Discussion Paper No.326

Effective Ambient Charges on Non-point Source Pollution in a Two-stage Bertrand Duopoly

Akio Matsumoto
Chuo University

Ferenc Szidarovszky
Corvinus University

December 2019

INSTITUTE OF ECONOMIC RESEARCH
Chuo University
Tokyo, Japan

Effective Ambient Charges on Non-point Source Pollution in a Two-stage Bertrand Duopoly*

Akio Matsumoto† Ferenc Szidarovszky‡

Abstract

This paper analyzes the effectiveness of the ambient charges for controlling emissions of non-point source pollutions. To this end, we construct a two-stage Bertrand duopoly game, in which optimal abatement technologies are chosen first and then the optimal prices as well as the optimal productions are determined. It is shown that the ambient charge is always effective at the second stage. Since the effect could be ambiguous at the first stage, this paper sheds light on the conditions under which the ambient charge becomes effective.

Keywords: Ambient charge, Bertrand duopoly competition, Non-point source pollution, Two stage game.

---

*The authors would like to thank participants at a seminar organized by the Institute of Economic Research of Chuo University and also thank Takaaki Ishida, Yusuke Kumahara and Hinata Shiratori for helpful discussions. The first author highly acknowledges the financial supports from the Japan Society for the Promotion of Science (Grant-in-Aid for Scientific Research (C) 18K0163) and Chuo University (Grant for Special Research). The usual disclaimers apply.

†Professor, Department of Economics, Chuo University, 742-1, Higashi-Nakano, Hachioji, Tokyo, 192-0393, Japan. akiom@tamacc.chuo-u.ac.jp

‡Professor, Department of Mathematics, Corvinus University, Budapest, Fövám tér 8, 1093, Hungary. szidarka@gmail.com

1 Introduction

Non-point source (NPS) pollution refers to the form of pollution in which neither the source nor the size of specific emissions can be observed. It includes farm surface runoff, water pollution contaminating river, lake and groundwater, air pollution that may arise health problems, ocean plastic pollution that recently came to focus, etc. It is emerging as a top threat to our living system including ecosystem. The key features of NPS pollution are its multiple sources that prevent from specifying individual contribution and its stochastic nature (e.g., weather effect) that makes accurate measuring difficult. In consequence, the conventional environmental policy implementation such as emission charge in form of Pigouvian tax, marketable emission permit, direct control on emission level etc. are not applicable. In order to control the total emissions of NPS pollution, Segerson (1988) proposes ambient based polices, according to which the regulator first determines an environmental standard level and then imposes uniform tax on the pollutants if the concentration level is above the standard level and pays uniform subsidies if it is below.

Much of the literature is theoretical. Further, a theoretical framework to study how to address NPS pollution is game-theoretic. This is because the payoff of one agent depends on the actions of the other agents. More specifically, it strictly depends on the actions of other agents whether an agent receives penalty or award regardless of its level. Ganguli and Raju (2012) model a Bertrand duopoly and numerically show that an increase of the ambient charge rate could increase the total concentration, which is called a "perverse" effect. Ishikawa, et al. (2019) extend the duopoly to an n-firm framework and theoretically show that the ambient charge is definitely effective in duopoly and triopoly, whereas for $n \geq 4$, the sign of the effect depends on the number of the firms involved and the degree of substitutability among the goods.

This paper is mainly concerned how a Bertrand firm changes its action in response to changes in the rate of the ambient charge and, in particular, reconsiders the ambient charge effect in a Bertrand duopoly in a two-stage theoretic framework in which the optimal abatement technologies are chosen first and then the optimal prices as well as the optimal outputs are selected. If we consider the first stage *short-run* in the sense that the levels of the abatement technologies are fixed and the second stage *long-run* in the sense that the technologies are strategic variables, then our main results can be mentioned as follows:

(i) The ambient charge is effective in controlling the NPS pollution in the short-run.

(ii) The conditions under which the ambient charge could be effective depend on the degree of substitutability, the demand elasticity with respect to the abatement technology and the ambient charge rate.

The rest of this paper is organized as follows. In Section 2, the optimal price strategies of Bertrand duopolistic firms are determined. It is shown that the total concentration of NPS pollution is negatively related to the rate of the ambient charge. In Section 3, the long-run optimal abatement technologies are determined. The issue is examined under the heterogenous firms in its first hald and under the homogenous firms in the second half. The parametric conditions to confirm the effectiveness of the ambient charge are demonstrated. In Section 4, concluding remarks and further research directions are presented.

2 Bertrand Equilibrium

We consider effects of an environmental policy on NPS pollution in a Bertrand duopoly market. Each firm produces a differentiated good. Demand functions for these goods are given by

$$\begin{aligned} q_i &= a - p_i + bp_j \\ q_j &= a + bp_i - p_j \end{aligned} \tag{1}$$

in the region of price space where quantities are non-negative. Here q_k denotes the quantity of good k produced by firm k and p_k is the unit price of this product for $k = i, j$. The goods are assumed to be substitutes (i.e., $b > 0$). Further, considering the duality of prices and quantities in a differentiated duopoly, we suppose on the following condition for the production differentiation parameter, b.

Assumption 1. $0 < b < 1$.

