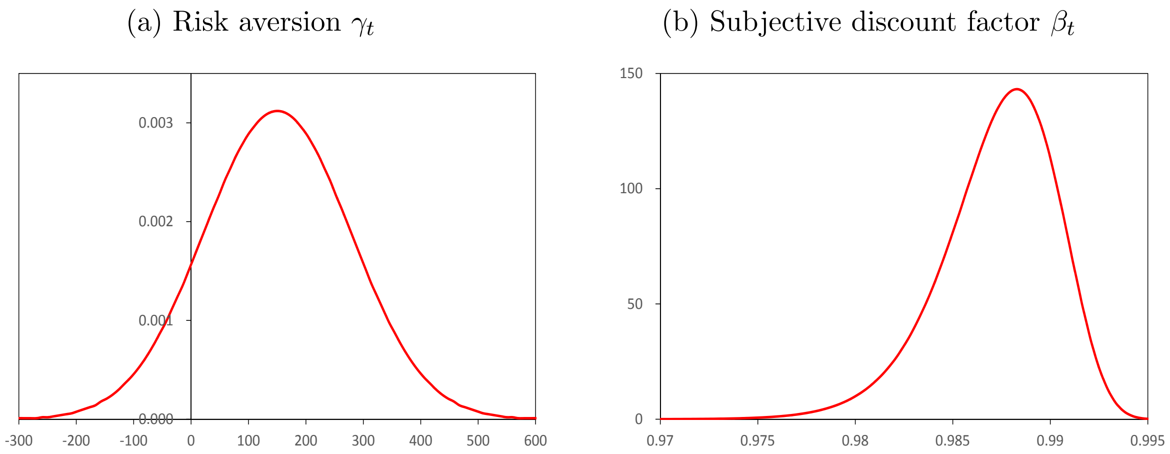


Figure 1: Unconditional distributions of state-dependent preferences
The distributions are drawn at Solution (e) of Table 3. The risk-aversion coefficient is specified as $\gamma_t = \mu_\gamma + b'_\gamma x_t$ and the subjective discount factor as $\beta_t = 1 - \exp\{\mu_\beta + b'_\beta x_t\}$, where x_t is a Gaussian state vector.

