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A Dynamic Optimization Approach to MMT-type Coordinated Fiscal and Monetary Stabilization Policy

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Abstract

In this paper, we aim to construct a dynamic Keynesian model incorporating the concepts of Modern Monetary Theory (MMT), a fiscal-dominant economic theory, and to use dynamic optimization to derive an optimal fiscal and monetary policy path. MMT is characterized by the dominance of fiscal policy over monetary policy and consider macroeconomic environment (inflation) being the limit of fiscal expansion, rather than a government debt ceiling, or a “sound” budget balance. We incorporate these MMT elements into the model, referring to previous studies. Ultimately, we confirm stability/instability at the equilibrium point and consider the validity of application to the fiscal and monetary policies in the real world. The conclusion suggests 1.) forward-looking environment where economic agents trust their central bank policy 2.) inflation target is sufficiently high, and 3.) nominal interest is sufficiently low. Under these economic conditions, the dynamic system converges to an equilibrium point, while confirming the point is “stable” and “determinate”. Stability implies that the economy settles at the equilibrium point without experiencing overheating or stagnation, and money supply and inflation expectations do not diverge upward or downward. This argument serves as a counterpoint to frequent criticisms of MMT, asserting that fiscal-dominant economic policies envisioned by MMT cannot stabilize the economy.

Keywords: MMT, Dynamic (in)stability, Dynamic Keynesian Model, Dynamic Optimization
JEL Classification Number: C61, E12, E31, E52, E62, E63

I. Introduction

Since the Global Financial Crisis, advanced economies had persisted in prolonged accommodative monetary policies, yet they struggled with continued low growth and unmet inflation targets. In the midst of these challenges, what gained prominence around 2018 was Modern Monetary Theory (MMT), which places a primary focus on fiscal policy. MMT traces its roots to Keynesian theory, but it has developed unique propositions regarding monetary perspectives and policy discussions. While MMT has faced criticism from mainstream economists and others[1], there appears to be limited agreement with its arguments, and support for its assertions is relatively scarce[2].

First of all, MMT, as the name suggests, begins with the question of what money is and how it is supplied within the private sector and how it collects back afterwards as tax payments. To explain briefly, according to Wray (1998), Wray (2015) and Kelton (2020), what sets MMT apart from conventional mainstream economics is the fundamental belief that the source of money circulation lies in government currency issuance through fiscal policy. This means that tax revenue is not a source of funding for the government budget in a nation that issues its own currency. While this implies that the government can execute fiscal policy freely, there are constraints in the form of limits on labor and resources, or supply constraints. If demand on goods exceeds supply constraints, in general, the economy enters an inflationary environment. MMT proponents acknowledge the need to set limits to prevent excessive inflation. Specifically, they propose Job Guarantee Program (JGP), which aims to continue government employment until full employment is achieved during times of insufficient demand. Conversely, if demand becomes excessive, the government controls private-sector employment needs and inflation rates by setting wages lower than those in the private sector. These are purely fiscal policy methodologies, and monetary policy plays no role here. The basic premise of MMT is that fiscal policy is the driver while monetary policy is passive. In the real world, monetary policy often seems dominant, which is the opposite of MMT's perspective.

Regarding the feasibility of the JGP, this paper does not delve into micro-level discussions. However, critics from both the public and mainstream economics often raise concerns about MMT, primarily related to fears of excessive inflation or the diversion of public debt due to an unlimited expansion of government deficits, even before assessing the viability of JGP. For instance, Globerman (2020) and Mankiw (2019) claim that controlling inflation is

[1] For instance, Krugman (2019) and Summers (2019).

[2] For instance, Mackintosh (2021).

challenging under the MMT regime.

As a response to these criticisms, investment-savings (IS) balance is considered useful, which is another fundamental aspect of MMT. This is because when considering the IS balance, as the government deficit increases, private savings such as households and businesses, should also increase by an equivalent amount (assuming we ignore foreign factors). It is unlikely that the private sector would hoard vast savings without engaging in consumption or investment. Abba Lerner, a key figure in MMT, also discussed this in Lerner (1943), stating that "as the national debt increases, it acts as a self-equilibrating force, gradually diminishing the further need for its growth and finally reaching an equilibrium level where its tendency to grow comes to a complete end. The greater the national debt, the greater the quantity of private wealth." He also pointed out that "an increase in private spending makes it less necessary for the government to undertake deficit financing to maintain total spending at a level that ensures full employment." Simply put, from these observations, whether or not we adopt the idea of MMT, even if fiscal deficits expand to address insufficient demand, there are limits to how much they can expand, and demand cannot keep increasing indefinitely, nor can inflation continue rising.

