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Reconsideration of MMT-type Fiscal and Monetary Stabilization Policy from a Viewpoint of Dynamic Keynesian Model

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Abstract

In this paper, the theoretical foundation of Modern Monetary Theory (MMT), which is often considered unconventional within mainstream economics, is used as the basis to examine the macroeconomic stability and instability through a dynamic Keynesian model. A common critique of MMT is the assertion that governments find it challenging to control inflation through fiscal policy. However, this study's findings indicate that by establishing credibility in inflation target and consistently pursuing proactive fiscal policies in response to deficient output gaps, inflation can be managed without escalating uncontrollably, leading to economic stabilization. Although the fundamental tenets of MMT may not heavily emphasize monetary policy, the significance of fiscal policy is reaffirmed by the conclusions drawn from this paper, ensuring that crucial aspects are not disregarded.

Keywords: MMT, Dynamic (in)stability, Dynamic Keynesian Model
JEL Classification Number: E12, E31, E52, E62, E63

I. Introduction

The global supply chain shock caused by the Covid-19 pandemic in 2020, coupled with the implementation of large-scale financial and fiscal policies, led to a rapid tightening of demand and supply, resulting in an unprecedented inflation surge in many countries. As of the present, advanced economies continue to experience high inflation rates, prompting central banks such as the Federal Reserve (Fed) to attempt containment through tightening monetary policies. While these efforts are expected to achieve convergence towards the inflation target, they are also anticipated to be accompanied by a sacrifice in employment and output. This approach of sacrificing employment maximization to maintain low inflation rates represents the mainstream perspective, significantly differing from the Modern Money Theory / Modern Monetary Theory (MMT), which originated from a subgroup of Post-Keynesian economics and is considered a non-mainstream approach.

MMT, structured by Abba Lerner, and currently advocated by scholars such as Randall Wray and Stephanie Kelton, fundamentally emphasizes proactive fiscal policies and passive monetary policies, always striving to maximize employment. The theory asserts that with the government sector's dynamic fiscal policy, maximum employment can be achieved at all times, effectively regulating employment movement between the public and private sectors through wage adjustments, demand adjustments, and controlling excessive price increases. Concepts of interest rates and investment are not crucial in MMT, which, it can be argued that, while the role of the government is equally important, it differs from Keynes (1936). Looking ahead, it is assumed that the trade-off with demand will dampen the inflation rate, gradually returning to pre-Covid conditions, thereby raising the need for MMT-oriented fiscal policy management to increase output, especially in countries like Japan, which may be caught in the liquidity trap.

Mainstream economists and the general public believe that governments should always demonstrate responsibility for the fiscal balance. They fear that without such responsibility, the "credibility" of government bonds may be damaged, leading to a surge in inflation rates and interest rates, eventually causing fiscal collapse or a decrease in private investment, commonly known as crowding out. As a result, many governments emphasize fiscal balance, with some like Japan aiming for a surplus in the primary balance (the balance between tax revenues, non-tax revenues, and expenditures excluding the cost of servicing government debt)[1] and the United States setting limits on debt[2] and government bond issuances and potentially enforcing mandatory spending cuts.

This perspective contrasts significantly with MMT. According to MMT theorists, one critical aspect is that governments with sovereign currencies, such as the United States and Japan, can issue their currency at will, unlike during the gold standard era. Consequently, they can never run out of financial resources. Therefore, MMT proposes that the focus should not be on fiscal balance but rather on inflation, and as long as inflation is not occurring, fiscal spending to fill demand shortfalls is desirable. This fundamental idea is elaborated on by scholars like Wray (2015) and Kelton (2020).

Additionally, it is essential to understand the basic concept of the investment-savings (IS) balance, which is often overlooked by the general public. In simple terms, when considering the domestic IS balance, if the government's fiscal balance "deteriorates," causing the deficit to expand, it will lead to an equal expansion of the private sector's surplus. This can be seen clearly when observing the sharp increase in government fiscal deficits through measures like cash transfers during and after the COVID-19 pandemic, alongside a significant increase in the private sector's surplus. It is important to note that when the government "borrows," it is borrowing from its own citizens. The explanation above is evident when we observe the IS balances in the US and Japan, as shown in Figures 1 and 2. Even if the government borrows from overseas, as long as the debt is denominated in its own currency, the government cannot default. Moreover, since citizens are required to pay taxes in the currency designated by the government, the idea of a loss of "currency credibility," as commonly believed, is not plausible.

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[1] Ministry of Finance (2023), "Japanese Public Finance Fact Sheet" https://www.mof.go.jp/english/policy/budget/budget/fy2023/02.pdf

[2] The White House (2023), "The Potential Economic Impacts of Various Debt Ceiling Scenarios", https://www.whitehouse.gov/cea/written-materials/2023/05/03/debt-ceiling-scenarios/