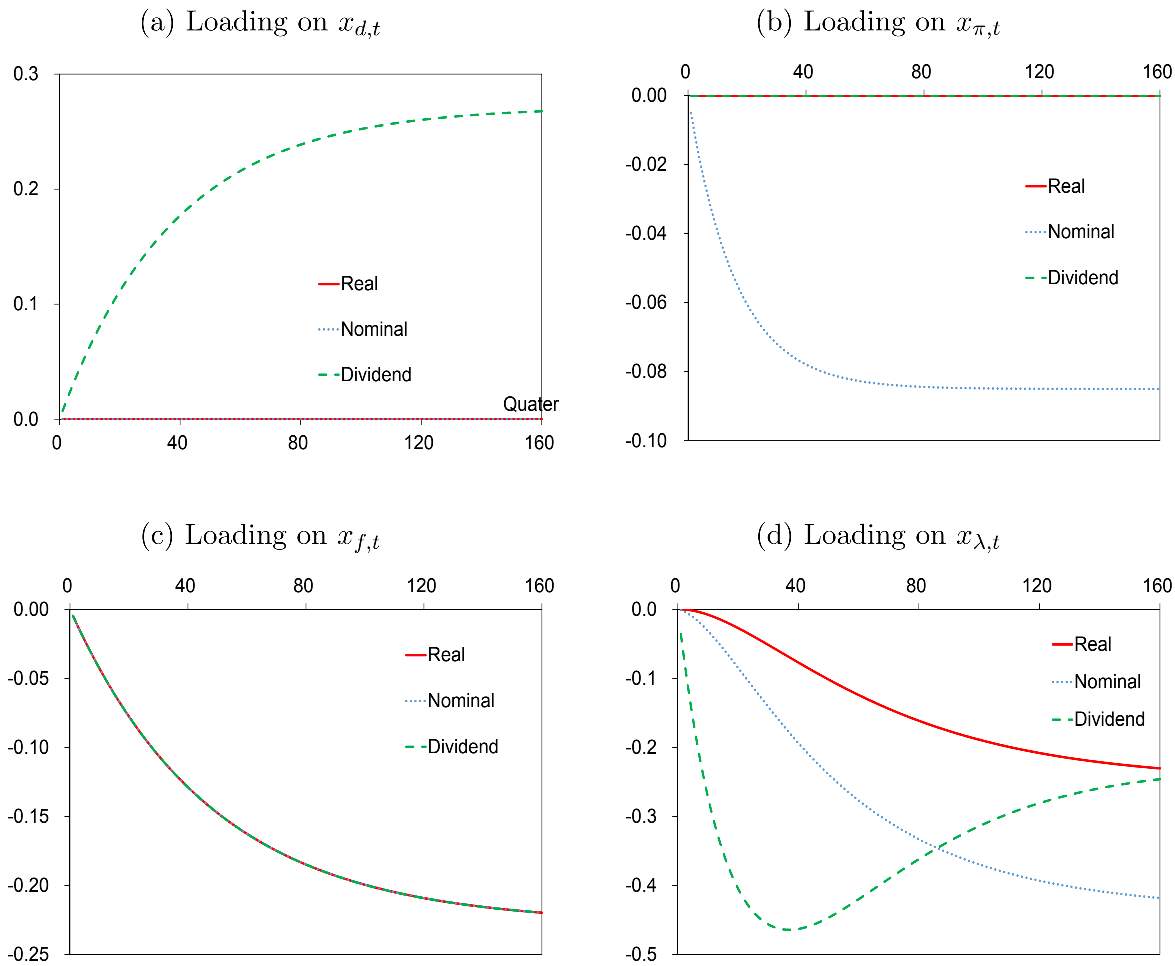


Figure 2: Log price loadings on state variables B_n^i $(i = \{R, N, D\})$

The log prices for real bonds (R), nominal bonds (N), and dividend strips (D) are given by $\ln P_{t,n}^i = A_n^i + B_n^{i\prime} x_t$ $(i = \{R, N, D\})$. Panel (a) plots against n (quarters) the loadings on $x_{d,t}$ (expected dividend growth), multiplied by the unconditional volatility of $x_{d,t}$ hence normalized in that they are the loadings on the factor with unit volatility. Panels (b)–(d) plot the analogous loadings on $x_{\pi,t}$ (expected inflation growth), $x_{f,t}$ (risk-free rate), and $x_{\lambda,t}$ (price of risks), respectively.