This paper, using mathematical equations, demonstrates that even in a macroeconomic model incorporating MMT principles, it does not lead to the divergence of inflation or fiscal deficits that mainstream critics often fear, but instead, reaches a stable equilibrium. Past studies with similar objectives that utilize MMT, such as Asada (2020) and Matsumoto (2023c), have also attempted to achieve similar outcomes. In addition to these past findings, this paper seeks to confirm whether the same stability at equilibrium can be achieved using dynamic optimization methods, which enables to draw an optimal fiscal path of the consolidated government.

This paper is particularly inspired by Matsumoto (2023c), which examine the stability/instability of a dynamic Keynesian model with a focus on MMT. This paper has common feature in a sense that dynamic equations for money supply, government expenditure, and the expected inflation rate are formulated. Here, we attempt to move a little step forward by incorporating a dynamic optimization analysis to investigate whether similar conclusions can be obtained.

As for other MMT-related papers, examples include Tanaka (2021), in which the author analyzes models incorporating functional fiscal policies of MMT while utilizing microeconomic and neoclassical frameworks to examine consumer utility maximization with utility functions and budget constraints and profit maximization for firms under monopolistic competition. The idea of functional fiscal policies, or “functional finance”, the term first described by Abba Lerner, is considered as the root of MMT. Lerner (1943) explains it “The principle of judging fiscal measures by the way they work or function in the economy", "not to any established traditional doctrine about what is sound or unsound". In contrast to Tanaka (2021), Asada (2020) investigates stability and instability of equilibrium points based on coordinated financial and fiscal policies by using dynamic Keynesian model.

Also, Asada, Demetrian, Zimka and Zimková (2023) use a four dynamic equation model consisting of fiscal policy, inflation expectations, money growth rate, and employment rate, in a dynamic Keynesian model incorporating MMT ideas, with active fiscal policy and passive monetary policy in financing the government debt. The study concluded that if the policy authorities pursue proactive fiscal and monetary policies and can garner credibility in their inflation target, it is possible to stabilize the economy.

The paper is structured as follows: Section II defines the variables and presents the model's construction of the goods and money markets. In Section III, the induced system of dynamic equations is presented. In Section IV, we present dynamic optimization method to analyze the dynamic equations. Section V examines the stability and instability of equilibrium points. Section VI considers the economic and policy implications of the previous analysis. Finally, Section VII gives concluding remarks.

II. Formulation of the IS-LM Part of the Model

In this chapter, we will explain the framework and nature of this paper. The symbols used are as follows, and the dot above a symbol denotes a derivative with respect to time.

$Y =$ real national income (real output). $K =$ real capital stock. $y = \frac{Y}{K} =$ output-capital ratio. $C =$ consumption expenditure. $c =$ marginal propensity to consume. $I =$ real private investment expenditure. $i = \frac{I}{K} =$ rate of investment. $G =$ real government expenditure. $g = \frac{G}{K} =$ government expenditure-capital ratio. $\rho =$ nominal rate of interest of public bonds. $p =$ price level. $T =$ real income tax. $\tau = \frac{T}{K} =$ real income tax-capital ratio. $B =$ nominal stock of public debt (public bond). $M =$ nominal money supply. $m = \frac{M}{PK}$ = money-capital ratio.

Here, we start with the equilibrium condition of the goods market.

$$Y = C + I + G \tag{1}$$

where,

$$C = c\left[Y + \rho\left(\frac{B}{P}\right) - T\right] \ ; \ 0 < c < 1 \tag{2}$$

By substituting Eq(2) to Eq(1), we have Eq(3).

$$Y = \frac{1}{1-c}\left\{c\left[\rho\left(\frac{B}{P}\right) - T\right] + I + G\right\} \tag{3}$$

We divide both sides of Eq(3) by K to derive the equilibrium condition of the goods market, which we call as an IS equation. Here, $b = \frac{B}{PK} =$ public debt-capital ratio. Rate of investment is a function of real expected rate of interest, where $i = i(\rho - \pi^e)$, and $\frac{di}{d(\rho-\pi^e)} < 0$.

