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Coordination and Imitation under Unawareness

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Abstract

This study focuses on discovered versions of coordination games with unawareness, and proposes a novel solution concept under unawareness named a successful-coordination equilibrium. In games with unawareness, coordination might fail because the models assume that agents are unaware of their realized actions. Then, when they observe the opponents' actions which they are unaware of, by teaching or asking their opponents how to play the opponents' actions, they might try to coordinate successfully. Although previous works propose models of discoveries, where players observe opponents' actions that they are unaware of and add these actions to their subjective games, they do not propose models of imitation that agents could add to the choices they did not have themselves, which was played by the opponents. We provide models of discoveries and imitations, showing that their revised games by discovering and imitating must propose a successful-coordination equilibrium.
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1 Introduction

This study focuses on coordination games with unawareness (or more strictly, symmetrical games with unawareness), discusses discoveries and imitations, and refines a cognitively stable generalized Nash equilibrium by proposing a successful-coordination equilibrium. Games with unawareness assume unaware players. Each player may be unaware of some actions, the opponents' view of

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games, or both.[1] Let us focus on a coordination game. In standard coordination games (without unawareness), an equilibrium means that coordination is successful. However, in coordination games with unawareness, an equilibrium might not mean that coordination is successful. Let us consider two players, Alice and Bob. They have two actions, X and Y. Suppose that Alice is aware of X but unaware of Y, whereas Bob is aware of Y but unaware of X. In other words, Alice can choose X but cannot choose Y, and Bob can choose Y but cannot choose X. Additionally, assume that both players commonly believe that Alice is aware of only X but unaware of Y, and Bob is aware of only Y but unaware of X. Then, Alice plays X and Bob plays Y, where Alice believes that Bob chooses Y while Bob believes that Alice chooses X. Both players' beliefs about the opponents' plays are correct even if both players miss coordination. Then, players try to improve the coordination failure. For example, Alice might teach Bob a way to play X or ask how to play Y, and then they try to coordinate.

To discuss cases like the above example, we need to models of discoveries and imitations. Recently, Schipper (2021) and Tada (2022) propose models of discoveries, where players observe opponents' actions, that they are unaware of, and add the actions to their subjective games.[2] In the above example, Alice is unaware of Y. With Bob playing Y, she observes and becomes aware of it. She then adds Y to her subjective game. However, since they discuss only discoveries, there are no models focusing on imitations. We present a model for observing and imitating such actions called the *imitative discovered game*. This model assumes that each player can observe and imitate *ways of playing* the opponents' actions, that is, each player can choose the opponents' played actions even if they could not choose them before observing such actions.

After analyzing imitative discovered games, we propose a successful-coordination equilibrium. In previous works, a generalized Nash equilibrium with stable belief hierarchies introduced by Sasaki (2017) or a self-confirming equilibrium introduced by Schipper (2021) to games with unawareness is used as a steady state equilibrium meaning that all agents confirm their correct beliefs about the opponents' plays. It seems that when players implement a generalized Nash equilibrium with stable belief hierarchies or self-confirming equilibrium, they play the same equilibrium in the next stage game. However, some generalized Nash equilibrium with stable belief hierarchies or self-confirming equilibrium induces coordination failure in a coordination game with unawareness. For example, Alice and Bob commonly believes that Alice is aware of X and unaware of Y and Bob is aware of Y and unaware of X. Then, both players can believe only that Alice plays X and Bob plays Y. Since Alice can implement only X and Bob can implement only Y, their beliefs are correct. Therefore, the play and beliefs are a generalized Nash equilibrium with stable belief hierarchies and a self-confirming

[1] As pointed out by Schipper (2014), unawareness is the lack of conception rather than the lack of information. He also provided a historical survey about unawareness and games with unawareness.

[2] There are differences between their models. Schipper (2021) uses Heifetz, Meier, and Schipper's (2013) model and discusses models of discoveries in extensive-form games. Tada (2022) uses a model similar to Perea (2018) and focus on only simultaneous-move games.

equilibrium. However, in the equilibrium, coordination obviously fails. Therefore, players might revise their subjective games. This case means that belief stability is different to stability of subjective games meaning that players do not have incentives not to revise their subjective games. This paper proposes a self-confirming equilibrium, where players only coordinate successfully. In a successful-coordination equilibrium, not only players' beliefs are correct but also their subjective games need not be revised for successful coordination. That is, the equilibrium notion is a refinement of generalized Nash equilibria with stable belief hierarchies and self-confirming equilibrium in games with unawareness.