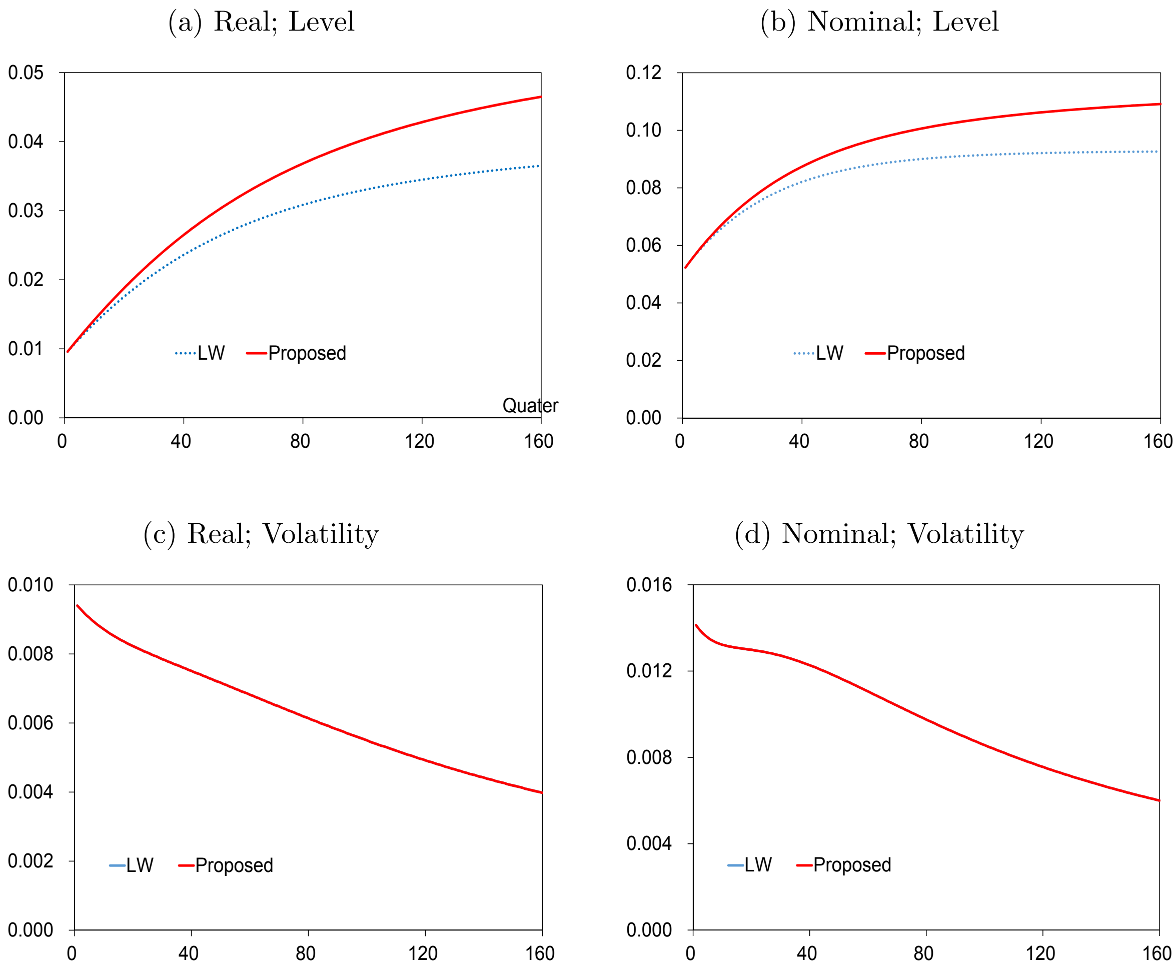


Figure 3: Average term structures of interest rates and volatilities
In Panels (a) and (b), the term structures of $4E[Y_{t,n}^i]$ ($i = \{R, N\}$) (annualized) are plotted against n (quarters), where $Y_{t,n}^i$ is the yield to maturity of a zero-coupon bond at time t. In Panels (c) and (d), the term structures of $\sqrt{4\text{var}[Y_{t,n}^i]}$ ($i = \{R, N\}$) (annualized) are plotted. The solid (dotted) line is for the proposed (LW) model.

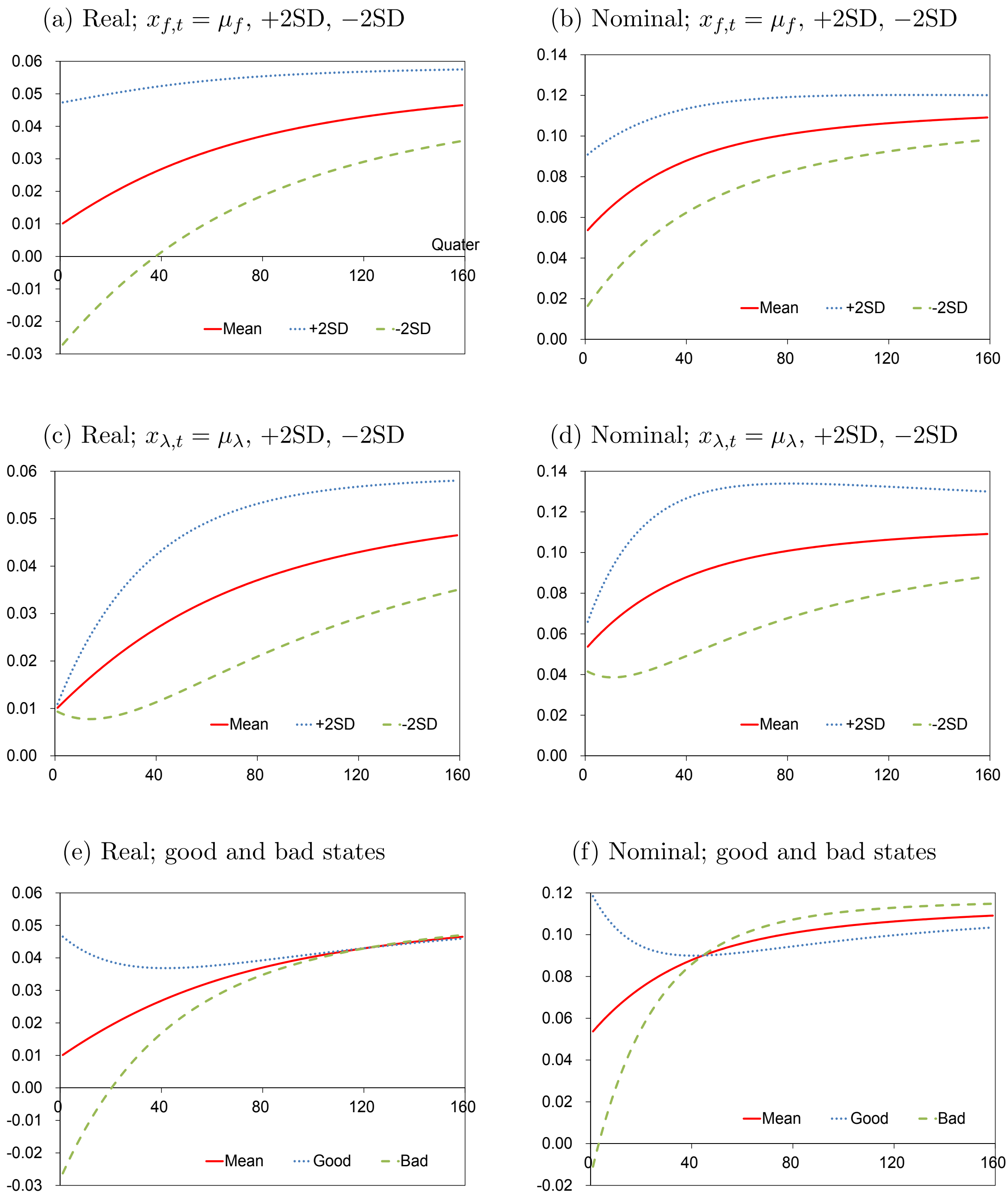
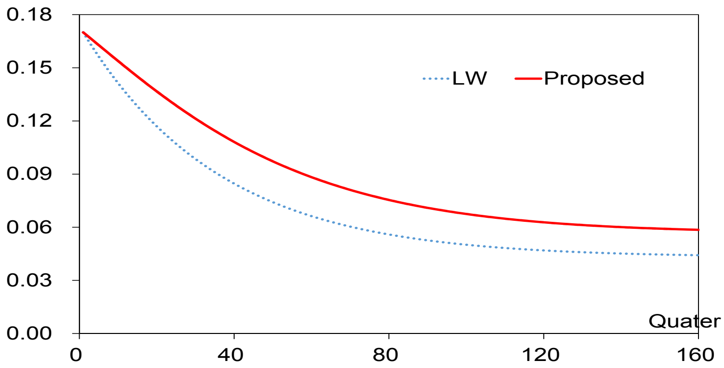
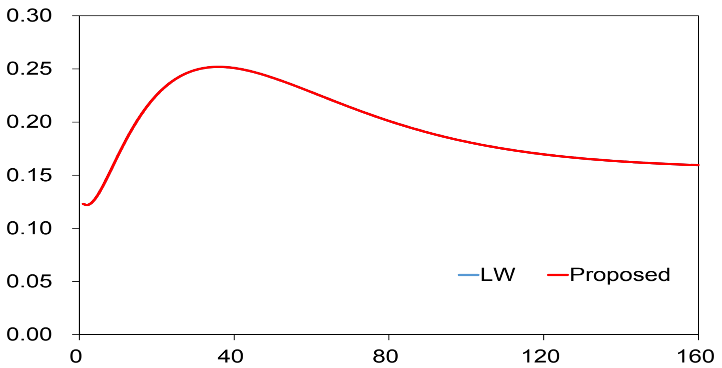