$$y = \frac{1}{1-c}\{c[\rho(y,m)b - \tau] + i(\rho(y,m) - \pi^e) + g\} \tag{4a}$$

We totally differentiate above equation with respect to y and g (We assume that τ is fixed for simplicity following Asada (2020) and Asada, Chiarella, Flaschel and Franke (2003)).

$$dy = \frac{c}{1-c}\left(\rho_y dy + i_\rho \rho_y\right) + \frac{1}{1-c} d_g$$

$$\left\{1 - \frac{c}{1-c}\left(1 + i_\rho\right)\rho_y\right\} dy = \frac{1}{1-c} dg$$

$$\frac{\partial y}{\partial g} = (\frac{c}{1-c})(\frac{1}{1 - \frac{c}{1-c}(1 + i_\rho)\rho_y})$$

Solving Eq(4a) with respect to y, we have Eq(4b).

$$y = y(g, m, b, \pi^e) \tag{4b}$$

Nominal interest rate above is defined by LM equation.

$$\frac{M}{p} = l(\rho)Y\ ;\ l_\rho < 0$$

$$m = l(\rho)y$$

$$\rho = \rho(y, m)$$

$$\rho_y = \frac{\partial \rho}{\partial y} = -\frac{l}{l_\rho y} > 0$$

$$\rho_m = \frac{\partial \rho}{\partial m} = \frac{1}{l_\rho y} < 0$$

(5)

Here, we assume the economy is facing a "liquidity trap", in which the economy is stagnant and interest rate is too low so that the increase in money supply is ineffective to nominal interest rate and output.

If $|l_\rho| \approx \infty$,

$$\rho_m \approx 0,\ y_m \approx 0 \text{ and } \rho_y \approx 0.$$

(6)

We determine the characteristics of each variable through total differentiation of Eq (4). Partial derivatives are given below.

$$y_g = \frac{\partial y}{\partial g} \approx \frac{1}{1-c} > 0 \tag{7}$$

$$y_m = \frac{\partial y}{\partial m} \approx 0 \tag{8}$$

$$y_{\pi^e} = \frac{\partial y}{\partial \pi^e} = -\frac{1}{1-c} i_{\rho-\pi^e} > 0 \tag{9}$$

III. Dynamic Equations

In this section, we derive the dynamic equations for each variable. Below are some of the definitional equations.

$$\dot{K} = I \tag{10}$$

$$\frac{\dot{K}}{K} = \frac{I}{K} = i(\rho - \pi^e) \tag{11}$$

$$\frac{\dot{P}}{P} = \pi \tag{12}$$

$$\pi = \varepsilon(y - \bar{y}) + \pi^e \quad ; \varepsilon > 0 \tag{13}$$

$$\frac{\dot{M}}{M} = \mu \tag{14}$$

Eq(10) is the rate of capital accumulation that is equivalent to investment expenditure. Eq(11) is the investment function of firms, which is based on the standard Keynesian theory[3]. The equation shows that investment is the decreasing function of the expected real rate of interest. Eq(12) represents the growth rate of price equals inflation rate. Eq(13) is the conventional linear "expectations-augmented Phillips curve", which ε is the reaction parameter from the output gap. $\bar{y}$ is the natural output-capital ratio, and π^e is inflation expectation variable. Eq(14) shows the growth rate of money.

Using Eqs(11) , (12) ,(14) and money-capital ratio m, dynamic law of the motion of the

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[3] See Keynes (1936), Asada and Ouchi (2009).

money-capital ratio can be described as below.

$$\frac{\dot{m}}{m} = \frac{\dot{M}}{M} - \frac{\dot{P}}{P} - \frac{\dot{K}}{K} = \mu - \pi - i(\rho - \pi^e)$$

$$\dot{m} = m[\mu - \pi - i(\rho - \pi^e)] \quad (15)$$

We can substitute each variable with equations we previously derived.

$$\dot{m} = [\mu + \varepsilon(\bar{y} - y(g, m, b, \pi^e)) - \pi^e - i(\rho(y(g, m, b, \pi^e), m) - \pi^e)]m \quad (16)$$