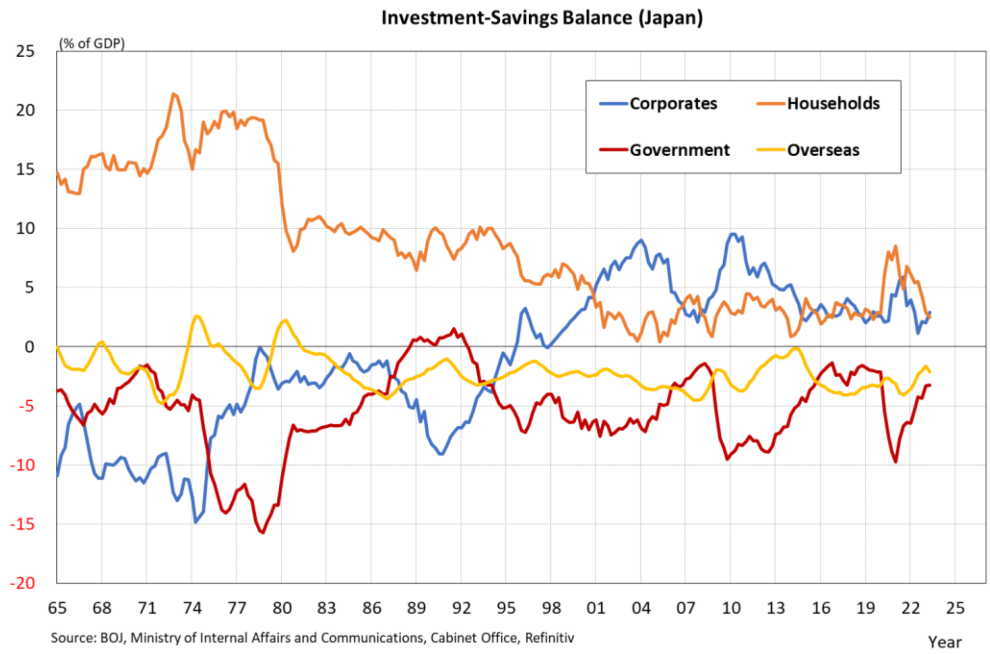

Figure 1

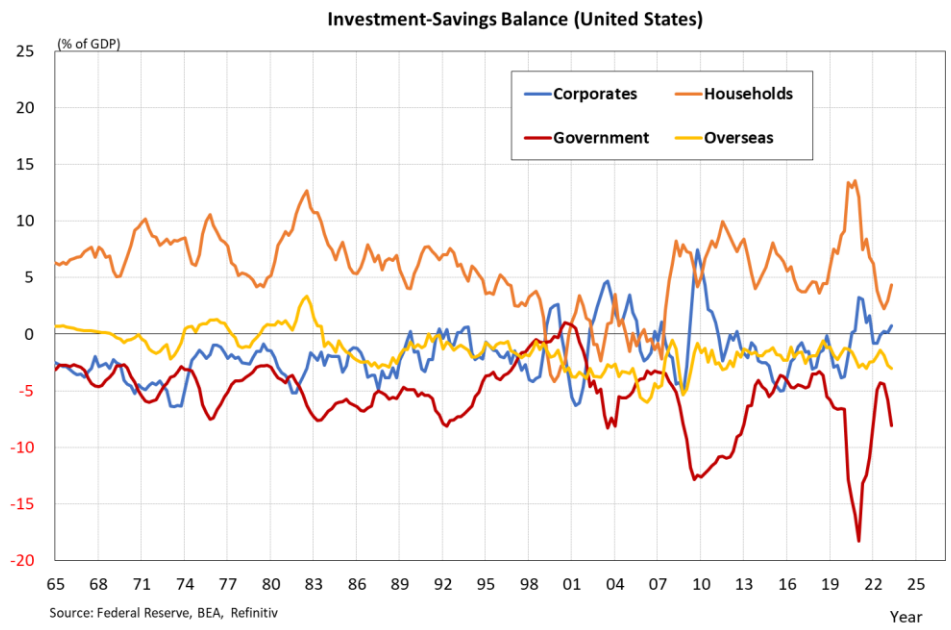

Figure 2

In any case, to give more credibility to the ideas of MMT, it is essential not only to rely on conceptual approaches, but also to use mathematical equations to demonstrate that under MMT-based fiscal and monetary policies, the economy can achieve a stable equilibrium rather than an unstable situation where inflation rates diverge. A common criticism of MMT, such as that presented by Globerman (2020) and Mankiw (2019), is that controlling inflation is challenging, making it difficult for the government to control inflation. Therefore, this paper aims to provide evidence for the attainment of such stability.

Academic literatures that focus on MMT includes Tanaka (2021), in which the author analyzes models incorporating functional fiscal policies of MMT while utilizing microeconomic and neoclassical frameworks to examine consumer utility maximization with utility functions and budget constraints and profit maximization for firms under monopolistic competition. The idea of functional fiscal policies, or "functional finance", the term first described by Abba Lerner, is considered as the root of MMT. Lerner (1943) explains it "The principle of judging fiscal measures by the way they work or function in the economy", "not to any established traditional doctrine about what is sound or unsound". In contrast to Tanaka (2021), Asada (2020) investigates stability and instability of equilibrium points based on coordinated financial and fiscal policies by using dynamic Keynesian model.

This paper is inspired by Asada (2020) and Asada, Demetrian, Zimka and Zimková (2024) that used a four dynamic equation model consisting of fiscal policy, inflation expectations, money growth rate, and employment rate, in a dynamic Keynesian model incorporating MMT ideas, with active fiscal policy and passive monetary policy in financing the government debt. The study concluded that if the policy authorities pursue proactive fiscal and monetary policies and can garner credibility in their inflation target, it is possible to stabilize the economy. We aim to achieve the same conclusion by simplifying the model discussed above into a three-variable (fiscal policy, inflation expectations, and money growth rate) framework, as utilized in Asada (2020). If we can obtain the same result, the significance of this paper lies in the ability to demonstrate the effectiveness of MMT-style policies in a more concise manner.

We employ a three dynamic equation model (fiscal policy, inflation expectations, and money growth rate) and analyze stability and instability at equilibrium points. Additionally, the paper explores whether the same conclusions can be reached when applying dynamic optimization techniques.

The paper is structured as follows: Section II defines the variables and presents the model's construction of the goods and money markets. In Section III, the induced system of dynamic equations is presented. Section IV analyzes the characteristics at the long-term equilibrium point. Section V examines the stability and instability of equilibrium points. Section VI considers the economic and policy implications of the previous analysis. Section VII proposes an alternative approach, with using dynamic optimization. Finally, Section VIII gives concluding remarks.

II. Formulation of the Model of the Goods and Money Markets

In this chapter, we will explain the framework and nature of this paper. The symbols used are as follows, and the dot above a symbol denotes a derivative with respect to time.