Figure 4: Conditional term structures of interest rates for the proposed model
Panels (a) and (b) plot the term structures when the risk-free-rate factor $x_{f,t}$ is above (+2SD) or below (−2SD) two standard deviations from the mean with the other factors fixed at the mean. Panels (c) and (d) plot the term structures when the price-of-risk factor $x_{\lambda,t}$ is above (+2SD) or below (−2SD) two standard deviations from the mean with the other factors fixed at the mean. Panels (e) and (f) plot the term structures when the economy is "Good" or "Bad" with a good (bad) state defined as when $x_{\lambda,t}$ is below (above) two standard deviations from the mean and the other factors are above (below) two standard deviations from the mean.

(a) Risk premiums



(b) Volatilities



(c) Sharpe ratios

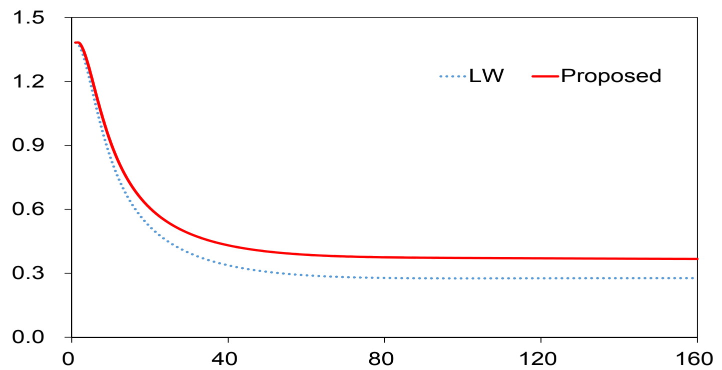


Figure 5: Average term structures of risk premiums, volatilities, and Sharpe ratios for dividend strips (annualized)

