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# Commitment games revisited

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**Abstract**

Commitments have positive-negative effects on games' outcomes, which vary depending on the conformation of the commitments. Renou (2009) investigates commitment in finite strategic form games mainly to the extent of pure strategies, and the major finding can be applied to mixed strategies. Here, there is little or no randomizing commitment. We reconsider commitment games that have a one-shot commitment before an action stage. There are two examples. The first has a mixed strategy equilibrium with players' randomizing in the commitment stage, which Pareto-dominates all the pure commitment equilibria. The other has no pure commitment equilibrium. In addition, we observe that some of the supports in a mixed strategy equilibrium might be the pure strategy equilibrium outcome after introducing multiple commitment stages.

JEL classification: C72, C73

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## 1 Review of Renou's Commitment Setting

Renou (2009) has derived various implications of commitment games. The analysis is novel since it considers committing not to a single action but to a set of actions. In the commitment stage (first stage), players simultaneously commit to subsets of their action sets, followed by the action stage (second stage) in which they choose actions among the restricted action sets. Renou (2009) investigates how introducing commitment influences outcomes in "status quo games" in a wide range of circumstances.

In this section, we verify commitment games as defined by Renou (2009) and the main results. Before playing a strategic form game (a status quo game,) players can make

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commitments simultaneously. A commitment does not have to be a single strategy. Instead, we may interpret commitment as the ability to eliminate some strategies in advance. A player can do nothing (commit to all strategies in the status quo game), eliminate one strategy (commit to all strategies but one), or keep a strategy and eliminate all others (commit to one strategy). A strategy profile of the status quo game made up by subgame perfect strategies is called "implementable."

Introducing a commitment device increases (or does not decrease, to be exact) pure strategy equilibria in all status quo games, since any pure Nash equilibrium is implementable in the status quo game. We confirm this by examining a simple $2 \times 2$ status quo game.

| | $a$ | $b$ |
|---|---|---|
| $a$ | $2, 2$ | $0, 0$ |
| $b$ | $3, 0$ | $1, 1$ |

This status quo game is dominance solvable and has the unique Nash equilibrium $(b, b)$. For example, it is a subgame perfect equilibrium path that both players commit to $\{b\}$ (eliminate $a$), followed by $(b, b)$. There is no profitable deviation for either player. If either player leaves $\{a, b\}$, the opponent player must play $b$ in the second stage, to which the player with $\{a, b\}$ chooses $b$ and the outcome remains $(b, b)$. Thus, this is a subgame perfect equilibrium path. In most commitment games, any pure Nash equilibria in the corresponding status quo games are implementable by all players committing to single action sets that include only the Nash equilibrium strategies.

Conversely, $(a, a)$, which is not a Nash equilibrium outcome in the status quo game is also implementable. Consider the path below. Player 1 eliminates dominant strategy $b$ while player 2 does nothing in the first stage. In the second stage, player 1 must play $a$. Player 2 compares payoff 2 with $(a, a)$ and payoff 0 with $(a, b)$, and chooses $a$, resulting in $(a, a)$. A potential deviation for player 1 is doing nothing or committing to $\{b\}$ in the first stage. However, the outcome of the deviation is $(b, b)$ in each case, and thus it is not a profitable deviation. Coming back to the case of player 1 committing to $\{a\}$, this path constructs a subgame perfect equilibrium. Thus, $(a, a)$ is implementable. In this manner, the introduction of the commitment stage weakly increases pure strategy equilibria.

# 2 Pareto Improvement through Randomized Commitments

In the last sentence on page 493, Renou (2009) states "although Theorem 2 is stated for pure strategies, it also applies to mixed strategies"This randomizing expansion requires attention to some details, although what is unspecified. We must define other players' randomized commitment $X^*_{-i}$ as weighted "restricted status quo games by strategies of other players, and player $i$ best responds in the first stage toward the randomized others' commitments." Notice that if we mistake $X^*_{-i}$ as "all pure commitments" on the equilibrium path, the latter claim of Theorem 2 does not hold.

Taking mixed strategies into account could Pareto-improve the outcome. Consider the next example.

| | $a$ | $b$ | $c$ | $d$ | $e$ |
|---|---|---|---|---|---|
| $a$ | $9, 10$ | $0, 11$ | $-1, 11$ | $0, 0$ | $-1, 0$ |
| $b$ | $0, -1$ | $6, 5$ | $0, 0$ | $0, 0$ | $0, 6$ |
| $c$ | $0, 0$ | $7, 0$ | $3, 4$ | $0, 5$ | $-1, 5$ |
| $d$ | $10, 0$ | $7, -1$ | $0, 1$ | $2, 2$ | $0, 0$ |
| $e$ | $10, -1$ | $0, -1$ | $4, 0$ | $3, 0$ | $1, 1$ |

$G_P$

This status quo game has the unique Nash equilibrium $(e, e)$. The implementable strategy profiles with pure commitments are $(d, d)$ and $(e, e)$ only. In the first stage, player 1 commits to $\{d\}$ and player 2 does nothing, and $(d, d)$ is realized.