We show that an imitative discovered game based on any coordination game with unawareness must have a successful-coordination equilibrium. Furthermore, we introduce notions of block games to coordination games with unawareness and reconstruct a block game called a coordination block game in which the players' action sets are the same. In an updated subjective game, some actions might be redundant because nobody plays them. Then, players might exclude such actions. A coordination block game is a smaller game that excludes unplayed actions. Each player would select an equilibrium among such coordination blocks in the game.

Related Literature

Feinberg (2021), Heifetz, Meier, and Schipper (2013), Halpern and Rêgo (2014), and Meier and Schipper (2014) are pioneering works on games with unawareness. Specifically, Heifetz, Meier, and Schipper (2013) and Halpern and Rêgo (2014) formulated extensive games with unawareness.

Meier and Schipper (2014) formulated Bayesian games with unawareness that are a generalization of Bayesian games. Another type-based model was proposed by Perea (2018) as a special case of the model by Meier and Schipper (2014). Note that the two models have different specifications. Meier and Schipper (2014) assumed that player's types are directly associated with beliefs about a structure of a game, whereas Perea (2018) did not make such an assumption. In his model, player's types are associated with beliefs about the structure of a game, but the types cannot be associated with the structure of the game itself.

The main solution concepts in games with unawareness have two approaches: equilibrium notions (Feinberg 2021; Čopič and Galeotti 2006; Ozbay 2007; Halpern and Rêgo 2014; Rẽgo and Halpern 2012; Grant and Quiggin 2013; Meier and Schipper 2014 Sasaki 2017; Schipper 2021; Kobayashi and Sasaki (2021)) and rationalizability notions (Heifetz, Meier, and Schipper 2013, 2021; Perea 2018; Guarino 2020). Recently, Tada (2022) proposed a generalization of the closedness-under-rational-behavior (CURB) notion proposed by Basu and Weibull (1991), which is one of the set-wise notions.

A split of our model is based on the literature about growing awareness, updating awareness, and discoveries, such as Karni and Vierø(2013, 2017), Schipper (2021), Galanis and Kotronis (2021), and Tada (2022). Studies in the literature indicate that agents additionally know information about states, events, consequences, actions, and so on, which they *were* previously unaware of. Note that our model refers to the growing awareness of the ways in which opponents play their game rather than the growing awareness of opponents' plays.

The rest of this paper is organized as follows. Section 2 presents the preliminaries of the study. In Section 3, we define a successful-coordination equilibrium and show the properties of this equilibrium. Section 4 proposes imitative discovered games and coordination block games. Finally, Section 5 presents the conclusions of the study.

2 Preliminaries

This section proposes a coordination game with unawareness, a more strictly symmetrical game with unawareness, and a cognitively stable generalized Nash equilibrium. Let $G = (I, A, u)$ be a standard n-person coordination game. $I = \{1, \ldots, n\}$ is a finite set of players, and $I_{-i} = I \setminus \{i\}$. $A = \times_{i \in I} A_i$, where A_i is a nonempty finite set of actions of i and $A_1 = \cdots = A_n$. Let $a_i \in A_i$ be i's action. $u = (u_i)_{i \in I}$, where $u_i : A \to \mathbb{R}$ is the utility function of i. For any $a = (a_1, \ldots, a_n) \in A$, $u_1(a) = \cdots = u_n(a) > 0$ if $a_1 = \cdots = a_n$, while $u_1(a) = \cdots = u_n(a) = 0$ otherwise.