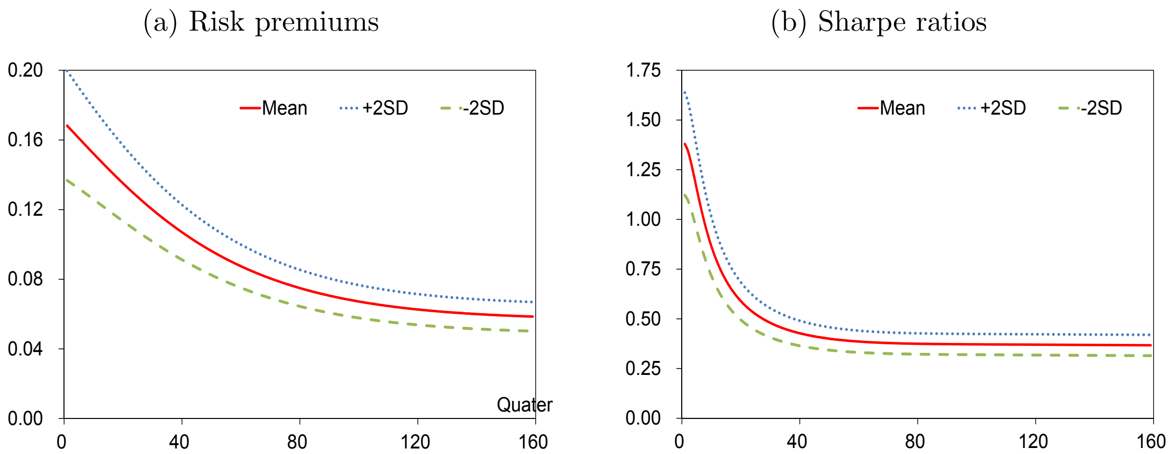


Figure 6: Conditional term structures of dividend risk premiums and Sharpe ratios for the proposed model (annualized)

The term structures are plotted when the price-of-risk factor $x_{\lambda,t}$ is above (+2SD) or below (−2SD) two standard deviations from the mean.

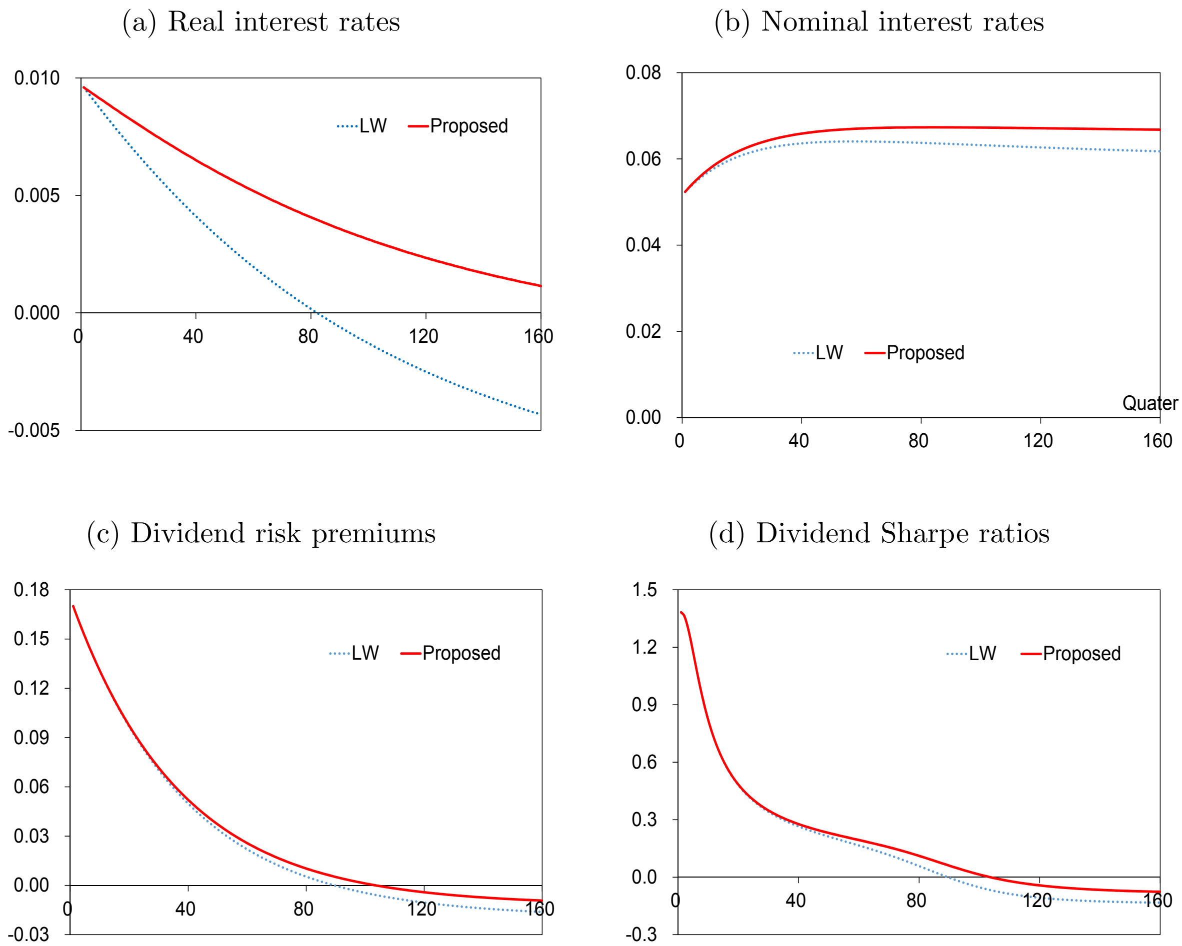


Figure 7: Average term structures of zero-coupon bonds and equities for $\rho_{dx3} = 0.1$
ρ_{dx3} stands for the correlation between innovations in dividend growth and risk-free-rate factor. It is first changed from -0.3 to 0.1 in the LW model, and then the parameters of the proposed model are re-calibrated in the same procedure as explained in Section 3.2.