There are no other implementable strategy profiles. For example, $(c, c)$ is not implementable. To make $(c, c)$ a Nash equilibrium at an action stage, player 1 must eliminate $e$ at the commitment stage, player 2, $d$ and $e$. However, player 1 has an incentive to keep $e$, strictly improving her payoff from 3 to 4. By rights, player 2 has a punishing option $e$ to avoid player 1's deviation (leaving $e$.) However, player 2 has to eliminate $e$ (and $d$) to elude her temptation to deviate to $e$ (or $d$) from $(c, c)$. A similar argument holds on $(a, a)$. Swapping the players explains that $(b, b)$ is not implementable. Most ("all" is accurate if there is no confusion) pure commitment pairs followed by mixed strategy equilibria at the action stages lead player 1's deviations to a single commitment $\{e\}$.

To the extent of randomized commitments, however, there is an equilibrium path in which both players commit

$$\left(\frac{1}{3}\{a, b\} + \frac{1}{3}\{a, b, c, d\} + \frac{1}{3}\{a, b, c, d, e\}, \frac{1}{3}\{a\} + \frac{1}{3}\{a, b, c\} + \frac{1}{3}\{a, b, c, d, e\}\right),$$

and plays $(a, a)$ if $(\{a, b\}, \{a\})$, $(d, a)$ if $(\{a, b, c, d\}, \{a\})$, $(e, a)$ if $(\{a, b, c, d, e\}, \{a\})$, $(b, b)$ if $(\{a, b\}, \{a, b, c\})$, $(c, c)$ if $(\{a, b, c, d\}, \{a, b, c\})$, $(e, c)$ if $(\{a, b, c, d, e\}, \{a, b, c\})$, $(b, e)$ if $(\{a, b\}, \{a, b, c, d, e\})$, $(d, d)$ if $(\{a, b, c, d\}, \{a, b, c, d, e\})$, and $(e, e)$ if $(\{a, b, c, d, e\}, \{a, b, c, d, e\})$. The implementable strategy profile is $1/9\,(a, a) + 1/9\,(d, a) + 1/9\,(e, a) + 1/9\,(b, b) + 1/9\,(c, c) + 1/9\,(e, c) + 1/9\,(b, e) + 1/9\,(d, d) + 1/9\,(e, e)$. The equilibrium payoff is $(5, 3)$.

Let us demonstrate using a concrete example. If player 1 commits to $\{a, b, c, d, e\}$, her payoff would be 10 in the case where player 2 commits to $\{a\}$, which leads to $(e, a)$; 4 in the case where player 2 commits to $\{a, b, c\}$, which leads to $(e, c)$; and 1 in the case where player 2 commits to $\{a, b, c, d, e\}$, which leads to $(e, e)$. The three cases each occur with a probability of 1/3. Thus, player 1's payoff from committing to $\{a, b, c, d, e\}$ is

$$\frac{1}{3} \cdot 10 + \frac{1}{3} \cdot 4 + \frac{1}{3} \cdot 1 = 5.$$

If player 1 commits to $\{a, b, c, d\}$, her payoff would be 10 in the case where player 2 commits to $\{a\}$, which leads to $(d, a)$; 3 in the case where player 2 commits to $\{a, b, c\}$, which leads to $(c,c)$; and 2 in the case where player 2 commits to $\{a,b,c,d,e\}$, which leads to $(d,d)$. As above, player 1's payoff from committing to $\{a,b,c,d\}$ is

$$\frac{1}{3}\cdot 10+\frac{1}{3}\cdot 3+\frac{1}{3}\cdot 2=5.$$

If player 1 commits to $\{a,b\}$, her payoff would be 9 in the case where player 2 commits to $\{a\}$, which leads to $(a,a)$; 6 in the case where player 2 commits to $\{a,b,c\}$, which leads to $(b,b)$; and 0 in the case where player 2 commits to $\{a,b,c,d,e\}$, which leads to $(b,e)$. Again, player 1's payoff from committing to $\{a,b\}$ is

$$\frac{1}{3}\cdot 9+\frac{1}{3}\cdot 6+\frac{1}{3}\cdot 0=5.$$

If player 1's deviation is $\{a,b,c\}$, for example, her expected payoff is (since each of $(a,a)$, $(c,c)$, and $(b,e)$ would occur with a probability of 1/3)

$$\frac{1}{3}\cdot 9+\frac{1}{3}\cdot 3+\frac{1}{3}\cdot 0=4,$$

which is not profitable. We can calculate player 2's payoff in a similar way.