We define coordination games with unawareness based on the studies by Kobayashi and Sasaki (2021) and Tada (2022), which are similar to that of Perea (2018).[3] [4] For any standard coordination game G, let $V = \times_{i \in I}(2^{A_i} \setminus \{\emptyset\})$ be the set of possible *views* of G. Similar to most previous works, we assume that the set of players is commonly known and each player's utility for each action profile does not depend on awareness. Let $v \in V$ and A_i^v be the set of actions of i in $v = \times_{j \in I} A_j^v$. Here, when player i is given v, i is aware of $a \in v$ and unaware of $a \in A \setminus v$. Let $\Gamma = (G, (T_i)_{i \in I}, (v_i)_{i \in I}, (b_i)_{i \in I})$ be a coordination game with unawareness, which is described as follows: for each $i \in I$,

- T_i is a finite and nonempty set of i's type, one of which is the type t_i^*.
- $v_i : T_i \to V$ is i's view function.
- $b_i : T_i \to T_{-i}$ is the belief function of i, where $T_{-i} = \times_{j \in I \setminus \{i\}} T_j$. If $b_i(t_i) = (t_j)_{j \in I \setminus \{i\}}$, then for each $j \in I \setminus \{i\}$, $v_j(t_j) \subseteq v_i(t_i)$.

[3] There is a difference between our approach and that of Perea (2018). Perea (2018) did not fix the belief hierarchies on views, and he dealt with probabilistic beliefs on awareness. By contrast, we assume that the "actual type" of players is fixed and given. Moreover, Perea (2018) did not deal with probabilistic beliefs.

[4] A type-based approach under unawareness is similar to that of Harsanyi (1967). However, as pointed out by Meier and Schipper (2014) and Perea (2018), Harsanyi's (1967) framework cannot present players' unawareness of actions. In his model, a player's action that they are unaware of is assigned to extremely low payoffs. Then, the rational player does not choose the action. However, if the player is not rational, they might play such an action. It contradicts the condition in which the player is unaware of such actions. By contrast, in a type-based approach under unawareness, each player cannot choose the actions that they are unaware of, even if they are irrational. Moreover, if the player is unaware of another player's actions, this player cannot reason that they can play the action.

Let us call G an objective game. An objective game can be interpreted as a "true game" in Γ. i's type, t_i, describes their view of the game and belief about the opponent's types. Given t_i, $v_i(t_i) = v$ implies that i is aware of v and unaware of $A \setminus v$, and $b_i(t_i) = (t_j)_{j \in I \setminus \{i\}}$ means that at t_i, i believes that the other types are $(t_j)_{j \in I \setminus \{i\}}$. Simultaneously, i believes that each view of j is $v_j(t_j)$. Let $b_i(t_i)(j)$ be j's type in $b_i(t_i)$. Each player may be unaware of some types of players, including their own.

In contrast to the previous literature, for simplicity, we define the strategies with a focus on pure actions only. For any $i \in I$, let $s_i : T_i \to A_i$. Then, given t_i, $s_i(t_i) \in A_i^{v_i(t_i)}$ is i's local action at t_i. Let $s_i = (s_i(t_i))_{t_i \in T_i}$ be i's generalized strategy, and let $s = (s_i)_{i \in I}$ be a generalized strategy profile. For any s, $s_i(t_i^*)$ is i's actual play. The set of players' actual play $A_i^{v_i(t_i^*)}$ may be a proper subset of i's *full* action set A_i. Then, the player i cannot implement $a_i \in A_i \setminus A_i^{v_i(t_i^*)}$.

Here, we define the generalized pure Nash equilibrium proposed by Halpern and Rêgo (2014) as follows: s^* is a generalized (pure) Nash equilibrium if, for any $i \in I$ and $t_i \in T_i$,

$$s_i^*(t_i) \in \arg\max_{x \in A_i^{v_i(t_i)}} u_i(x, (s_j^*(b_i(t_i)(j)))_{j \in I_{-i}}).$$

A generalized Nash equilibrium is interpreted as an equilibrium in beliefs. However, as shown by Schipper (2014), because games with unawareness suppose the unawareness of players' actions, a generalized Nash equilibrium might consist of wrong beliefs. Then, players who have such wrong beliefs might revise their subjective games, and they might choose different actions from the immediately preceding stage in the game. To avoid such issues, Sasaki (2017) proposed a notion of cognitive stability or stable belief hierarchies. This notion represents that in an equilibrium satisfying cognitive stability or stable belief hierarchies, all participants' beliefs about the opponents' plays are correct. Although Sasaki (2017) distinguished a notion of stable belief hierarchies from a notion of cognitive stability, he showed that the two notions are equivalent. Let us define cognitive stability as follows: A generalized Nash equilibrium s^* is cognitively stable if for any $i \in I$ and $t_i \in T_i$

$$s_i^*(t_i) = s_i^*(t_i^*).$$

Cognitive stability represents whether all players' beliefs about the opponents' play are correct. In a cognitively stable generalized Nash equilibrium, all players' local actions are the same. It means that each player's belief is correct.