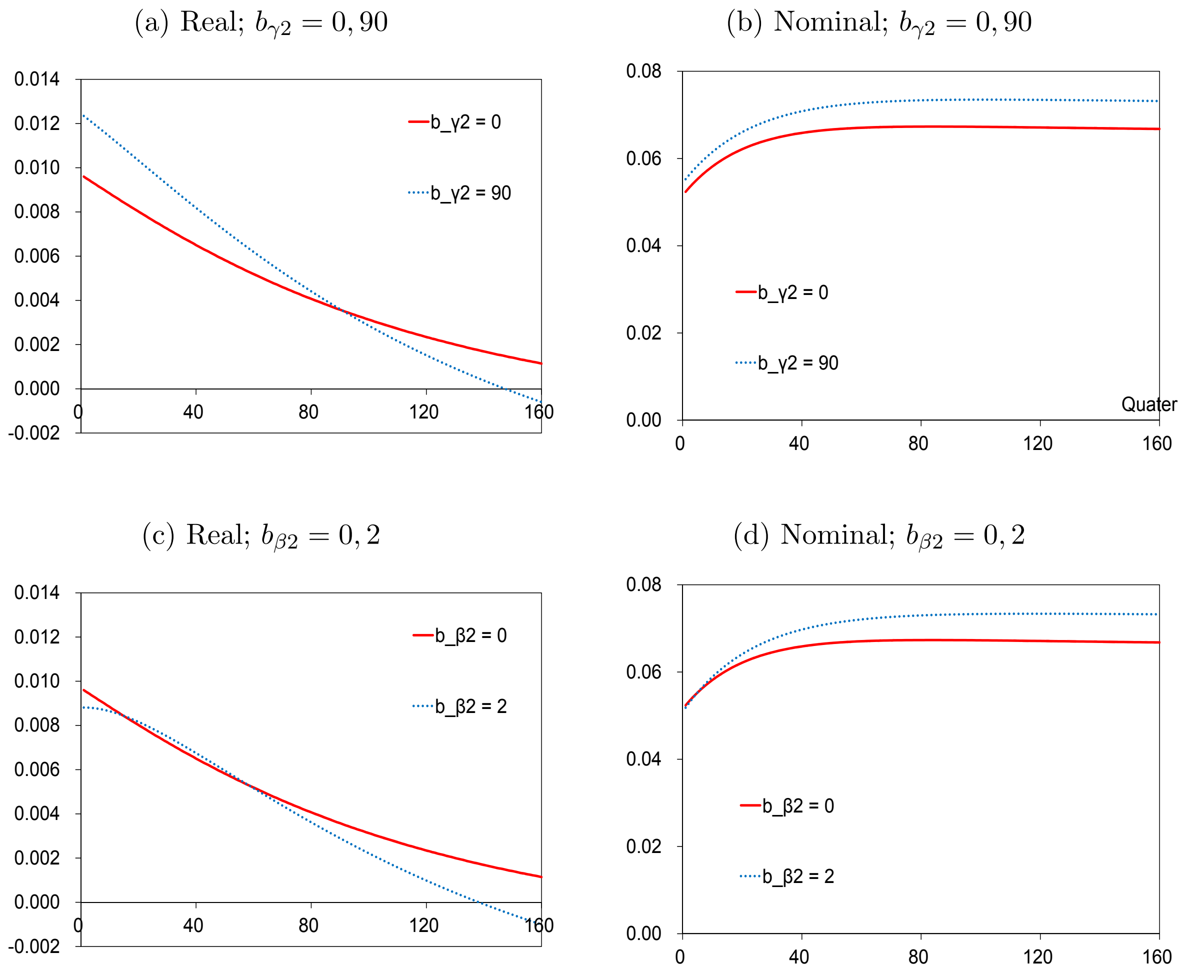


Figure 8: Average term structures of interest rates for $b_{\gamma 2} = 90$ or $b_{\beta 2} = 2$
$b_{\gamma 2}$ is the coefficient of the expected inflation-growth factor $x_{\pi,t}$ in the risk aversion γ_t and $b_{\beta 2}$ is the analogous coefficient in the log subjective discount rate $\ln(1 - \beta_t)$. These coefficients, originally set at zero, are changed as indicated above while the other parameters are held fixed at the values presented in Table 5. In each panel, the plot labeled as $b_{\gamma 2} = 0$ or $b_{\beta 2} = 0$ is the same as shown in Figure 7(a) (real) or Figure 7(b) (nominal).