The dynamic equation of inflation expectation is described below. This equation represents the dynamics of inflation expectations by capturing the distinction between forward-looking and backward-looking expectation formations using θ. If θ is close to 1, it signifies a forward-looking expectation formation with a strong influence from the inflation target, whereas if it is closer to zero, it indicates a backward-looking expectation formation influenced by the actual inflation rate. Therefore, θ can be referred to as the "credibility parameter" with respect to the policy authority's inflation target. This formulation originates from Asada, Chiarella, Flaschel and Franke (2003) and Asada (2020). γ can be interpreted as a reaction parameter to inflation expectation from both of the gaps of inflation target and output.

$$\dot{\pi}^e = \gamma[\theta(\bar{\pi} - \pi^e) + (1 - \theta)(\pi - \pi^e)] \ ; \ (\gamma > 0, 0 \leq \theta \leq 1)$$

$$\dot{\pi}^e = \gamma\{\theta(\bar{\pi} - \pi^e) + (1 - \theta)[\varepsilon[y(g, m, b, \pi^e) - \bar{y}]]\} \quad (17)$$

From these dynamic equations, we can rewrite as below.

$$\dot{m} = F_1(g, m, b, \pi^e)$$

$$\dot{\pi}^e = F_2(g, m, b, \pi^e) \quad (18)$$

Next, we formulate the equation that also symbolizes the characteristic of MMT. Eq(19) below is the “budget constraint” of the consolidated government, including the central bank. implying that the government deficit must be financed through the issuance of new high-powered money (monetary base) or government bonds. We acknowledge that the MMT proponents do not have the idea of “budget constraint” of government since they can always print their own money to finance the debt, but we still think this equation meaningfully describe their basic idea. The formulation is based on Asada (2020) and Mitchell, Wray, and

Watts (2019).

$$\dot{H} + \dot{B} = PG + \rho B - PT \tag{19}$$

Where, $H =$ high-powered money (monetary base). $B =$ nominal stock of public debt (public bond). This simply tells that the government expenditure including the interest payment of government bond is financed with the issuance of government bonds, high-powered money, or tax.

Money supply is described below, as the product of high-powered money and ν, which is the money multiplier (constant). These are differentiated with respect to time.

$$M = \nu H \quad ; \nu > 1 \tag{20}$$

$$\dot{M} = \nu \dot{H}$$

$$\dot{H} = \frac{1}{\nu}\dot{M}$$

$$= \frac{1}{\nu}\mu M \tag{21}$$

Below is the growth rate of nominal public debt stock.

$$\mu_B = \frac{\dot{B}}{B}\ ,\ \dot{B} = \mu_B B \tag{22}$$

Thus, it can be rewritten as below.

$$\frac{1}{\nu}\mu M + \mu_B B = PG + \rho B - PT \tag{23}$$

We divide this by price and capital to transform into a real term with respect to capital.

$$\frac{1}{\nu}\mu\left(\frac{M}{PK}\right) + \mu_B\left(\frac{B}{PK}\right) = \frac{G}{K} + \rho\left(\frac{B}{PK}\right) - \tau$$

$$\frac{1}{\nu}\mu m + \mu_B b = g + \rho b - \tau \tag{24}$$

We assume that $\tau = \frac{T}{K}$ is constant for simplicity following Asada (2020) and Asada, Chiarella, Flaschel and Franke (2003). We also assume that the government controls the issuance of public debt, which means that they could keep its level constant, as described in Eq(25). Eq(26) is the equation of growth rate of nominal public debt stock.

$$b = \frac{B}{PK} = \bar{b} \tag{25}$$

$$0 = \frac{\dot{b}}{b} = \frac{\dot{B}}{B} - \frac{\dot{P}}{P} - \frac{\dot{K}}{K} = \mu_B - \pi - i(\rho - \pi^e) = \mu_B - \varepsilon(y - \bar{y}) - \pi^e - i(\rho - \pi^e)$$

$$\mu_B = \varepsilon(y - \bar{y}) + \pi^e + i(\rho - \pi^e)$$
$$= \varepsilon\big(y(g, m, \bar{b}, \pi^e) - \bar{y}\big) + \pi^e + i\big(\rho(g, m, \bar{b}, \pi^e) - \pi^e\big) \tag{26}$$

We can insert Eq(26) into Eq(24).

$$\frac{1}{v}\mu m + [\varepsilon(y - \bar{y}) + \pi^e + i(\rho - \pi^e)]\bar{b} = g + \rho\bar{b} - \tau$$

$$\mu m = v\Big\{g + \rho(y(g, m, \bar{b}, \pi^e), m)\bar{b} - \tau$$
$$- \Big[\big[\varepsilon\big(y(g, m, \bar{b}, \pi^e) - \bar{y}\big) + \pi^e + i\big(\rho(y(g, m, \bar{b}, \pi^e)) - \pi^e\big)\big]b\Big]\Big\} \tag{27}$$

This is inserted into Eq(16).