Justification for Assumption 1 is given as follows. From (1), the inverse demand or price functions are obtained as

$$\begin{aligned} p_i &= \alpha - \beta q_i - \gamma q_j \\ p_j &= \alpha - \gamma q_i - \beta q_j \end{aligned} \tag{2}$$

where

$$\alpha = \frac{a}{1-b}, \ \beta = \frac{1}{1-b^2} \text{ and } \gamma = \frac{b}{1-b^2}.$$

Notice that α denotes the maximum price of both goods and thus is positive, implying that $1 > b$ should hold.[1] The goods are substitutes, independent or

[1] There is another way to show this condition. Following Singh and Vives (1984), we can derive the exact forms of the linear functions given in (2) as the optimal solutions that solve a net utility maximizing problem of the representative consumer,

$$U(q_i, q_j) - (p_i q_i + p_j q_j)$$

where U is the utitlity function,

$$U(q_i, q_j) = \alpha (q_i + q_j) - \frac{1}{2} \left(\beta q_i^2 + 2\gamma q_i q_j + \beta q_j^2\right)$$

with the parameter conditions, $\beta^2 - \gamma^2 > 0$ and $\beta - \gamma > 0$, both of which correspond to $1 - b^2 > 0$ and $1 - b > 0$ in our framework.

complements according to whether $\gamma \gtreqless 0$ or

$$\frac{b}{1-b^2} \gtreqless 0. \tag{3}$$

Hence the goods are substitutes if $\gamma > 0$ or $b > 0$. The differentiation parameter therefore should satisfy the following constraint:

$$0 < b < 1.$$

If $b = 1$, then inverse demand functions do not exit since α, β and γ are not defined, and from (1)

$$p_i - p_j = a - q_i = q_j - a,$$

implying that p_i and p_j cannot be uniquely determined. The parameter ratio $\gamma^2/\beta^2 = b^2$ expresses the degree of product differentiation ranging from zero when the goods are independent and to unity when the goods are perfect substitutes. The above strict inequalities imposed on b eliminate these extreme cases.

Each firm produces output as well as emits pollutions and it is assumed that one unit of production emits one unit of pollution. However using an abatement technology denoted by ϕ_k, firm k can reduce the actual amount of pollution to $\phi_k q_k$ by abating $(1-\phi_k)q_k$. The technology is subject to $0 \leq \phi_k \leq 1$ with a pollution-free technology if $\phi_k = 0$ (i.e., no pollution) and a fully-discharged technology if $\phi_k = 1$ (i.e., no abatement). Since the pollutions are non-point source, the government can measure the total quantity of pollution, $\sum_k \phi_k q_k$ but cannot identity individual contributions to it. To control such NPS pollution, the government carries out the environmental policy that imposes uniform ambient charges θ on the total quantity and has an exogenously determined environmental standard $\bar{E}$. The government will, according to θ times the difference between $\sum_k \phi_k q_k$ and $\bar{E}$, charge the penalty if the difference is positive and award the subsidy if negative. θ is positive and measured in some monetary unit per emission. It is then not necessarily less than unity.

The profit functions of firms i and j are

$$\begin{aligned} \pi_i(p_i, p_j) &= q_i p_i - \eta_i q_i - \theta\left(\phi_i q_i + \phi_j q_j - \bar{E}\right), \\ \pi_j(p_i, p_j) &= q_j p_j - \eta_j q_j - \theta\left(\phi_i q_i + \phi_j q_j - \bar{E}\right), \end{aligned} \tag{4}$$

where η_k is the marginal production cost for firm $k = i, j$. Substituting the demand functions (1) into (4) and then solving the resultant first-order conditions of profit maximization for firms i and j present the optimal prices,

$$\begin{aligned} p_i^*(\phi_i, \phi_j) &= \frac{1}{4-b^2}\left\{a(2+b) + 2\eta_i + b\eta_j + \theta\left[(2-b^2)\phi_i - b\phi_j\right]\right\} \\ p_j^*(\phi_i, \phi_j) &= \frac{1}{4-b^2}\left\{a(2+b) + 2\eta_j + b\eta_i + \theta\left[(2-b^2)\phi_j - b\phi_i\right]\right\}. \end{aligned} \tag{5}$$

Denoting the marginal cost associated with producing an additional unit of output by $c_k = \eta_k + \theta\phi_k$, we assume the following to ensure positive equilibrium quantities and prices,[2]

Assumption 2. $a > c_k$ for $k = i, j$.

Substituting the optimal prices into the demand functions presents the optimal productions,

$$\begin{aligned} q_i^*(\phi_i, \phi_j) &= a - p_i^*(\phi_i, \phi_j) + bp_j^*(\phi_i, \phi_j) \\ &= \frac{1}{4-b^2}\left[a(2+b) - (2-b^2)\eta_i + b\eta_j - 2\theta\phi_i + b(3-b^2)\theta\phi_j\right] > 0 \end{aligned} \tag{6}$$

where the last inequality is due to $2a - 2\eta_i - 2\theta\phi_i = 2(a - c_i) > 0$. The positive optimal product of firm j is also shown in the same way (i.e., $q_j^*(\phi_i, \phi_j) > 0$).

The total pollution at the equilibrium is

$$E^*(\phi_i, \phi_j) = \phi_i q_i^*(\phi_i, \phi_j) + \phi_j q_j^*(\phi_i, \phi_j). \tag{7}$$

Differentiationg $E^*(\phi_i, \phi_j)$ with respect to θ yields, after arranging the terms,

$$\frac{\partial E^*(\phi_i, \phi_j)}{\partial \theta} = -\frac{2\left[\phi_i^2 - b(3-b^2)\phi_i\phi_j + \phi_j^2\right]}{4-b^2} \tag{8}$$

where for $\phi_i \neq \phi_j$,

$$\begin{aligned} \phi_i^2 - b(3-b^2)\phi_i\phi_j + \phi_j^2 &\geq \phi_i^2 - 2\phi_i\phi_j + \phi_j^2 \\ &= \left(\phi_i - \phi_j\right)^2 > 0 \end{aligned}$$

and for $\phi_i = \phi_j = \phi$,

$$\phi_i^2 - b(3-b^2)\phi_i\phi_j + \phi_j^2 = \left[2 - b(3-b^2)\right]\phi^2 \geq 0$$

and the last equality holds only when $b = 1$, which is eliminated by Assumption 1. Therefore we have the following result.