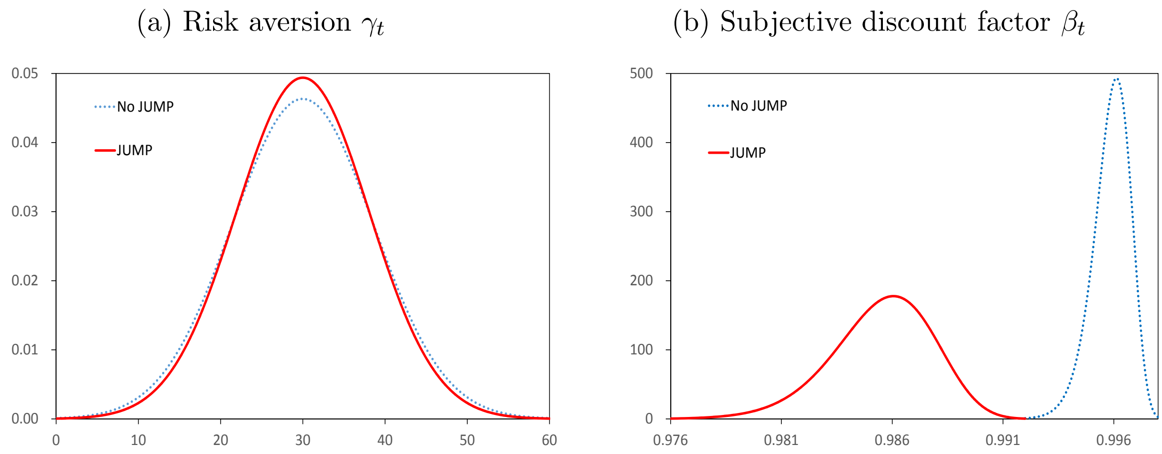


Figure 9: Unconditional distributions of state-dependent preferences with and without jumps when the mean and volatility of $x_{\lambda,t}$ are reduced

The distributions are drawn at the parameter values given (partially) in Table 6. The risk-aversion coefficient is specified as $\gamma_t = \mu_\gamma + b'_\gamma x_t$ and the subjective discount factor as $\beta_t = 1 - \exp\{\mu_\beta + b'_\beta x_t\}$, where x_t is a Gaussian state vector.