$Y =$ real national income (real output). $K =$ real capital stock. $y = \frac{Y}{K} =$ output-capital ratio. $C =$ consumption expenditure. $c =$ marginal propensity to consume. $I =$ real private investment expenditure. $i = \frac{I}{K} =$ rate of investment. $G =$ real government expenditure. $g = \frac{G}{K} =$ government expenditure-capital ratio. $\rho =$ nominal rate of interest of public bonds. $p =$ price level. $T =$ real income tax. $B =$ nominal stock of public debt (public bond). $M =$ nominal money supply. $m = \frac{M}{pK}$ = money-capital ratio.

Here, we start with the equilibrium condition of the goods market.

$$Y = C + I + G \tag{1}$$

where,

$$C = c\left[Y + \rho\left(\frac{B}{P}\right) - T\right] \ ; \ 0 < c < 1 \tag{2}$$

$$T = \tau\left[Y + \rho\left(\frac{B}{P}\right)\right] \ ; \ 0 < \tau < 1 \tag{3}$$

We derive Y of Eq(4) by substituting Eq(2) and Eq(3) into Eq(1).

$$Y = \frac{1}{1 - c(1-\tau)}\left[c(1-\tau)\rho\left(\frac{B}{P}\right) + I + G\right] \tag{4}$$

We divide both sides of Eq(4) by K. This is the expanded version of the equilibrium condition of the goods market, which we may call as an IS equation.

$$y = \frac{1}{1 - c(1-\tau)}[c(1-\tau)\rho(y,m)b + i(\rho(y,m) - \pi^e) + g]$$

$$= y(g, m, b, \pi^e) \tag{5}$$

Nominal interest rate is derived by LM equation.

$$m = L(y,\rho)\ ;\ L_y \geq 0, L_y \leq 0$$

$$\rho = \rho(y,m);\ \rho_m \geq 0\ , \rho_m \geq 0 \tag{6}$$

We determine the characteristics of each variable through partial differentiations of Eq(5) with respect to each variable. Partial derivatives are given below.

$$y_g = \frac{dy}{dg} = \frac{1}{1-c(1-\tau)-\rho_y(c(1-\tau)b+i_{\rho-\pi^e})} > 0 \tag{7}$$

$$y_m = \frac{dy}{dm} = \frac{\rho_m\left[cb(1-\tau)+i_{\rho-\pi^e}\right]}{1-c(1-\tau)-\rho_y(c(1-\tau)b+i_{\rho-\pi^e}} \geq 0 \tag{8}$$

$$y_{\pi^e} = \frac{dy}{d\pi^e} = -\frac{i_{\rho-\pi^e}}{1-c(1-\tau)-\rho_y(c(1-\tau)b+i_{\rho-\pi^e})} > 0 \tag{9}$$

$$y_b = \frac{dy}{db} = \frac{c(1-\tau)\rho(y,m)}{1-c(1-\tau)-\rho_y(c(1-\tau)b+i_{\rho-\pi^e})} \geq 0 \tag{10}$$

III. Dynamic Equations

In this section, we derive the dynamic equations for each variable. Below are some of the definitional equations.

$$\dot{K} = I \tag{11}$$

$$\frac{\dot{K}}{K} = \frac{I}{K} = i(\rho-\pi^e);\ i_{\rho-\pi^e} = \frac{\partial i}{\partial(\rho-\pi^e)} < 0 \tag{12}$$

$$\frac{\dot{P}}{P} = \pi \tag{13}$$

$$\pi = \varepsilon(y-\bar{y}) + \pi^e \quad ; \varepsilon > 0 \tag{14}$$

$$\frac{\dot{M}}{M} = \mu \tag{15}$$

Eq(11) is the rate of capital accumulation that is equivalent to investment expenditure. Eq(12) is the investment function of firms, which is based on the standard Keynesian theory[3]. This equation shows that investment is the decreasing function of the expected real rate of interest. Eq(13) represents the growth rate of price equals inflation rate. Eq(14) is the conventional linear "expectations-augmented Phillips curve", which ε is the reaction parameter from the output gap. $\bar{y}$ is the natural output-capital ratio, and π^e is inflation expectation variable. Eq(15) shows the growth rate of money.

[3] See Keynes (1936), Asada and Ouchi (2015).

Using Eq(12) , (13) ,(15) and money-capital ratio m, dynamic law of the motion of the money-capital ratio can be described as below.

$$\frac{\dot{m}}{m} = \frac{\dot{M}}{M} - \frac{\dot{P}}{P} - \frac{\dot{K}}{K} = \mu - \pi - i(\rho - \pi^e) \tag{16}$$

Eq(16) can be rewritten as follows.

$$\dot{m} = \mu m - \pi m - i(\rho - \pi^e)m \tag{17}$$

We can substitute above variables with equations already mentioned.

$$\dot{m} = [\mu + \varepsilon(\bar{y} - y(g, m, b, \pi^e)) - \pi^e - i(\rho(y(g, m, b, \pi^e), m) - \pi^e)]m \tag{18}$$

We formulate below the dynamic equation of government's fiscal policy, which follows MMT's doctrine of attaining full employment and price stability[4]. Unlike Asada(2020), we use output-capital ratio as a surrogate variable of the rate of employment.

$$\dot{g} = \alpha_1(\bar{y} - y) + \alpha_2\{\bar{\pi} - \pi\} \ ; \ (\alpha_1 > 0,\ \alpha_2 > 0, \bar{\pi} > 0)$$
$$\dot{g} = \alpha_1\big(\bar{y} - y(g, m, b, \pi^e)\big) + \alpha_2\{\bar{\pi} - \varepsilon[y(g, m, b, \pi^e) - \bar{y}] - \pi^e\} \tag{19}$$