Theorem 1 *Under Assumptions 1 and 2, the ambient charge is effective in controlling the total amount of NPS pollution,*

$$\frac{\partial E^*(\phi_i, \phi_j)}{\partial \theta} < 0.$$

[2]In particular, under Assumption 2,

$$\begin{aligned} ab + b\eta_j - \theta b\phi_j &= b\left(a + \eta_j - \theta\phi_j\right) \\ &> b(\eta_j + \theta\phi_j + \eta_j - \theta\phi_j) \\ &= 2b\eta_j > 0. \end{aligned}$$

With this inequality we have $p_i^* > 0$. In the same way, $p_j^* > 0$ can be shown.

We verify the individual responses to a change in θ. Differentiating the optimal production of each firm gives

$$\frac{\partial q_i^*(\phi_i,\phi_j)}{\partial\theta} = \frac{b(3-b^2)\phi_j - 2\phi_i}{4-b^2},$$

$$\frac{\partial q_j^*(\phi_i,\phi_j)}{\partial\theta} = \frac{b(3-b^2)\phi_i - 2\phi_j}{4-b^2},$$

from both of which we can derive the zero-response curves of firms i and j,

$$\phi_j = \frac{2}{b(3-b^2)}\phi_i \Leftrightarrow \frac{\partial q_i^*(\phi_i,\phi_j)}{\partial\theta} = 0$$

and

$$\phi_j = \frac{b(3-b^2)}{2}\phi_i \Leftrightarrow \frac{\partial q_j^*(\phi_i,\phi_j)}{\partial\theta} = 0.$$

where under Assumption 1,

$$\frac{b(3-b^2)}{2} < \frac{2}{b(3-b^2)}.$$

Theorem 2 *Although the total pollution is negatively related to a change in the ambient charge rate, the individual response could be "perverse,"*

$$\frac{\partial q_i^*(\phi_i,\phi_j)}{\partial\theta} < 0 \text{ and } \frac{\partial q_j^*(\phi_i,\phi_j)}{\partial\theta} > 0 \quad \text{if } \frac{2}{b(3-b^2)}\phi_i < \phi_j,$$

$$\frac{\partial q_i^*(\phi_i,\phi_j)}{\partial\theta} < 0 \text{ and } \frac{\partial q_j^*(\phi_i,\phi_j)}{\partial\theta} < 0 \quad \text{if } \frac{b(3-b^2)}{2}\phi_i < \phi_j < \frac{2}{b(3-b^2)}\phi_i,$$

$$\frac{\partial q_i^*(\phi_i,\phi_j)}{\partial\theta} > 0 \text{ and } \frac{\partial q_j^*(\phi_i,\phi_j)}{\partial\theta} < 0 \quad \text{if } \phi_j < \frac{b(3-b^2)}{2}\phi_i.$$

If firm i has a more efficient abatement technology than firm j to the extent that $2\phi_i/b(3-b^2) < \phi_j$ holds, then it has an individual perverse reaction. On the other hand, if the technology of firm j is much more efficient satisfying $\phi_j < b(3-b^2)\phi_i/2$, then firm j increases its pollution when the rate of ambient charge increases. Otherwise, the ambient charge is effective in controlling the individual pollution. Notice that the government is unable to observe these individual responses.

3 Optimal Abatement Technology

Given the rate of ambient charge θ and the optimal decisions at the second stage, each firm now determines the optimal abatment technology at the first stage. This section is divided into two subsections. In Section 3.1, a selection of the abatement technology is considered when the firms are heterogeneous in the sense that their production costs are different and then in Section 3.2, the same issue is discussed when the firms are homogeneous.

3.1 Heterogenous Firms

Substituting the optimal prices in (5) and the optimal productions in (6) into the profit functions in (4) and subtracting the implmentation cost of the abatement technology yield the reduced form of the profit functions of the firms,

$$\begin{aligned} \pi_i^*(\phi_i, \phi_j) &= q_i^* p_i^* - \eta_i q_i^* - \theta\left(\phi_i q_i^* + \phi_j q_j^* - \bar{E}\right) - (1-\phi_i)^2 \\ \pi_j^*(\phi_i, \phi_j) &= q_j^* p_j^* - \eta_j q_j^* - \theta\left(\phi_i q_i^* + \phi_j q_j^* - \bar{E}\right) - (1-\phi_j)^2 \end{aligned} \tag{9}$$

where the arguments of the optimal prices and productions are omitted for notational simplicity. Differentiating $\pi_i^*(\phi_i, \phi_j)$ with respect to ϕ_i presents the first-order condition for firm i,

$$\frac{\partial \pi_i^*}{\partial \phi_i} = \frac{\partial \pi_i^*}{\partial p_i}\frac{\partial p_i^*}{\partial \phi_i} + \frac{\partial \pi_i^*}{\partial p_j}\frac{\partial p_j^*}{\partial \phi_i} + \left.\frac{\partial \pi_i^*}{\partial \phi_i}\right|_{p_i^*, p_j^*:const} = 0$$

where

$$\begin{aligned} \frac{\partial \pi_i^*}{\partial p_i} &= a - 2p_i^* + bp_j^* + \eta_i - \theta\left(-\phi_i + b\phi_j\right) = 0, \\ \frac{\partial p_i^*}{\partial \phi_i} &= \frac{\theta\left(2-b^2\right)}{4-b^2}, \\ \frac{\partial \pi_i^*}{\partial p_j} &= bp_i^* - b\eta_i - \theta\left(b\phi_i - \phi_j\right), \\ \frac{\partial p_j^*}{\partial \phi_i} &= -\frac{\theta b}{4-b^2}, \\ \left.\frac{\partial \pi_i^*}{\partial \phi_i}\right|_{p_i^*, p_j^*:const} &= 2(1-\phi_i) - \theta\left(a - p_i^* + bp_j^*\right). \end{aligned}$$