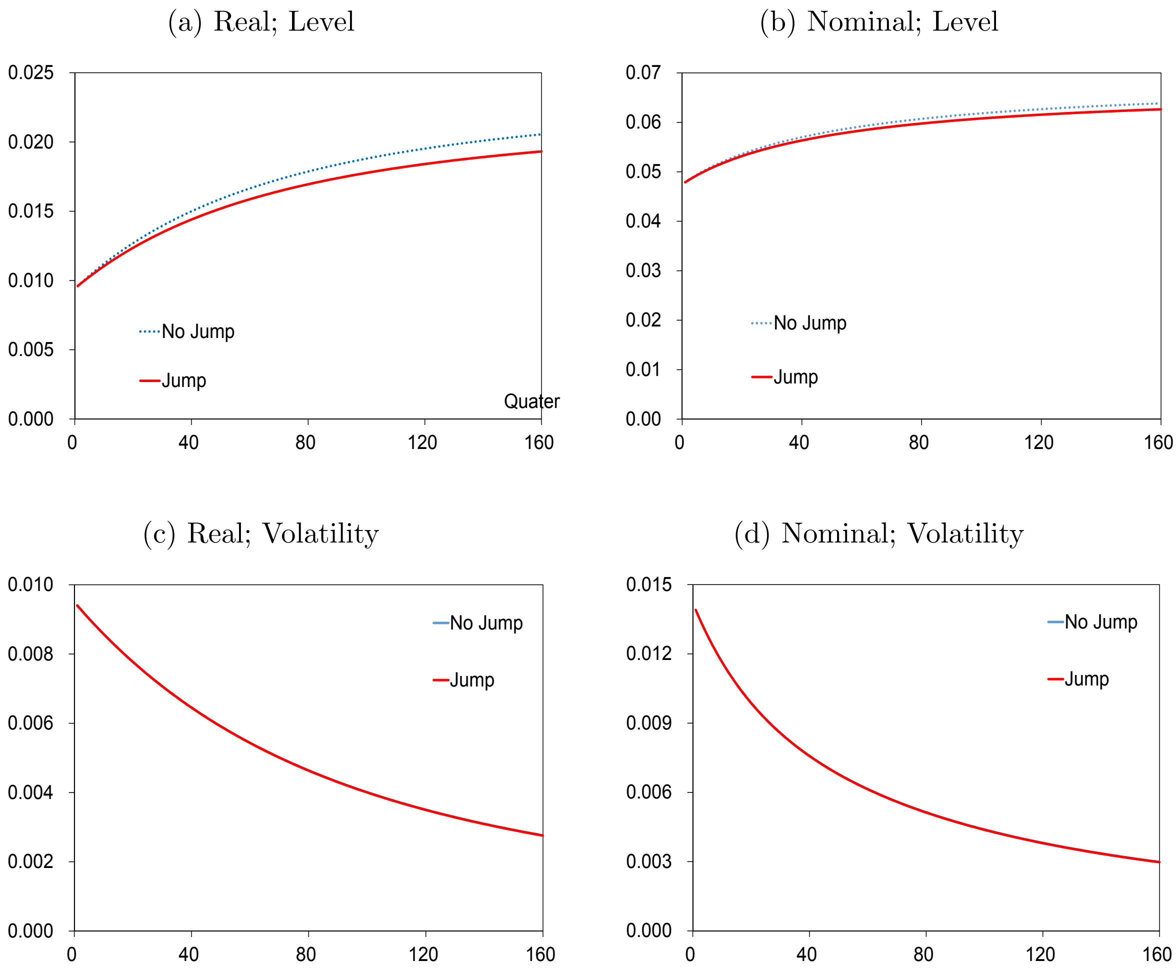


Figure 10: Average term structures of interest rates and volatilities with and without jumps when the mean and volatility of $x_{\lambda,t}$ are reduced

In panels (a) and (b), the term structures of $4E[Y^i_{t,n}]$ $(i = \{R, N\})$ (annualized) are plotted against maturity n (quarters), where $Y^i_{t,n}$ is the yield to maturity of a zero-coupon bond at time t. In panels (c) and (d), the term structures of $\sqrt{4\mathrm{var}[Y^i_{t,n}]}$ $(i = \{R, N\})$ (annualized) are plotted. They are drawn at the parameter values given (partially) in Table 6.

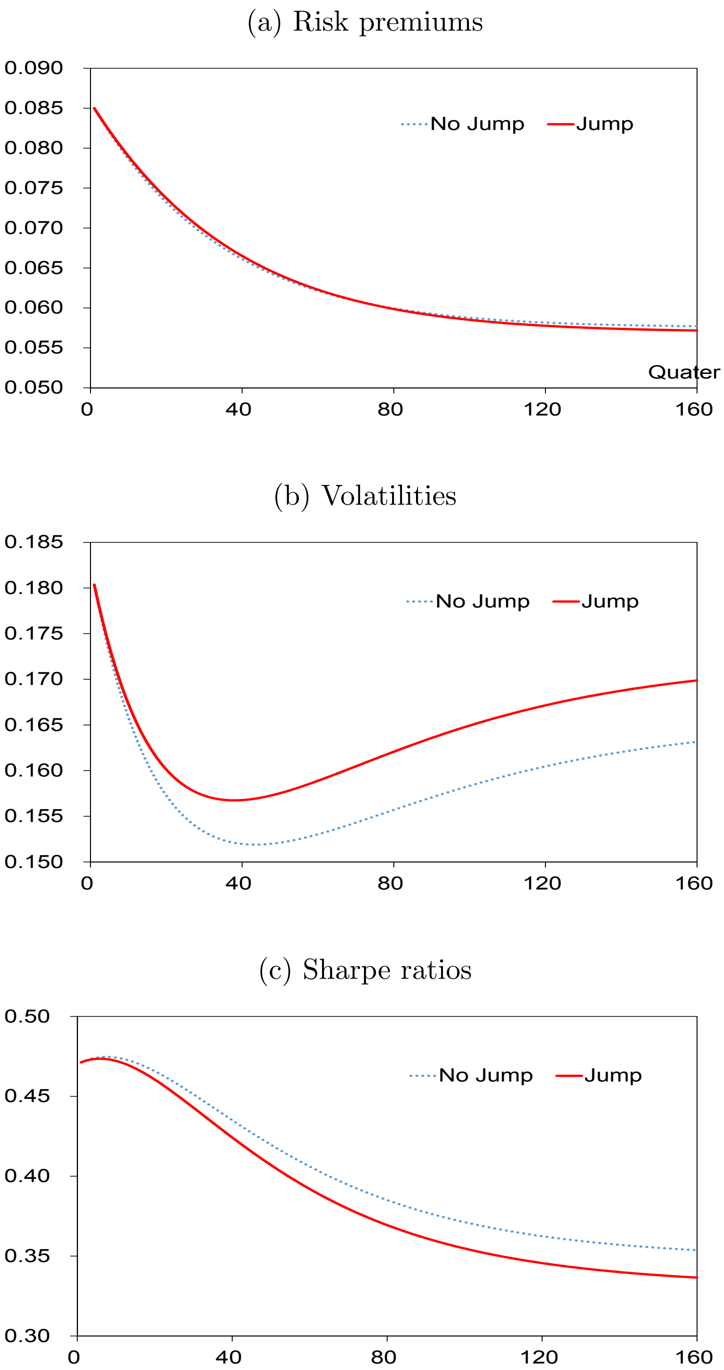


Figure 11: Average term structures of risk premiums, volatilities, and Sharpe ratios for dividend strips (annualized) with and without jumps when the mean and volatility of $x_{\lambda,t}$ are reduced

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