3 Successful-Coordination Equilibrium

Cognitive stability is the concept of checking the correctness of beliefs in a played equilibrium. In any cognitively stable generalized Nash equilibrium, all players confirm that their beliefs are correct; while in cognitively unstable generalized Nash equilibrium, some players confirm that their beliefs are incorrect.

In many classes of games with unawareness, it seems that in a cognitive stable generalized Nash equilibrium all players need not revise their subjective views, and in a cognitively unstable generalized Nash equilibrium some players needs to revise their subjective view. In other words, cognitive stability seems to be a concept of the stability of subjective views. However, in some games with unawareness, especially coordination games with unawareness, players who play with a cognitively stable equilibrium might revise their subjective views.

Example 1. Consider two people, Alice, and Bob. They face the following coordination game, which is an objective game.

$v^0 =$

Alice / Bob	X	Y
X	1, 1	0, 0
Y	0, 0	1, 1

In v^0, there exist two pure Nash equilibria, (X, X) and (Y, Y).

Here, let us assume the following about Alice's belief in this game.

- Alice can implement her action X, but she cannot do the other action Y because she does not know how to play Y.
- Alice knows that Bob can choose his actions X and Y if he knows how to play them.
- Alice knows that Bob can choose only Y, and she knows that Bob does not know how to play X; hence, she knows that Bob cannot choose X.
- Alice supposes that Bob believes that it is common knowledge that Alice can choose only X, Bob can choose only Y, and the others' actions cannot be played.

In addition, let us assume the following about Bob's belief in this game.

- Bob can implement his action Y, but he cannot do the other action X because he does not know how to play X.
- Bob knows that Alice can choose her actions X and Y if she knows how to play them.
- Bob knows that Alice can choose only X, and he knows that she does not know how to play Y; hence, he knows that Alice cannot choose Y.
- Bob supposes that Alice believes that it is common knowledge that Alice can choose only X, Bob can choose only Y, and the others' actions cannot be played.

Then, Alice's first-order view of this game is as follows:

$v^1 =$

Alice / Bob	X	Y
X	1, 1	0, 0

;

Bob's first-order view of this game is as follows:

$v^2 =$

Alice / Bob	Y
X	0, 0
Y	1, 1

; and

Both players' second- or higher-order views of this game are as follows:

$v^3 =$

Alice / Bob	Y
X	0, 0

.

In this example, the mathematical formulation is as follows. Denote Alice by A and Bob by B. Suppose that $T_A = \{t_A^*, t_A\}$ and $T_B = \{t_B^*, t_B\}$ such that

$v_A(t_A^*) = v^1$ and $b_A(t_A^*) = t_B$;
$v_A(t_A) = v^3$ and $b_A(t_A) = t_B$;
$v_B(t_B^*) = v^2$ and $b_B(t_B^*) = t_A$; and
$v_B(t_B) = v^3$ and $b_B(t_B) = t_A$.

Suppose that each player is rational. Each player doesn't need to believe that the opponent is rational. Alice then performs the best response to (X, Y) in v^3 as X in v^1. In addition, Bob performs the best response to (X, Y) in v^3 as Y in v^2. Their beliefs and decisions consist of the following generalized strategy profile: $s^* = ([s_A(t_A^*) = X, s_A(t_A) = X], [s_B(t_B^*) = Y, s_B(t_B) = Y])$. The generalized strategy profile is a cognitively stable generalized Nash equilibrium. In the play, both players confirm the correctness of their beliefs.

However, each player knows that the equilibrium play is not a Nash equilibrium in each first-order subjective view. Alice is aware of the Nash equilibrium (X, X) in v^1, and Bob is aware of the Nash equilibrium (Y, Y) in v^2. In the generalized Nash equilibrium, coordination is not successful. □

This example shows that we need to distinguish between stability of beliefs and stability of subjective views in a coordination game. It seems that previous works interpreted unawareness of actions as *unawareness of the existence of the actions*. Therefore, previous works might have considered the absence of any issue because Alice and Bob are unaware of the objective game v^0. In other words, as both players are unaware of what they face in the coordination game, they cannot know that their coordination is unsuccessful.