Also, the dynamic equation of inflation expectation is described as below. This equation represents the dynamics of inflation expectations by capturing the distinction between forward-looking and backward-looking expectation formations using θ . If θ is close to 1, it signifies a forward-looking expectation formation with a strong influence from the inflation target, whereas if it is closer to zero, it indicates a backward-looking expectation formation influenced by the actual inflation rate. Therefore, θ can be referred to as the "credibility parameter" with respect to the policy authority's inflation target. This formulation originates from Asada, Chiarella, Flaschel and Franke (2012) and Asada (2020).

$$\dot{\pi}^e = \gamma[\theta(\bar{\pi} - \pi^e) + (1 - \theta)(\pi - \pi^e)] \ ; \ (\gamma > 0, 0 \leq \theta \leq 1)$$
$$\dot{\pi}^e = \gamma\{\theta(\bar{\pi} - \pi^e) + (1 - \theta)\big[\varepsilon[y(g, m, b, \pi^e) - \bar{y}]\big]\} \tag{20}$$

From the above dynamic equations, we can rewrite as below.

$$\dot{g} = F_1(g, m, b, \pi^e)$$

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[4] See Wray (1998), Wray (2015), Kelton (2020).

$$\dot{m} = F_2(g, m, b, \pi^e)$$
$$\dot{\pi}^e = F_3(g, m, b, \pi^e) \quad (21)$$

Next, we formulate the equation that also symbolizes the characteristic of MMT. Eq(22) below is the "budget constraint" of the government, implying that the government deficit must be financed through the issuance of new high-powered money (monetary base) or government bonds. We acknowledge that the MMT proponents do not have the idea of "budget constraint" of government since they can always print their own money to finance the debt, but we still think this equation sufficiently describes their basic idea. The formulation is based on Asada (2020) and Mitchell, Wray, and Watts (2019).

$$\dot{H} + \dot{B} = PG + \rho B - PT \quad (22)$$

Where, $H =$ high-powered money (monetary base). $B =$ nominal stock of public debt (public bond). This simply tells that the government expenditure including the interest payment of government bond is financed with the issuance of government bonds, high-powered money, or tax.

Here, we substitute below equations to Eq(22) and obtain Eq(25).

$$T = \tau(Y + \rho \tfrac{B}{P}) \quad (23)$$
$$PT = \tau(pY + \rho B) \quad (24)$$
$$\dot{H} + \dot{B} = PG + \rho B - \tau(PY + \rho B) \quad (25)$$

Also, money supply is described below, which is the product of high-powered money and ν, which is the money multiplier (constant). Differentiation with respect to time is shown in Eq (27).

$$M = \nu H \quad ; \nu > 1 \quad (26)$$
$$\dot{M} = \nu \dot{H}$$
$$\dot{H} = \frac{1}{\nu}\dot{M}$$
$$= \tfrac{1}{\nu}\mu M \quad (27)$$

Eq(28) is the growth rate of nominal stock of public debt.

$$\mu_B = \frac{\dot{B}}{B} \tag{28}$$

Eq(25) can be rewritten as below.

$$\frac{1}{v}\mu M + \mu_B B = PG + \rho B - \tau(PY + \rho B) \tag{29}$$

We divide above equation by price and capital to transform into a real term.

$$\frac{1}{v}\mu\left(\frac{M}{PK}\right) + \mu_B\left(\frac{B}{PK}\right) = \frac{G}{K} + \rho\left(\frac{B}{PK}\right) - \tau\left(\frac{Y}{K} + \rho\frac{B}{PK}\right)$$

$$\frac{1}{v}\mu m + \mu_B b = g + (1-\tau)\rho b - \tau y \tag{30}$$

We assume that the government controls the issuance of public debt, which also means that they could keep the level of bconstant, as described in Eq(31). Transformation of this equation is shown in Eq(32).

$$b = \frac{B}{PK} = \bar{b} \tag{31}$$

$$0 = \frac{\dot{b}}{b} = \frac{\dot{B}}{B} - \frac{\dot{P}}{P} - \frac{\dot{K}}{K} = \mu_B - \pi - i(\rho - \pi^e) = \mu_B - \varepsilon(y - \bar{y}) - \pi^e - i(\rho - \pi^e)$$

$$\mu_B = \varepsilon(y - \bar{y}) + \pi^e + i(\rho - \pi^e) \tag{32}$$

We can insert Eq(32) into Eq(30).

$$\frac{1}{v}\mu m + [\varepsilon(y - \bar{y}) + \pi^e + i(\rho - \pi^e)]\bar{b} = g + (1-\tau)\rho b - \tau y$$

$$\mu m = v\Big\{g + (1-\tau)\rho\big[y\big(g, m, \bar{b}, \pi^e\big), m\big]\bar{b} - \tau y\big(g, m, \bar{b}, \pi^e\big)$$

$$- \Big[\Big[\varepsilon\big(y\big(g, m, \bar{b}, \pi^e\big) - \bar{y}\big) + \pi^e + i\big(\rho(y\big(g, m, \bar{b}, \pi^e\big)) - \pi^e\big)\Big]b\Big]\Big\} \tag{33}$$

Thus, we now have three dynamic equations, which are real money-capital ratio, real government expenditure-capital ratio, and inflation expectation, respectively from Eq(18), (19), (20), together with the "budget constraint" equation (33).