The same condition is obtained for firm j. These first-order conditions are arranged as

$$\begin{aligned} 2(4-b^2+2\theta)(4-b^2-2\theta)\phi_i + b(4+b-b^2)(4-b-b^2)\theta^2\phi_j &= m_i \\ b(4+b-b^2)(4-b-b^2)\theta^2\phi_i + 2(4-b^2+2\theta)(4-b^2-2\theta)\phi_j &= m_j \end{aligned} \tag{10}$$

with

$$\begin{aligned} m_i &= 2(2-b)^2(2+b)^2 - 4\theta\left[a(2+b) - (2-b^2)\eta_i + b\eta_j\right], \\ m_j &= 2(2-b)^2(2+b)^2 - 4\theta\left[a(2+b) - (2-b^2)\eta_j + b\eta_i\right]. \end{aligned}$$

The second-order conditions for firms i and j are

$$\frac{\partial^2 \pi_i^*}{\partial \phi_i^2} = \frac{\partial^2 \pi_j^*}{\partial \phi_j^2} = -2(4 - b^2 + 2\theta)(4 - b^2 - 2\theta) < 0$$

where one sufficient condition is

$$\theta < \frac{4 - b^2}{2}.$$

Solving (10) for the abatement technologies presents

$$\begin{aligned} \phi_i^e &= \frac{2(4 - b^2 + 2\theta)(4 - b^2 - 2\theta)m_i - b(4 + b - b^2)(4 - b - b^2)\theta^2 m_j}{(4 - b^2)\Delta_1 \Delta_2}, \\ \phi_j^e &= \frac{2(4 - b^2 + 2\theta)(4 - b^2 - 2\theta)m_j - b(4 + b - b^2)(4 - b - b^2)\theta^2 m_i}{(4 - b^2)\Delta_1 \Delta_2} \end{aligned} \tag{11}$$

with

$$\Delta_1 = 2(2 - b)^2(2 + b) - (1 - b)\left(4 - 6b - b^2 + b^3\right)\theta^2,$$

$$\Delta_2 = 2(2 - b)(2 + b)^2 - (1 + b)\left(4 + 6b - b^2 - b^3\right)\theta^2.$$

It can be verified that $\Delta_1 < 0$ and/or $\Delta_2 < 0$ could be possible if θ is larger than unity. For the sake of analytical simplicity we make the following assumption by changing the monetary unit or the measure unit of emission, under which the second-order condition for the profit maximizing outputs hold,

Assumption 3. $\theta < 1$.

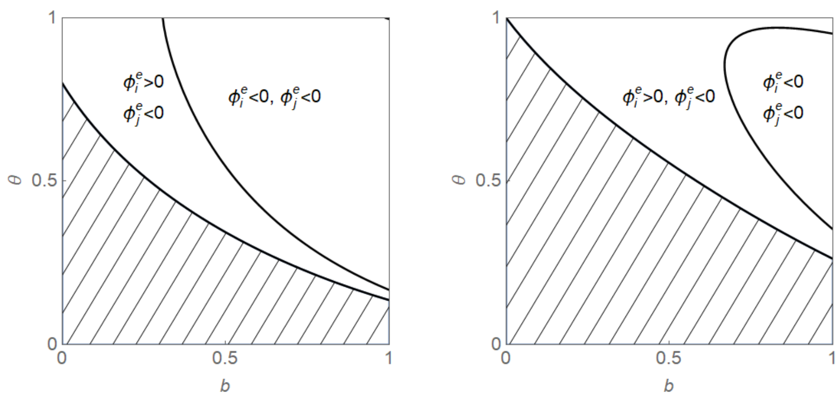
As is seen in (11), the optimal abatement technologies ϕ_i^e and ϕ_j^e depend on the five parameter values of a, b, θ, η_i and η_j. It is not easy to analytically determine dependecy of the optimal technology on these parameters. One exception is that the following holds regardless of the values of other parameters,

$$\phi_i^e \to 1 \text{ and } \phi_j^e \to 1 \text{ as both of } b \text{ and } \theta \to 0.$$

Further it is numerically verfied that given the values of (a, η_i, η_j), there are loci of b and θ such that

$$\phi_i^e = 0 \text{ and } \phi_j^e = 0.$$

Specifying the values of the parameters, we illustrate the two loci of $\phi_i^e = 0$ and $\phi_j^e = 0$ in Figure 1. The downward-sloping curve that is a boundary of the shaded region is the $\phi_j^e = 0$ locus and $\phi_j^e > 0$ below the curve. The other curve located above is the $\phi_i^e = 0$ locus and $\phi_i^e > 0$ below the curve. Hence, $0 < \phi_i^e < 1$ and $0 < \phi_j^e < 1$ are established in the shaded region. In the region between these two curves, $\phi_i^e < 0$ and $\phi_j^e > 0$ whereas $\phi_i^e < 0$ and $\phi_j^e > 0$ in the region above or right to the upper curve. There is a case in which the loci of $\phi_i^e = 0$ and $\phi_j^e = 0$ are located outside the feasible region of (b, θ) for some combinations of the parameter values.