However, Alice and Bob may revise their subjective views if we interpret unawareness of actions as the *unawareness of how to implement the actions*. Even if players are aware of the existence of actions, they cannot perform them because they do not know how to do so. Let X be the action "going to Alice's house," Let Y be the action "going to Bob's house." In other words, assume that Example 1 is a meeting game. Then, under our interpretation, this example plays out as follows: Both players know the objective game v^0. That is, they know that they must play the coordination game. However, since Alice does not know how to go to Bob's house, she does not perform that action. Similarly, since Bob does not know how to go to Alice's house, he does not perform that action. Moreover, the two are aware of the opponent's unawareness. That is, Alice is aware that Bob is unaware of how to get to her house, and Bob is aware that Alice is unaware of how to get to his house. Both Alice and Bob then stay home, and they believe that the other person has also done the same. This implementation is a cognitively stable generalized Nash equilibrium. However, under our interpretation, they are aware of not only the fact that the cognitively stable generalized Nash equilibrium is not a coordination play but also why they play the non-coordination play. That is, they are aware that they are unaware of how to get to each other's house. Then, Alice may ask Bob how to get to his house, or she may guide him about how to get to her house. Further, Bob may ask Alice how to get to her house, or he may guide Alice about how to get to his house. Then, their subjective views would be revised, and they would not play the same cognitively stable generalized Nash equilibrium.

For games with unawareness other than coordination games, when a cognitively stable generalized Nash equilibrium is played, all players would play the same cognitively stable generalized Nash equilibrium in the next stage game. However, for coordination games with unawareness, under the above interpretation, in some cognitively stable generalized Nash equilibrium players might not implement the same equilibrium in the next stage game, like Example 1. To consider only equilibria that deal with successful coordination, we propose a *successful-coordination equilibrium* that is a refinement of the cognitively stable generalized Nash equilibrium.

Definition 1. In a coordination game with unawareness Γ, s^* is a successful-coordination equilibrium if

1. for any $i \in I$ and $t_i \in T_i$,

$$s_i^*(t_i) \in \arg \max_{x \in A_i^{v_i(t_i)}} u_i(x, (s_j^*(b_i(t_i)(j)))_{j \in I_{-i}});$$

2. for any $i \in I$ and $t_i \in T_i$, $s_i^*(t_i) = s_i^*(t_i^*)$; and

3. $s_1(t_1^*) = \cdots = s_n(t_n^*)$.

The first and second conditions respectively require our definitions of generalized Nash equilibria and cognitive stability to be applied. The third condition requires that the coordination be successful. In a coordination game with unawareness, the third condition is necessary to avoid instances like that illustrated in Example 1.

We can easily deduce the following remarks:

Remark 1. In every coordination game *without* unawareness, any successful-coordination equilibrium is a Nash equilibrium, and vice versa.

Remark 2. A successful-coordination equilibrium may not exist. See Example 1.

Remark 3. Every successful-coordination equilibrium is a cognitively stable generalized Nash equilibrium. This is obvious from their definitions. However, the converse does not hold. See Example 1.

A successful-coordination equilibrium has the following properties. Although our results are obvious, each property suggests the existence of successful coordination equilibria.

Proposition 1. In a coordination game with unawareness Γ, for any $i \in I$, if $A_i^{v_i(t_i^*)} = A_i$, then every cognitively stable generalized Nash equilibrium is a successful-coordination equilibrium.

Before proving this proposition, we refer to Sasaki's (2017) proposition. Although his proposition includes a generalized mixed strategy profile, we restrict his result to pure strategies.

Proposition 2 (Sasaki 2017)**.** In a simultaneous move game with unawareness Γ, for any $i \in I$, if $A_i^{v_i(t_i^*)} = A_i$, then in any cognitively stable generalized Nash equilibrium, the actual plays of all players are Nash equilibria in G.