$$\dot{m} = \nu\Big\{g + (1-\tau)\rho\big[y\big(g,m,\bar{b},\pi^e\big),m\big]\bar{b} - \tau y\big(g,m,\bar{b},\pi^e\big) - \Big[\big[\varepsilon\big(y\big(g,m,\bar{b},\pi^e\big) - \bar{y}\big) + \pi^e + i\big(\rho\big(y\big(g,m,\bar{b},\pi^e\big)\big) - \pi^e\big)\big]b\Big]\Big\} - \big[\varepsilon\big(y\big(g,m,\bar{b},\pi^e\big) - \bar{y}\big) + \pi^e\big]m - i\Big[\rho\Big(y\big(g,m,\bar{b},\pi^e\big)\Big) - \pi^e\Big]m = F_1(m,g,\pi^e)$$

$$\dot{g} = \alpha_1\big(\bar{y} - y(g,m,b,\pi^e)\big) + \alpha_2\{\bar{\pi} - \varepsilon\big[y\big(g,m,\bar{b},\pi^e\big) - \bar{y}\big] - \pi^e\} = F_2(m,g,\pi^e)$$

$$\dot{\pi}^e = \gamma\Big\{\theta(\bar{\pi} - \pi^e) + (1-\theta)\Big[\varepsilon\big[y\big(g,m,\bar{b},\pi^e\big) - \bar{y}\big]\Big]\Big\} = F_3(m,g,\pi^e)$$

(34)

IV. Characteristics of the Long-run Equilibrium Solution

We now turn to identifying the characteristics of each dynamic equations' long-run equilibrium solution. The long-run equilibrium of the equations in Eq(34) are written as

$$\dot{m} = \dot{g} = \dot{\pi}^e = 0\,. \tag{35}$$

By substituting this to the dynamic equations in Eq(34), we attain the long-run equilibrium of the system as follows.

$$\pi^e = \bar{\pi} = \pi^* \tag{36}$$

$$y(g,m,\bar{b},\pi^e) = \bar{y} = y^* \tag{37}$$

The left hand side of Eq(37) is equivalent to Eq(5). The total differentiation of the equations $\dot{m} = F_1(m,g,\bar{\pi}) = 0$ and IS equation of Eq(37), $F_3(m,g,\bar{\pi}) = 0$ are shown below, using the unknown variables of g and m.

$$dF_1 = \left(\frac{\partial F_1}{\partial g}\right)^* dg + \left(\frac{\partial F_1}{\partial m}\right) dm = 0\,; \quad \left.\frac{dm}{dg}\right|_{F_1=0} = \left(\frac{\left(\frac{\partial F_1}{\partial g}\right)}{\left(\frac{\partial F_1}{\partial m}\right)}\right) > 0$$

Where $\left(\frac{\partial F_1}{\partial g}\right) = \nu > 0$ and $\frac{\partial F_1}{\partial m} = \rho_m\big[(1-\tau)\bar{b} - i_{\rho-\bar{\pi}}\big] - \bar{\pi} - i_{\rho-\bar{\pi}}\rho_m m - i(\rho(\bar{y},m) - \bar{\pi}) < 0$

(37)

$$dF_3 = \left(\frac{\partial F_3}{\partial g}\right) dg + \left(\frac{\partial F_3}{\partial m}\right) dm = 0\,; \quad \left.\frac{dm}{dg}\right|_{F_3=0} = -\left(\frac{\left(\frac{\partial F_3}{\partial g}\right)}{\left(\frac{\partial F_3}{\partial m}\right)}\right) < 0$$

(38)

The partial derivatives $\frac{\partial F_3}{\partial g}$ and $\frac{\partial F_3}{\partial m}$ are already solved in Eq(7) and (8).

The image of equilibrium is shown below in Figure 3.

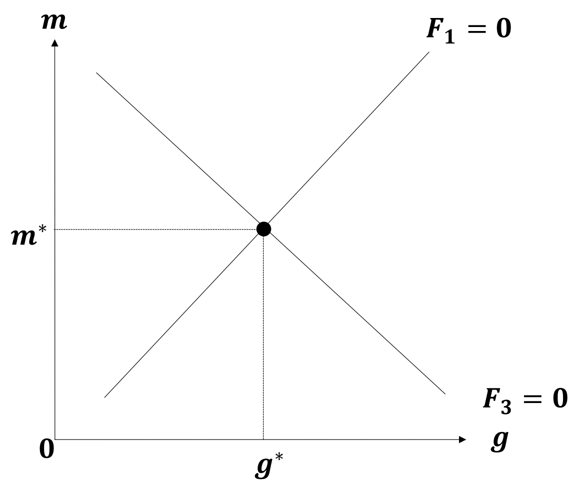

Figure 3

V. Local Stability/Instability of the Long-run Equilibrium Point

In this section, we define the local stability/instability of the long-run equilibrium point. We have the following Jacobian matrix of the three-dimensional dynamic system at the equilibrium point. The approach/methodology was inspired by the past study, such as Asada (2024), Asada and Semmler (1995), Asada et al. (2003), and Asada and Ouchi (2009). The Jacobian matrix of the system (34) at the equilibrium point becomes as follows.

$$J = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \tag{39}$$

The partial derivatives of this system are shown below. After the derivation, long-run equilibrium conditions of Eq(36) and (37) are substituted. Here, we assume the economy is facing a "liquidity trap", in which the economy is stagnant and interest rate is too low so that the increase in money supply is ineffective to nominal interest rate and output. Therefore, we have $\rho_m \approx 0$, $y_m \approx 0$ and $\rho_y \approx 0$.

$$\begin{aligned} F_{11} &= \nu\{(\rho_y y_m + \rho_m)\bar{b}[(1-\tau) - i_{\rho-\pi^e}] - y_m(\tau + \varepsilon\bar{b})\} - m[\varepsilon y_m + i_{\rho-\pi^e}(\rho_y y_m + \rho_m)] \\ &\quad - [\varepsilon(y(g, m, \bar{b}, \pi^e) - \bar{y}) + \pi^e] - i[\rho(y(g, m, \bar{b}, \pi^e), m) - \pi^e] \\ &\approx -\bar{\pi} - i[\rho(y(g, m, \bar{b}, \pi^e), m) - \bar{\pi}] < 0 \end{aligned} \tag{40}$$