(A) $a = 10, \eta_i = 8, \eta_j = 5$ (B) $a = 5, \eta_i = 3, \eta_j = 1$

Figure 1. Dividion of the (b, θ) plane

Substituting ϕ_i^e and ϕ_j^e into the optimal prices obtained at the second stage (that is, $p_k^*(\phi_i, \phi_j)$ for $k = i, j$ in (5)) presents the optimal prices with the optimal technologies,

$$p_i^e = p_i^*(\phi_i^e, \phi_j^e) \text{ and } p_j^e = p_j^*(\phi_i^e, \phi_j^e)$$

both of which are substituted into the demand functions (1) to obtain the corresponding optimal outputs

$$q_i^e = a - p_i^e + bp_j^e \text{ and } q_j^e = a + bp_i^e - p_j^e.$$

The mutual size relation between these optimal values become prescribed by the magnitude relation of the production costs:

$$\phi_i^e - \phi_j^e = \frac{4(1+b)\theta}{\Delta_2}(\eta_i - \eta_j), \tag{12}$$

$$p_i^e - p_j^e = \frac{2(2-b)(2+b) - (1-b)b(1+b)\theta^2}{\Delta_2}(\eta_i - \eta_j) \tag{13}$$

where the denominator is positive since it is greater than the following under $\theta < 1$,

$$2(2-b)(2+b) - (1-b)b(1+b) = 8 - b - 2b^2 + b^3 > 0$$

and

$$q_i^e - q_j^e = -(1+b)\left(p_i^e - p_j^e\right). \tag{14}$$

Therefore we have the following results concerning the optimal strategy

Theorem 3 *A firm with a higher production cost adopts a worse technology, sets a higher price and produces less output than a firm with a lower production cost.*

Theorems 2 and 3 imply that a firm with a lower production cost might perversely react to a change in the ambient charge while a firm with a higher production cost reacts cooperatively.

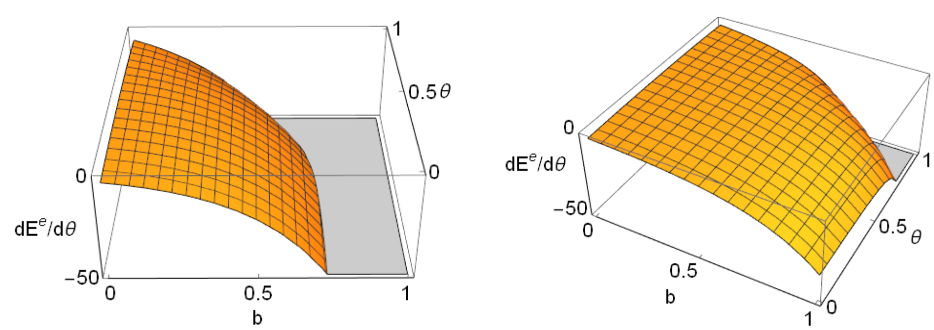
It may be intractable to analytically deal with the effects caused by a change in the rate of the ambient charge on the total concentration of NPS pollution. Instead, using the same parameter specification to illustrate Figures 1(A) and 1(B), we numerically reveal the controllability with the ambient charge. The total concentration is defined as

$$E^e(\theta) = \phi_i^e q_i^e + \phi_j^e q_j^e.$$

The forms of the derivative of $E^e(\theta)$ with respect to θ is long and clumsy, we would not present them but illustrate them in Figure 2. It can be seen that

$$\frac{dE^e(\theta)}{d\theta} < 0 \text{ for } 0 < b < 1 \text{ and } 0 < \theta < 1.$$

Although these are numerical examples and thus there could be a case in which the direction of inequality is reversed, we have shown a possibility that the ambient charge can control the emission of NPS pollution.



(A) $a = 10, \eta_i = 8, \eta_j = 5$ (B) $a = 5, \eta_i = 3, \eta_j = 1$

Figure 2. Numerical confirmation of $\frac{dE^e}{d\eta} < 0$

3.2 Homogeneous Firms

We now turn attention to the homogeneous firms that have the same marginal production cost:

Assumption 4. $\eta_i = \eta_j = \eta$.

Solving (10) for ϕ_i and ϕ_j gives the optimal levels of the abatment technologies,

$$\phi_i^*(\theta) = \phi_j^*(\theta) = \phi^e(\theta) = \frac{A - B\theta}{A - C\theta^2} \tag{15}$$

where A, B and C are defined as

$$\begin{aligned} A &= 2(2-b)^2(2+b) > 0 \\ B &= 4[a - (1-b)\eta] > 0 \\ C &= (1-b)f(b) \end{aligned}$$

with

$$f(b) = 4 - 6b - b^2 + b^3.$$

Lemma 1 *Under Assumption 1, the function $f(b)$ defined on the unit interval $[0,1]$ with $f(0) = 4$ and $f(1) = -2$ is decreasing in b and crosses the horizontal axis at $b_0 \simeq 0.642$ at which $f(b_0) = 0$, implying that*

$$f(b) > 0 \text{ for } 0 < b < b_0 \text{ and } f(b) < 0 \text{ for } b_0 < b < 1.$$

Concerning the value of $\phi^e(\theta)$ for $b_0 < b < 1$, we have the following result.