Proof. Suppose that for any $i \in I$, $A_i^{v_i(t_i^*)} = A_i$. A cognitively stable generalized Nash equilibrium s^* is given. That is, for any $(i, t_i) \in I \times T_i$, $u_i(s_i^*(t_i), (s_j^*(b_i(t_i)(j)))_{j \in I_{-i}}) = u_i(s_i^*(t_i), (s_j^*(t_j^*))_{j \in I_{-i}}) = u_i((s_j^*(t_j^*))_{j \in I})$, that is, every participant's actual play best responds to the others' actual play. Suppose $((s_j^*(t_j^*))_{j \in I})$ is not a Nash equilibrium in G, that is, there exist $(i, a_i) \in I \times A_i$ such that $a_i \neq s_i^*(t_i^*)$ and $u_i(a_i, (s_j^*(t_j^*))_{j \in I_{-i}}) > u_i((s_j^*(t_j^*))_{j \in I})$. However, since s^* is a cognitively stable generalized Nash equilibrium, it is a contradiction. Hence, $((s_j^*(t_j^*))_{j \in I})$ is a Nash equilibrium in G. Here, because t_i^* denotes i's actual type, $((s_j^*(t_j^*))_{j \in I})$ refers to all players' actual plays. □

Proof of Proposition 1. Suppose that for any $i \in I$, $A_i^{v_i(t_i^*)} = A_i$. A coordination game with unawareness is a special case of static game with unawareness. Therefore, by Proposition 2, in every cognitively stable generalized Nash equilibrium s^*, the actual play of all players $(s_i^*(t_i^*))_{i \in I}$ is a Nash equilibrium in the standard coordination game G. In any standard coordination game, a Nash equilibrium $a^* = (a_1^*, \ldots a_n^*)$ satisfies $a_1^* = \cdots = a_n^*$. By definition of an actual play, for any $i \in I$, since $s_i(t_i^*) = a_i^*$, $s_1^*(t_1^*) = \cdots = s_n^*(t_n^*)$. Therefore, s^* satisfies every condition of Definition 1. □

Proposition 1 suggests that if every player is aware of every player's action, then the converse of Remark 2 holds.

Proposition 3. Suppose that $\bigcap_{i \in I} \bigcap_{t_i \in T_i} v_i(t_i) \neq \emptyset$ in a coordination game with unawareness Γ. If some $a \in \bigcap_{i \in I} \bigcap_{t_i \in T_i} v_i(t_i)$ is a Nash equilibrium in G, then there exists a successful-coordination equilibrium.

Proof. Suppose that $\bigcap_{i \in I} \bigcap_{t_i \in T_i} v_i(t_i) \neq \emptyset$ in Γ and that some $a = (a_1, \ldots, a_n) \in \bigcap_{i \in I} \bigcap_{t_i \in T_i} v_i(t_i)$ is a Nash equilibrium in G. As Γ is a coordination game, the

Nash equilibrium in G satisfies $a_1 = \cdots = a_n$. For any $(i, t_i) \in I \times T_i$, let $s_i(t_i) = a_i$. Then, the generalized strategy profile s satisfies the conditions of Definition 1. That is, the s is a successful-coordination equilibrium. □

Proposition 3 is a special case of Sasaki (2017, Proposition 2).

4 Discovery and Imitation of Actions

This section presents a model of discoveries and imitations under unawareness based on the model of *endogenously* discovered games (Schipper 2021; Tada 2022) as follows.[5] [6]

Definition 2. $\Gamma' = (G, (T'_i)_{i\in I}, (v'_i)_{i\in I}, (b'_i)_{i\in I})$ is an *imitative discovered game* with $s = (s_i)_{i\in I}$ in $\Gamma = (G, (T_i)_{i\in I}, (v_i)_{i\in I}, (b_i)_{i\in I})$ if for any $(i, t_i) \in I \times T_i$, there exists $t'_i \in T'_i$ such that

1. $v'_i(t'_i) = \times_{j\in I}[A_j^{v_i(t_i)} \bigcup_{k\in I} s_k(t^*_k)]$, where t^*_k is k's actual type; and
2. for any $(j, t_j) \in I_{-i} \times T_j$ with $b_i(t_i)(j) = t_j$, there exists t'_j such that $b'_i(t'_i)(j) = t'_j$ and $v'_j(t'_j) = \times_{k\in I}[A_k^{v_j(t_j)} \bigcup_{h\in I} s_h(t^*_h)]$, where t^*_h is k's actual type.

When all players observe each other's plays, the first condition suggests that each player not only gains knowledge of the opponents' feasible actions, but also discovers (or 'learns') a way of playing such actions. The second condition suggests that as supposed by each player, every player commonly believes that all players gain knowledge of the others' feasible actions and discover a way of playing such actions.