We assume here the investment rate, $i[\rho(y(g, m, \bar{b}, \pi^e), m) - \bar{\pi}]$ is sufficiently small, because of the stagnant economy.

$$F_{12} = v\{1 + y_g[-\varepsilon - \tau - i_{\rho-\pi^e}]\} - m(y_g\varepsilon) \tag{41}$$

$$F_{13} = v\{-\tau y_{\pi^e} + \bar{b}[\rho_y y_{\pi^e}[(1-\tau) - i_{\rho-\pi^e}] - \varepsilon y_{\pi^e} - 1 + i_{\rho-\pi^e}]\} - m[y_{\pi^e}(\varepsilon + i_{\rho-\pi^e}\rho_y) + 1 - i_{\rho-\pi^e}] < 0 \tag{42}$$

$$F_{21} = -\alpha_1 y_m - \alpha_2 \varepsilon y_m < 0 \tag{43}$$

$$F_{22} = -\alpha_1 y_g - \alpha_2 \varepsilon y_g < 0 \tag{44}$$

$$F_{23} = -\alpha_1 y_{\pi^e} - \alpha_2(\varepsilon y_{\pi^e} + 1) < 0 \tag{45}$$

$$F_{31} = \gamma[(1-\theta)\varepsilon y_m] > 0 \tag{46}$$

$$F_{32} = \gamma[(1-\theta)\varepsilon y_g] > 0 \tag{47}$$

$$F_{33} = \gamma[-\theta + (1-\theta)\varepsilon y_{\pi^e}] > 0 \tag{48}$$

The characteristic equation of the Jacobian matrix is shown below.

$$\Gamma(\lambda) \equiv |\lambda I - J| = \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0\,, \tag{49}$$

where

$$a_1 = -traceJ_3 = -F_{11}(\varepsilon) - F_{22}(\alpha_1, \alpha_2) - F_{33}(\gamma, \theta)\ , \tag{50}$$

$$a_2 = sum\ of\ all\ principal\ second\ order\ minors\ of\ J_3$$

$$= \begin{vmatrix} F_{11}(\varepsilon) & F_{12}(\varepsilon) \\ F_{21}(\alpha_1,\alpha_2) & F_{22}(\alpha_1,\alpha_2) \end{vmatrix} + \begin{vmatrix} F_{11}(\varepsilon) & F_{13}(\varepsilon) \\ F_{31}(\gamma,\theta) & F_{33}(\gamma,\theta) \end{vmatrix} + \begin{vmatrix} F_{22}(\alpha_1,\alpha_2) & F_{23}(\alpha_1,\alpha_2) \\ F_{32}(\gamma,\theta) & F_{33}(\gamma,\theta) \end{vmatrix} \tag{51}$$

$$a_3 = -detJ_3 = -\begin{vmatrix} F_{11}(\varepsilon) & F_{12}(\varepsilon) & F_{13}(\varepsilon) \\ F_{21}(\alpha_1,\alpha_2) & F_{22}(\alpha_1,\alpha_2) & F_{23}(\alpha_1,\alpha_2) \\ F_{31}(\gamma,\theta) & F_{32}(\gamma,\theta) & F_{33}(\gamma,\theta) \end{vmatrix}\ . \tag{52}$$

The stability condition of this system due to Routh-Hurwitz conditions for a three-dimensional system is that all roots of the characteristic equation Eq(49) have negative real parts if and only if the set of inequalities

$$a_1 > 0,\ a_3 > 0, a_1 a_2 - a_3 > 0\ \ , \tag{53}$$

is satisfied.

[Proposition 1]

Suppose that the parameter values θ and α_1, α_2 are sufficiently small and γ is sufficiently large. The equilibrium point of the dynamic system Eq(34) is then unstable.

[Proof]

When θ is close to zero, F_{33} turns out to become positive.

$$F_{33} = \gamma[\varepsilon y_{\pi^e}] > 0 \tag{54}$$
$$\qquad (+)$$

In this case, one of the stability conditions would not be satisfied.

$$a_1 = -traceJ_3 = -F_{11}(\varepsilon) - F_{22}(\alpha_1, \alpha_2) - F_{33}(\gamma, \theta) \tag{55}$$
$$\qquad (\text{-}) \qquad (\text{-}) \qquad (+)$$

Additionally, when α_1, α_2 are also sufficiently small and γ is sufficiently large, from Eq(44) and (54), it is more certain that a_1 would be negative. □

[Proposition 2]

Suppose that the parameter values θ is close to 1, α_1, α_2 and γ are sufficiently large. The equilibrium point of the dynamic system Eq(34) is then stable.

[Proof]

Under this condition, we can figure that all of the roots of the characteristic equation Eq(49) have negative real parts.

$$a_1 = -traceJ_3 = -F_{11}(\varepsilon) - F_{22}(\alpha_1, \alpha_2) - F_{33}(\gamma, \theta) > 0 \tag{56}$$
$$\qquad (\text{-}) \qquad (\text{-}) \qquad (\text{-})$$

$$a_2 = \begin{vmatrix} (-) & (+) \\ (-) & (-) \end{vmatrix} + \begin{vmatrix} (-) & (-) \\ 0 & (-) \end{vmatrix} + \begin{vmatrix} (-) & (-) \\ 0 & (-) \end{vmatrix} = (+) - (-) + (+) + (+) > 0 \tag{57}$$

$$a_3 = -\begin{vmatrix} (-) & (+) & (-) \\ (-) & (-) & (-) \\ 0 & 0 & (-) \end{vmatrix} = -\big((-) - (+)\big) > 0 \tag{58}$$

We consider below whether $a_1a_2 - a_3 > 0$.