This outcome is interpreted as follows. If the status quo game is played, the players play $(e,e)$ and the payoff for player 1 is 1. Although $(d,d)$ is more desirable for player 1 than $(e,e)$, $(d,d)$ is not played in the status quo game since player 1 has an incentive to play $e$ against player 2's $d$. Player 1 needs to remove this temptation by eliminating $e$ in the first stage. If player 1 eliminates $e$, then $(e,d)$ would be zero expectation, so that player 2 has an incentive to commit to $\{a,b,c\}$, (where in a subgame $\{a,b,c,d\}\times\{e\}$, not $(b,e)$ but $(d,e)$ would be played,) resulting in $(c,c)$, through which both players' payoffs improve from 2 to 3 or 4. Now, the best reply for player 1 against player 2's $\{a,b,c\}$ is committing not to $\{a,b,c,d\}$ but to $\{a,b\}$, since player 2's temptation to play $d$ or $e$ has disappeared. If player 1 commits to $\{a.b\}$, then player 2's best reply is further eliminating $b$ and $c$ into $\{a\}$. However, against player 2's $\{a\}$ , player 1 has an incentive to do nothing in the first stage. If player 1 plays $\{a,b,c,d,e\}$, one of player 2's best replies is doing nothing in the first stage. In summary, the first stage seems to be a matching-pennies-like situation.

# 3 Multiple Commitment Stages

Let us assume that the number of commitment stages increases from one to four. In the $n$-stage commitment game of $G_P$, denoted by $\Gamma^n(G_P)$, $(b,b)$ and $(c,c)$ continues to be the implementable strategy profile. In addition, $\Gamma^4(G_P)$ has an implementable strategy profile $(a,a)$ that is Pareto efficient. In the first commitment stage, player 1 eliminates $e$ while player 2 does nothing. In the second commitment stage, after observing player 1's elimination of $e$, player 2 commits to $\{a,b,c\}$, followed by $(c,c)$. Player 2's commitment demonstrates her temptation to play $e$ if player 1 plays $b$; thus player 1 eliminates $c$ and $d$ with relief, playing $(b,b)$. By the same reasoning, player 2 can make a single commitment to $\{a\}$ resulting in $(a,a)$ in the next period.

When there is only one commitment stage, the best reply for player 1 to player 2's $\{a\}$ is $e$. However, player 1 has already eliminated $e$. Hence,$(a, a)$ is realized in the action stage. Bade et al. (2009) asserts that the number of commitment stages does not matter. However, the class of the strategic form game with finite strategy sets results in different priorities. The example above and the examples in Dutta and Ishii (2016) indicate that the number of the commitment stages alters the outcomes of commitment games.

# 4 No Pure Commitment Equilibria

Renou (2009) states that it considers pure strategies mainly. When players randomize their commitments, the calculation of the lower bound of payoffs mentioned in the main theorem changes. To avoid the problem, a natural setting is "pure commitments and mixed actions" as in Dutta and Ishii (2016). Unfortunately, there is a status quo game that has no such equilibrium.

| | $a$ | $b$ | $c$ |
|---|---|---|---|
| $a$ | $5, 7$ | $4, 8$ | $0, 9$ |
| $b$ | $1, 6$ | $8, 5$ | $3, 3$ |
| $c$ | $13, 0$ | $12, 2$ | $2, 4$ |

$G_N$

There is no subgame perfect equilibrium with pure commitments in Renou's (2009) setting. For example, $(b, b)$ following $\{a, b\} \times \{b, c\}$ is not implementable, since player 2 has an incentive to commit to $\{a\}$. Additionally, $(c, c)$ following $\{a, c\} \times \{a, c\}$ is not implementable, since player 1 has an incentive to commit to $\{b, c\}$, followed by a mixed strategy equilibrium that improves player 1's payoff. The latter case suggests how difficult it is to construct a pure commitment equilibrium in $\Gamma(G_N)$.

The status quo game has the unique equilibrium $(.5b + .5c, .2b + .8c)$. Since $c$ dominates $a$, player 1 can play $a$ with positive probability only by eliminating $c$ on the equilibrium path. In truth, player 1 has an incentive to eliminate $c$ in the status quo game, resulting in $(.5a + .5b, .5a + .5b)$. Then, consider the commitment to $\{a, b\}$ by player 1. The unique best reply for player 2 is a commitment to $\{a\}$. Player 1's best reply to $\{a\}$ is not $\{a, b\}$ but keeping option $c$. The above best-reply-sequence shows that $\Gamma(G_N)$ also has a matching-pennies-like structure. In addition, there is no pure commitment equilibrium.

As in the case of $\Gamma^4(G_P)$, $\Gamma^2(G_N)$ has a subgame perfect equilibrium outcome $(a, a)$ by player 2 committing to $\{a\}$ after observing player 1 eliminating $c$. Still, having multiple commitment stages is key here.

# 5 Concluding Comments about Multiple Commitment Stages

Renou (2009) mainly considers pure strategies, and produces various results. However, there is a situation in which the mixing expansion brings Pareto improvement. Of course, considering pure strategies is sufficient in many circumstances. By contrast, a game with no pure strategy equilibrium needs mixed equilibria. In particular, an equilibrium with mixed commitments does not tend to be intuitively comprehensive. In this paper, we consider that including multiple commitment stages leads to "reading" from mixed equilibria to pure equilibria. When our priority is intuitive interpretation, it is better that mixed equilibria are possible in the action stage and commitment must be pure as in Dutta and Ishii (2016).

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