Theorem 4 *In case of $b_0 < b < 1$, if $\theta \leq A/B$, then*

$$0 \leq \phi^e(\theta) < 1.$$

Proof. The condition $\theta \leq A/B$ and Lemma 1 with $b_0 < b < 1$ lead to $0 \leq A - B\theta$ and $C < 0$. Hence $0 \leq A - B\theta < A < A - C\theta^2$ that is divided by $A - C\theta^2$ to have

$$0 \leq \frac{A - B\theta}{A - C\theta^2} < 1$$

so

$$0 \leq \phi^e(\theta) < 1.$$

This completes the proof. ■

Notice that if $\theta \leq 3/2a$, then the condition $\theta \leq A/B$ is always satisfied over the interval $(0,1)$.

We then draw attention to the value of $\phi^e(\theta)$ for $0 < b < b_0$. In case of $0 < b < b_0,\ C > 0$ due to Lemma 1. The difference between the denominator and the numerator of the last expression in (15) is

$$\left(A - C\theta^2\right) - (A - B\theta) = C\theta\left(\frac{B}{C} - \theta\right). \tag{16}$$

The ratios A/B and B/C depend on b. As is shown shortly, the former is decreasing in b in the unit interval and the latter is increasing in $b \in (0, b_0)$. In particular, differentiating A/B with respect to b gives

$$\frac{d}{db}\left(\frac{A}{B}\right) = -\frac{(2-b)\left[a(2+3b)+(2-b+2b^2)\eta\right]}{2[a-(1-b)\eta]^2}. \tag{17}$$

Due to Assumption 1, $2-b+2b^2 > 0$ and $2-b > 0$. Therefore $a(2+3b)+(2-b+2b^2)\eta > 0$ for $0 < b < 1$, implying A/B is decreasing in b over interval $(0,1)$. Next, denoting $a = \eta + n$ with $n > 0$ and then differentiating B/C with respect to b gives

$$\frac{d}{db}\left(\frac{B}{C}\right) = \frac{4\left\{2[5-5b-3b^2+2b^3]n+(4-5b^2-4b^3+3b^4)\eta\right\}}{(1-b)^2(4-6b-b^2+b^3)^2} \tag{18}$$

where it can be shown that

$$5-5b-3b^2+2b^3 > 0 \text{ and } 4-5b^2-4b^3+3b^4 > 0 \text{ for } 0 < b < b_0.$$

Hence B/C is increasing in b. It is clear that

$$\frac{A}{B} = \frac{4}{a-\eta} \text{ if } b = 0 \text{ and } \frac{A}{B} = \frac{3}{2a} \text{ if } b = 1$$

and

$$\frac{B}{C} = a-\eta \text{ if } b = 0 \text{ and } \text{limit}_{b\to b_0}\frac{B}{C} = \infty.$$

It is also apparent that for $b = 0$,

$$\frac{A}{B} \lesseqqgtr \frac{B}{C} \text{ according to } 2 \lesseqqgtr a-\eta.$$

Lemma 2 *If $a-\eta \geq 2$, then we have*

$$\frac{A}{B} < \frac{B}{C} \text{ for } b \in (0, b_0)$$

and if $a-\eta < 2$, then there is a $b_1 \in (0, b_0)$ such that

$$\frac{A}{B} > \frac{B}{C} \text{ for } b \in (0, b_1),\ \frac{A}{B} < \frac{B}{C} \text{ for } b \in (b_1, b_0)$$

and

$$\frac{A}{B} = \frac{B}{C} \text{ holds for } b = b_1.$$

Proof. If $a-\eta \geq 2$, then $A/B \leq B/C$ for $b = 0$. Since A/B is decreasing in $b \in (0,1)$ by (17) and B/C is increasing in $b \in (0, b_0)$ by (18), the first result is shown. On the other hand, if $a-\eta < 2$, then the dowanward-sloping AC curve crosses the upward-sloping B^2 curve only once from above at some $b \in (0, b_0)$ that is denoted by b_1. It is apparent that

$$\frac{A}{B} > \frac{B}{C} \text{ for } b \in (0, b_1) \text{ and } \frac{A}{B} < \frac{B}{C} \text{ for } b \in (b_1, b_0).$$

This completes the proof. ■

We are now ready to determine the optimal level of the abatement techonology in case of $0 < b < b_0$.

Theorem 5 *Assume* $0 < b < b_0$. *If* $a - \eta \geq 2$ *and* $\theta \leq A/B$ *or if* $a - \eta < 2$ *and* $\theta \leq \min[A/B, B/C]$, *then*

$$0 \leq \phi^e(\theta) < 1.$$

Proof. $0 < b < b_0$ implies $C > 0$. Further, the first part of Lemma 2 and $\theta \leq A/B$ imply $\theta \leq A/B < B/C$ leading to $\theta < B/C$. By (16), we have

$$0 \leq A - B\theta < A - C\theta^2.$$

By the second part of Lemma 2, $B/C < A/B$ in $(0, b_1)$ and thus $\theta \leq B/C$ implying $\theta \leq A/B$. In the same way, $A/B < B/C$ over (b_1, b_0) and thus $\theta \leq A/B$ that leads to $\theta \leq B/C$. In both intervals, by (16) with $C > 0$, we have the same inequality condition,

$$0 \leq A - B\theta < A - C\theta^2$$

that is divided by $A - C\theta^2$ to obtain

$$0 \leq \phi^e(\theta) < 1.$$

This completes the proof. ■

Theorems 1 and 2 clarify the conditions for which the optimal level of the abatement technology satisfies $0 < \phi^e(\theta) < 1$. We now turn attention to sensitivity of $\phi^e(\theta)$ to a change in θ. Differentiating $\phi^e(\theta)$ of (15) with respect to θ gives