$$a_1a_2 - a_3 = [-F_{11}(\varepsilon) - F_{22}(\alpha_1, \alpha_2) - F_{33}(\gamma, \theta)][F_{11}(\varepsilon)F_{22}(\alpha_1, \alpha_2) - F_{12}(\varepsilon)F_{21}(\alpha_1, \alpha_2) + F_{11}(\varepsilon)F_{33}(\gamma, \theta) - F_{13}(\varepsilon)F_{31}(\gamma, \theta) + F_{22}(\alpha_1, \alpha_2)F_{33}(\gamma, \theta) - F_{23}(\alpha_1, \alpha_2)F_{32}(\gamma, \theta)] - [F_{11}(\varepsilon)F_{22}(\alpha_1, \alpha_2)F_{33}(\gamma, \theta) - F_{12}(\varepsilon)F_{21}(\alpha_1, \alpha_2)F_{33}(\gamma, \theta)] \tag{59}$$

We organize above equation to gather equations with α_1, α_2.

$$a_1a_2 - a_3 = -F_{22}(\alpha_1, \alpha_2)^2F_{11}(\varepsilon) + F_{22}(\alpha_1, \alpha_2)F_{12}(\varepsilon)F_{21}(\alpha_1, \alpha_2) - F_{22}(\alpha_1, \alpha_2)^2F_{33}(\gamma, \theta) + F_{22}(\alpha_1, \alpha_2)F_{23}(\alpha_1, \alpha_2)F_{32}(\gamma, \theta) + G(\alpha_1, \alpha_2, \gamma, \theta) \tag{60}$$

We apply each equation previously attained to Eq (57).

$$a_1a_2 - a_3 = \alpha_1{}^2\left[-y_m{}^2F_{11}(\varepsilon) + y_gy_mF_{12}(\varepsilon) - y_g{}^2F_{33}(\gamma, \theta)\right] + \alpha_2{}^2\left[-\varepsilon^2y_m{}^2F_{11}(\varepsilon) - \varepsilon^2y_g{}^2F_{33}(\gamma, \theta)\right] + \alpha_1\alpha_2\left[-2y_m{}^2\varepsilon F_{11}(\varepsilon) + 2y_gy_m\varepsilon F_{12}(\varepsilon) - 2y_g{}^2\varepsilon F_{33}(\gamma, \theta)\right] > 0$$

For instance, parameter value of θ close to 1 and sufficiently large parameter value of γ in $F_{33}(\gamma,\theta)$ would work to exceed the other negative values. □

Also, we can point out that even in the case when either of the parameter value of α_1 (α_2) is fixed, sufficiently large value of the other $\alpha_2(\alpha_1)$ can stabilize the system, since those are squared.

VI. Policy Implications / Economic Interpretations

Here, we consider the economic and policy implications of the analysis conducted thus far. First, this analysis is conducted under the premise of an economic situation where economic activity is stagnant and trapped in a liquidity trap, as mentioned earlier. In Proposition 1, it is assumed that the parameter θ is small (close to zero), α_1 ・ α_2 are also small, and γ has a large value. This implies that people have backward-looking inflation expectations (adaptive expectations) with a high degree of responsiveness to these expectations. At the same time, the responsiveness to fiscal policy concerning demand and inflation gaps (deviations from the inflation target) is low. In the first place, under these underlying conditions, it cannot be said that the fiscal policy stance is of a MMT nature, which implies not being preoccupied with factors like employment.

Consequently, the long-term equilibrium point is shown to be unstable in such a situation, which means growth of money and inflation expectation of the public could diverge indefinitely. Some individuals may refer to this kind of situation as hyperinflation, and prominent scholar Hyman Minsky describes it as an open-type inflation, where both nominal wages and prices continue to rise[5]. Due to the lack of confidence in the integrated government's inflation target and the significant impact of backward-looking expectations, even in the presence of demand deficiency, the government may not sufficiently implement fiscal policy.

On the other hand, as seen in Proposition 2, under the assumption that the parameter θ is

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[5] Minsky (1986) points out that the condition for becoming an open-ended inflation is the removal of the inflation barrier through "the existence of large and growing demands for consumer goods that are financed outside of wage incomes received in direct productive channels."

close to 1, $\alpha_1 \cdot \alpha_2$, and γ have large values, it suggests that people have faith in the inflation target and have strong responsiveness to inflation expectations concerning deviations from the target. Additionally, the responsiveness to fiscal policy concerning demand and inflation gaps is high.

Therefore, if the integrated government commits clearly to the inflation target and implements proactive fiscal policy to address demand deficiency, the long-term equilibrium point becomes stable.

Abba Lerner, who is considered a foundational figure in MMT, stated in Lerner (1943) that, "*as the national debt increases it acts as a self-equilibrating force, gradually diminishing the further need for its growth and finally reaching an equilibrium level where its tendency to grow comes to a complete end. The greater the national debt, the greater the quantity of private wealth*." He also pointed out that, "*an increase in private spending makes it less necessary for the government to undertake deficit financing to maintain total spending at a level that ensures full employment.*" This, too, represents a simple yet crucial perspective for the possibility of the government achieving full employment through fiscal policy while simultaneously controlling inflation and stabilizing the economy.