Example 1 (Continued.) Suppose Alice and Bob play $s^* = ([s_A(t^*_A) = X, s_A(t_A) = X], [s_B(t^*_B) = Y, s_B(t_B) = Y])$. Then, according to the first condition, Alice adds Bob's action Y to not only Bob's choice but also Alice's choice in her subjective view v^1; and Bob adds Alice's action X to not only Alice's choice but also Bob's choice in his subjective view v^2. Moreover, as both players suppose that each of them commonly believes that they gain knowledge of each other's feasible actions and discovers a way of playing such actions according to the second condition, both players add actions X and Y to their respective choice in each other's second or any higher-order view v^3. Then, each agent's first and any higher-order view is replaced with v^0.

This imitative discovered game $\Gamma' = (G, (T'_A, T'_B), (v'_A, v'_B), (b'_A, b'_B))$ is formulated as follows:

[5] Karni and Vierø(2013, 2017) discussed cases in which agents discover their own new feasible actions. However, in their model, such actions are not *endogenously* discovered but rather *exogenously* discovered. In other words, such actions are given to agents by modelers.

[6] Unlike Schipper (2021) and Tada (2022)), we do not deal with *discovery processes*. As indicated by one of the main results, only one update of imitation is required in our model.

$T_A = \{t'_A\}$ and $T_B = \{t'_B\}$;
$v_A(t'_A) = v^0$ and $b_A(t'_A) = t'_B$; and
$v_B(t'_B) = v^0$ and $b_B(t'_B) = t'_B$.

In Γ', Alice and Bob can choose two actions X and Y. □

Interestingly, any imitative discovered game has the following property:

Proposition 4. Given any n-person coordination game with unawareness and any generalized strategy profile, the imitative discovered game has a successful-coordination equilibrium.

Proof. It is obvious. □

The above proposition means that it leads to an existence of a successful-coordination equilibrium in which each player revises their subjective view with just one. In Example 1, there exist two successful coordination equilibria in Γ': $s'_1 = (s'_A(t'_A) = X, s_B(t'_B) = X)$ and $s'_2 = (s'_A(t'_A) = Y, s_B(t'_B) = Y)$.

After players discover revised subjective games in the imitative discovered game, which set of actions do the players pay attention to? It seems to be redundant that a player rationalizes actions based on their subjective view. Let us consider the following example:

Example 2. Consider the following objective game played by Colin (C) and David (D):

$v_O =$

Colin / David	α	β	γ	δ
α	1, 1	0, 0	0, 0	0, 0
β	0, 0	1, 1	0, 0	0, 0
γ	0, 0	0, 0	1, 1	0, 0
δ	0, 0	0, 0	0, 0	1, 1

.

Here, suppose that Colin believes the following view is a common belief:

$v_C =$

Colin / David	α	β
α	1, 1	0, 0

.

In contrast, David believes the following view as a common belief:

$v_D =$

Colin / David	γ	δ
β	0, 0	0, 0
δ	0, 0	1, 1

.

Let us formulate this game $\Gamma = (G, (T_C, T_D), (v_C, v_D), (b_C, b_D))$ as follows:

$T_C = \{t^*_C, t_C\}$ and $T_D = \{t^*_D, t_D\}$;
given t^*_C, $v_C(t^*_C) = v_C$ and $b_C(t^*_C) = t_D$;
given t_C, $v_C(t_C) = v_D$ and $b_C(t_C) = t^*_D$;

given t_D^*, $v_D(t_D^*) = v_D$ and $b_D(t_D^*) = t_C$; and
given t_D, $v_D(t_D) = v_C$ and $b_D(t_D) = t_C^*$.

Suppose that both players implement a generalized strategy profile $s^* = ([s_C(t_C^*) = \alpha, s_C(t_C) = \beta], [s_D(t_D^*) = \gamma, s_D(t_D) = \alpha])$. In the strategy profile, the actual play is (α, γ). Then, the imitative discovered game $\Gamma' = (G, (T'_C, T'_D), (v'_C, v'_D), (b'_C, b'_D))$ is formulated as follows:

$T'_C = \{t'_C, t''_C\}$ and $T'_D = \{t'_D, t''_D\}$;
given t'_C, $v'_C(t'_C) = v'_C$ and $b'_C(t'_C) = t''_D$;
given t''_C, $v'_C(t''_C) = v'_D$ and $b'_C(t''_C) = t'_D$;
given t'_D, $v'_D(t'_D) = v'_D$ and $b'_D(t'_D) = t''_C$; and
given t''_D, $v'_D(t''_D) = v'_C$ and $b'_D(t''_D) = t'_C$, where

$v'_C =$

Colin / David	α	β	γ
α	1, 1	0, 0	0, 0
γ	0, 0	0, 0	1, 1

, and

$v'_D =$

Colin / David	α	γ	δ
α	1, 1	0, 0	0, 0
β	0, 0	0, 0	0, 0
γ	0, 0	1, 1	0, 0
δ	0, 0	0, 0	1, 1

.

Then, Colin knows that there exist two Nash equilibria in v'_C, (α, α) and (γ, γ), whereas David knows that there exist three Nash equilibria in v'_D, (α, α), (γ, γ), and (δ, δ). Note that Colin is not aware that David's view is v'_D and that David is not aware that Colin's view is v'_C.

Each player must select one of those equilibria in each other's view. Here, let us focus on David. Although he knows there are three equilibria, it seems odd that he includes all equilibria in his choices because δ is played by neither Colin nor David. □

After the players imitate the opponents' plays and revise their views, they might exclude redundant actions that nobody plays. Then, the players might reconstruct their subjective views to exclude such actions. To provide such representation, we use *block game* notions proposed by Myerson and Weibull (2015). A block is a Cartesian product of nonempty subsets of players' actions. Let us first focus on actions that each player observes and imitates, and then let us define a coordination block game as follows.[7]

Definition 3. Given any coordination game without unawareness $G = (I, A, u)$ and any block $T = \times_{i \in I} T_i \in V$, $G^T = (I, T, u^T)$ is a *coordination block game* if

[7]Tada (2022) proposed a similar notion as a realizable CURB block game. In a realizable CURB block game, the block is CURB.

1. $T_i = \cdots = T_n$; and
2. $u^T = (u_i^T)_{i \in I}$, where $u_i^T(a) = u_i(a)$ for any $i \in I$ and $a \in T$.

Here, T is called a coordination block.

Then, the following proposition holds.

Proposition 5. Given any n-person coordination game Γ and any generalized strategy profile s, let $T \in V$ be a block such that for any $i \in I$, $A_i^T = \bigcup_{j \in I} s_j(t_j^*)$. Then, the block game $G^T = (I, T, u^T)$ is a coordination block game.

Proof. It is obvious. □

In the case of Example 2, as Colin and David play (α, γ), they focus on α and γ. That is, David excludes δ from his choices. Then, the coordination block is $\{\alpha, \gamma\} \times \{\alpha, \gamma\}$, and the coordination block game is

$v_T =$

Colin / David	α	γ
α	1, 1	0, 0
γ	0, 0	1, 1

.

Hence, David can restrict his choices. Then, in their equilibrium selection, both players focus on α and γ.

In some coordination games (with unawareness), some players are unaware of some of the choices, and the set of choices might be too large. Hence, in the first play, players might not be able to select some specific (successful coordination) equilibrium, or they might not be able to restrict action sets to a coordination block. However, by discovering and imitating only the actions taken in the first play, players can restrict their actions to some specific coordination blocks.

5 Conclusion

We focus on coordination games with unawareness and propose models, where players imitate the opponents' play for successful coordination. In some coordination games with unawareness, no successful-coordination equilibrium might exist, although there exists a cognitively stable generalized Nash equilibrium. However, in the real word, it does not seem that people leave coordination failure behind. They teach or ask the opponents for a way to play their actions, and imitate these actions.

This study proposes a model of discoveries and imitations, where each player observes all the players' plays and imitates the approaches of such plays, and shows that the revised game must have a successful-coordination equilibrium. Moreover, when each player focuses on the played actions by imitating the approaches to such played actions, they might reconstruct a coordination block game, where each player's action set consists of every component of the played action profile.

We assume that each player can imitate the approaches of the opponents' plays. However, imitation is not always successful. Players might not understand how to play the opponents' actions. For example, a teacher teaches how to study, but students might not know how to study. When you are summoned to a strange coffee shop, it can be difficult to get there even if you are told how to get there. Future research needs to examine when imitation is possible.

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