$$\frac{d\phi^e(\theta)}{d\theta} = -\frac{g(\theta)}{\left(A - C\theta^2\right)^2} \tag{19}$$

where

$$g(\theta) = BC\theta^2 - 2AC\theta + AB$$

with

$$\begin{aligned} g'(\theta) &= 2BC\theta - 2AC \\ g'(0) &= -2AC \begin{cases} < 0 & \text{if } C > 0 \text{ or } 0 < b < b_0, \\ > 0 & \text{if } C < 0 \text{ or } b_0 < b < 1. \end{cases} \end{aligned}$$

and for the discriminant of $g(\theta)$,

$$\frac{D}{4} = ABC^2\left(\frac{A}{B} - \frac{B}{C}\right). \tag{20}$$

The sign of the derivative in (19) is determined by the sign of $g(\theta)$.

Theorem 6 *In case of $b_0 < b < 1$, if $\theta \leq A/B$, then*

$$\frac{d\phi^e(\theta)}{d\theta} < 0.$$

Proof. In this case, Lemma 1 gives $C < 0$ and then $D > 0$, implying that equation $g(\theta) = 0$ has two real roots,

$$\theta_{\pm} = \frac{A}{B} \pm \sqrt{\frac{A}{B}\left(\frac{A}{B} - \frac{B}{C}\right)}.$$

The smaller root θ_- is negative and the larger root θ_+ is positive and larger than A/B. It can be checked that $g(0) > 0$, $g'(0) > 0$ and $g(\theta) > 0$ for $\theta \leq A/B$. Therefore, (19) with $g(\theta) > 0$ leads to the negative derivative. ■

We now determine the sign of the derivative in case of $0 < b < b_0$.

Theorem 7 *If $a - \eta \geq 2$ or if $a - \eta < 2$ and $b_1 < b < b_0$, then*

$$\frac{d\phi^e(\theta)}{d\theta} < 0$$

whereas if $a - \eta < 2$ and $0 < b < b_1$, then

$$\frac{d\phi^e(\theta)}{d\theta} < 0 \text{ for } 0 \leq \theta < \theta_1$$

and

$$\frac{d\phi^e(\theta)}{d\theta} \geq 0 \text{ for } \theta_1 \leq \theta \leq \frac{B}{C}$$

where

$$\theta_1 = \frac{A}{B} - \sqrt{\frac{A}{B}\left(\frac{A}{B} - \frac{B}{C}\right)} > 0.$$

Proof. Lemma 1 yields $C > 0$ as $0 < b < b_0$. Lemma 2 implies $A/B < B/C$ under the conditions given in the first part. This inequality indicates $D < 0$ by (20), implying that $g(\theta) > 0$ for all $\theta \geq 0$. Therefore $d\phi^e(\theta)/d\theta < 0$. Lemma 2 implies $A/B > B/C$ under the conditions in the second part, leading to $D > 0$ by (20). Hence $g(\theta) = 0$ has two real solutions, smaller of which is θ_1 and less than B/C, since

$$\theta_1 - \frac{B}{C} = \sqrt{\frac{A}{B} - \frac{B}{C}} \left(\sqrt{\frac{A}{B} - \frac{B}{C}} - \sqrt{\frac{A}{B}} \right) \leq 0.$$

Hence $g(\theta) \geq 0$ for $\theta \leq \theta_1$. Therefore $d\phi^e(\theta)/d\theta < 0$. Lastly, $g(\theta) \leq 0$ for $\theta_1 \leq \theta \leq B/C$. Therefore (19) implies $d\phi^e(\theta)/d\theta \geq 0$ where the equality holds for $\theta = \theta_1$. ■

We have determined the optimal level of the abatement technology for the two symmetric firms. Accordingly, the optimal prices under the optimal technologies are determined by inserting $\phi^e(\theta)$ into the forms in (5),

$$p_k^*(\phi_i^*(\theta), \phi_j^*(\theta)) = p_k^*(\phi^e(\theta), \phi^e(\theta)) \text{ for } k = i, j$$

that is now denoted by $p^e(\theta)$,

$$p^e(\theta) = \frac{a + \eta + \theta\,(1 - b)\,\phi^e(\theta)}{2 - b}. \tag{21}$$

Notice that the derivative of $p^e(\theta)$ consists of two parts,

$$\frac{\partial p^e(\theta)}{\partial \phi^e} = \left. \frac{\partial p^e(\theta)}{\partial \phi^e} \right|_{\theta:const} \text{ and } \frac{\partial p^e(\theta)}{\partial \theta} = \left. \frac{\partial p^e(\theta)}{\partial \theta} \right|_{\phi^e(\theta):const}.$$

The optimal production under the optimal technology is obtained by the demand function (1) with $p^e(\theta)$,

$$q^e(\theta) = a - (1 - b)p^e(\theta). \tag{22}$$

The total amount of pollutions emitted by the two firms is the double of individually emitted pollutions,

$$E^e(\theta) = 2\phi^e(\theta)q^e(\theta). \tag{23}$$

We are now concerned with the changes of $E^e(\theta)$ in response to a change in the rate of the ambient charge, θ. If the value of θ increases (i.e., $\Delta\theta > 0$), then there are two sorts of effects, the technological effect caused by a change in $\phi^e(\theta)$ and the production effect caused by a change in θ. Suppose an increase of θ and consider the production effect first. Keeping the abatement technology fixed, the firm pushes the price up by $\Delta p^e > 0$ via (21) and thus, via the downward-sloping demand function (22), decreases output as well as emission levels (i.e., $\Delta q^e < 0$). Therefore, the production effect is negative. The change in emission caused by the production effect is