VII. Alternative Approach: Dynamic Optimization

The analysis results in this study show that an economic model incorporating MMT elements can achieve a long-term equilibrium solution. However, an alternative approach using dynamic optimization methods could also be considered. Through a dynamic optimization approach, we might be able to derive a same conclusion, with government's utility maximization under certain constraints on fiscal policy management. For instance, considering an analysis using the Hamiltonian in the optimal control theory, the following equation could be considered, inheriting the equations we have covered in this paper:

$$\max_{g(t)} \int_0^\infty -\{\xi(\pi-\bar{\pi})^2+(1-\xi)(y-\bar{y})^2\}e^{-rt}\,dt$$

$$= \max_{g(t)} \int_0^\infty -\Big\{\xi\big[\varepsilon\big(y(g,m,\bar{b},\pi^e)-\bar{y}\big)+\pi^e-\bar{\pi}\big]^2$$

$$-(1-\xi)\big(y(g,m,\bar{b},\pi^e)-\bar{y}\big)^2\Big\}e^{-rt}\,dt\,;\;\; 0<\xi<1$$

$s.t.$

$$\dot{m} = \nu\Big\{g+(1-\tau)\rho\big[y(g,m,\bar{b},\pi^e),m\big]\bar{b}-\tau y(g,m,\bar{b},\pi^e)$$

$$-\Big[\big[\varepsilon\big(y(g,m,\bar{b},\pi^e)-\bar{y}\big)+\pi^e+i\big(\rho(y(g,m,\bar{b},\pi^e))-\pi^e\big)\big]b\Big]\Big\}$$

$$-\big[\varepsilon\big(y(g,m,\bar{b},\pi^e)-\bar{y}\big)+\pi^e\big]m-i\Big[\rho\Big(y\big(g,m,\bar{b},\pi^e\big)\Big)-\pi^e\Big]m$$

$$= f_1(g(t),m(t),\pi^e(t))$$

$$\dot{\pi}^e = \gamma\big\{\theta(\bar{\pi}-\pi^e)+(1-\theta)\big[\varepsilon[y(g,m,b,\pi^e)-\bar{y}]\big]\big\}=f_2(g(t),m(t),\pi^e(t))$$

(59)

Current value Hamiltonian and Pontryagin's maximum principle conditions[6] are described below, where ϕ_1 and ϕ_2 are the costate variables corresponding to two dynamic constraints in Eq(59).

$$H = -\xi\big[\varepsilon\big(y(g,m,\bar{b},\pi^e)-\bar{y}\big)+\pi^e-\bar{\pi}\big]^2-(1-\xi)\big(y(g,m,\bar{b},\pi^e)-\bar{y}\big)^2$$
$$+\phi_1(t)f_1\big(g(t),m(t),\pi^e(t)\big)+\phi_2(t)f_2(g(t),m(t),\pi^e(t))$$

$$\operatorname*{Max}_{g(t)} H(g,m,\pi^e)\; for\; all\; t\in[0,\infty]$$

$$\dot{m}(t)=\frac{\partial H(t)}{\partial\phi_1(t)}=f_1(g(t),m(t),\pi^e(t))\quad [equation\; of\; motion\; for\; m]$$

$$\dot{\pi}^e=\frac{\partial H(t)}{\partial\phi_2(t)}=f_2(g(t),m(t),\pi^e(t))\qquad [equation\; of\; motion\; for\; \pi^e]$$

$$\dot{\phi}_1(t)=-\frac{\partial H(t)}{\partial m(t)}+r\phi_1(t)\qquad [equation\; of\; motion\; for\; \phi_1]$$

$$\dot{\phi}_2(t)=-\frac{\partial H(t)}{\partial \pi^e(t)}+r\phi_2(t)\qquad [equation\; of\; motion\; for\; \phi_2]$$

6 Explanation on Pontryagin's maximum principle conditions and dynamic optimization are available from Chiang (1992), Chiang and Wainwright (2005)

$$\lim_{t\to\infty} \phi_1 e^{-rt} = 0\,, \quad \lim_{t\to\infty} \phi_2 e^{-rt} = 0 \qquad [transversality\ conditions]$$

(60)

Here, the deviation from the inflation target and the demand-supply gap are integrated as the government's loss function, which represents the objective function to maximize utility by minimizing it. The idea is similar to the works of Matsumoto (2023a), Matsumoto (2023b), and Taylor (1989). Solving these equations would then verify the dynamic stability of the long-term equilibrium point. If dynamic systems' stability can be achieved using such methods, it would add credibility to MMT-based fiscal policy management as a realistic economic policy. Combining the findings of this study with those of Asada (2020) and others could lead to a certain level of persuasion. This should be considered as a topic for future research.

VIII. Concluding Remarks

In this paper, considering the inclusion of monetary policy, the original essence of MMT's perspective, which primarily focuses on fiscal policy, takes on a slightly different nuance. The conclusion reached is that gaining confidence in achieving the inflation target is one of the conditions for stabilizing dynamic systems. However, in reality, financial markets and others pay attention to central bank inflation targets and inflation trends. Given that the conclusion of this paper has resulted in the importance of an active fiscal policy stance, it is unlikely to deviate significantly from the essence of MMT or the key points it advocates.

What proponents of MMT assert is that in times of deficient demand, the government can sustain full employment by directly increasing the number of employments, a concept known as the Job Guarantee Program (JGP). However, when delving into concrete policies like this, other aspects such as the practicality of policy implementation come into consideration. Nevertheless, the crux of MMT lies in the notion that a government with the sovereign authority to issue its own currency will never exhaust its budget until it faces inflation due to supply shortages, making it perpetually capable of addressing demand deficiencies. Nonetheless, as previously mentioned, a common critique of MMT is that an excessively aggressive fiscal policy could render inflation uncontrollable. In contrast, demonstrating mathematically, as in this paper, that inflation remains controllable even with the adoption of MMT-like policies represents an important albeit imperfect stride.

Prominent economists, like Krugman (2019) and Summers (2019), have expressed critical views of MMT, and it cannot be readily affirmed that MMT garners widespread support in the general public. However, considering the actual scenario in the US and Japan where substantial government debt levels did not lead to high inflation until the emergence of the Covid-19 pandemic, it can be argued that MMT's assertions hold sway up to a certain extent. Nevertheless, there are perspectives, as reflected by Mackintosh (2021), that acknowledge a concordance between the real-world situation and MMT's contentions. To further establish the credibility of MMT's arguments, it will be crucial to progressively accumulate scholarly assessments over a period of time.

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