$$e_1 = \phi^e(\theta)\frac{\Delta q^e}{\Delta \theta} < 0.$$

Next, taking θ as given, we consider the technology effect. Suppose that the firm replaces the existing abatement technology with more efficient one (i.e., $\Delta\phi^e < 0$). This is the direct effect and decreases emission (i.e., $q^e\Delta\phi^e < 0$). Since the decrease in ϕ^e also pushes the price down via the optimal price (21) (i.e., $\Delta p^e < 0$) that then generates more production and more emissions via the demand function (21). This is the indirect effect and positive (i.e., $\phi^e \cdot \Delta q^e/\Delta p^e \cdot \Delta p^e/\Delta\phi^e$). The technology effect is the sum of these direct and indirect effects. The initial change in the technology is thought to be induced by the change in θ. Thus dividing the sum of these changes presents the total change in emission due to the technology effect

$$e_2 = q^e \frac{\Delta\phi^e}{\Delta\theta} + \phi^e \frac{\Delta q^e}{\Delta p^e}\frac{\Delta p^e}{\Delta\phi^e}\frac{\Delta\phi^e}{\Delta\theta}.$$

Notice that the first term is negative and the second term is positive. The sign of the total effect caused could be the demand on the relative magnitute of the opposite signed effects. These intuitive arguments can be confirmed by differentiating (23) with respect to θ,

$$\begin{aligned}\frac{1}{2}\frac{dE^e(\theta)}{d\theta} &= \frac{d\phi^e(\theta)}{d\theta}q^e(\theta) + \phi^e(\theta)\frac{dq^e(\theta)}{dp^e(\theta)}\left\{\frac{\partial p^e(\theta)}{\partial\phi^e(\theta)}\frac{d\phi^e(\theta)}{d\theta} + \frac{\partial p^e(\theta)}{\partial\theta}\right\} \\ &= q^e(\theta)\left(1 + \frac{\phi^e(\theta)}{q^e(\theta)}\frac{dq^e(\theta)}{d\phi^e(\theta)}\right)\frac{d\phi^e(\theta)}{d\theta} + \phi^e(\theta)\frac{dq^e(\theta)}{dp^e(\theta)}\frac{\partial p^e(\theta)}{\partial\theta} \qquad (24) \\ &= q^e(\theta)\left(1-\varepsilon\right)\frac{d\phi^e(\theta)}{d\theta} - \frac{(1-b)^2}{2-b}\left[\phi^e(\theta)\right]^2\end{aligned}$$

where ε denotes the elasticity of demand with respect to the abatement technology ϕ evaluated at the optimal point and is defined as

$$\varepsilon = -\frac{\phi^e(\theta)}{q^e(\theta)}\frac{dq^e(\theta)}{d\phi^e(\theta)}.$$

By Lemmas 3, 4 and the last form in (24), we have the following results.

Theorem 8 *The ambient charge is effective in controlling NPS pollutions,*

$$\frac{dE^e(\theta)}{d\theta} < 0$$

if the demand is inelastic (i.e.,$\varepsilon < 1$) and one of the following conditions hold,

(i) $b_0 < b < 1$ *and* $\theta \leq \frac{A}{B}$,

(ii) $0 < b < b_0$ *and* $a - \eta \geq 2$,

(iii) $b_1 < b < b_0$ *and* $a - \eta < 2$,

(iv) $0 < b < b_1,\ a - \eta < 2$ *and* $0 < \theta < \theta_1$

or the demand is elastic (i.e., $\varepsilon > 1$*) and*

$$(v)\ 0 < b < b_1,\ a - \eta < 2 \text{ and } \theta_1 < \theta < \frac{B}{C}$$

or if the elasticity is unity.

4 Concluding Remarks

This paper is mainly concerned with whether the ambient charge, environmental policy implementation, is effective in controlling the total emission of NPS pollution in a Bertrand duopolistic framework. To this end, we first examined the ambient charge effect in short-run in which the abatement technnlogies are fixed. Theorem 1 analytically demonstrated that the total emission of NPS pollution falls in response to an increase in the rate of the ambient charge. We then turned attention to the effect in the long-run in which the firms are able to adjust their abatement technologies ranging from zero (i.e., no abatement) to unity (i.e., no pollution). After selecting the optimal abatement technology, we specified the conditions for $0 \leq \phi^e(\theta) < 1$ in Theorems 3 and 4 and for $d\phi^e(\theta)/d\theta < 1$ in Theorems 5 and 6. Finally, the conditions under which the ambient charge becomes effective were summarized in Theorem 7.

In future studies, we will verify the effectiveness of the ambient charge in two more general cases: first without Assumption 3 (i.e., symmetry) and second symmetric oligopolies with more firms.

References

Ganguli, S., and Raju, S., Perverse environmental effects of ambient charges in a Bertrand duopoly, *Journal of Environmental Economics and Policy*, 1, 289-296, 2012.

Ishikawa, T., Matsumoto, A., and Szidarovszky, F., Regulation of non-point source pollution under n-firm Bertrand competition, *Environmental Economics and Policy Studies*, 21, 579-597, DOI: 10.1007/s10018-019-00243-9, 2019.

Segerson, K., Uncertainty and incentives for nonpoint pollution control, *Journal of Environmental Economics and Management* 15, 87-98, 1988.

Singh, N. and Vives, X., Price and quantity competition in a differentiated duopoly, *Rand Journal of Economics* 15, 546-554, 1984.