We now have two dynamic equations, which are real money-capital ratio and inflation expectation, respectively from Eq(16) and (17),

$$\dot{m} = v\Big\{g + \rho\big[y(g, m, \bar{b}, \pi^e), m\big]\bar{b} - \tau$$
$$- \Big[\big[\varepsilon\big(y(g, m, \bar{b}, \pi^e) - \bar{y}\big) + \pi^e + i\big(\rho(y(g, m, \bar{b}, \pi^e)) - \pi^e\big)\big]b\Big]\Big\}$$
$$- m\{\varepsilon\big(y(g, m, \bar{b}, \pi^e) - \bar{y}\big) + \pi^e + i\big(\rho(y(g, m, \bar{b}, \pi^e)) - \pi^e\big)\}$$
$$= f_1(g(t), m(t), \pi^e(t))$$

$$\dot{\pi}^e = \gamma\Big\{\theta(\bar{\pi} - \pi^e) + (1 - \theta)\Big[\varepsilon\big[y\big(g, m, \bar{b}, \pi^e\big) - \bar{y}\big]\Big]\Big\} = f_2(g(t), m(t), \pi^e(t)) \tag{28}$$

IV. Dynamic Optimization

From here, we use the dynamic equations formalized earlier to solve for the maximization (minimization) of the objective functional, or a social loss functional, through dynamic optimization. Our objective functional is aimed at minimizing the inflation gap and output gap[4]. Within the dynamic system of given money and inflation expectations, government expenditure serves as the control variable.

$$\max_{g(t)} \int_0^{\infty} -\{\xi(\pi-\bar{\pi})^2+(1-\xi)(y-\bar{y})^2\}e^{-rt}\,dt$$

$$= \max_{g(t)} \int_0^{\infty} -\left\{\xi\left[\varepsilon\left(y\left(g,m,\bar{b},\pi^e\right)-\bar{y}\right)+\pi^e-\bar{\pi}\right]^2-(1-\xi)\left(y\left(g,m,\bar{b},\pi^e\right)-\bar{y}\right)^2\right\}e^{-rt}\,dt \ ;$$

$$0<\xi<1$$

$s.t.$

$$\dot{m} = \nu\left\{g+\rho\left[y\left(g,m,\bar{b},\pi^e\right),m\right]\bar{b}-\tau\right.$$
$$-\left[\left[\varepsilon\left(y\left(g,m,\bar{b},\pi^e\right)-\bar{y}\right)+\pi^e+i\left(\rho\left(y\left(g,m,\bar{b},\pi^e\right)\right)-\pi^e\right)\right]b\right]\Big\}$$
$$-m\left\{\varepsilon\left(y\left(g,m,\bar{b},\pi^e\right)-\bar{y}\right)+\pi^e+i\left(\rho\left(y\left(g,m,\bar{b},\pi^e\right)\right)-\pi^e\right)\right\}$$
$$= f_1(g(t),m(t),\pi^e(t))$$
$$\dot{\pi}^e = \gamma\left\{\theta(\bar{\pi}-\pi^e)+(1-\theta)\left[\varepsilon\left[y\left(g,m,\bar{b},\pi^e\right)-\bar{y}\right]\right]\right\} = f_2(g(t),m(t),\pi^e(t)) \ , \tag{29}$$

where r is the discount rate that is treated as a positive parameter.

Current value Hamiltonian and Pontryagin's maximum-principle conditions[5] are described below.

$$H = -\xi\left[\varepsilon\left(y\left(g,m,\bar{b},\pi^e\right)-\bar{y}\right)+\pi^e-\bar{\pi}\right]^2-(1-\xi)\left(y\left(g,m,\bar{b},\pi^e\right)-\bar{y}\right)^2$$
$$+\phi_1(t)f_1\left(g(t),m(t),\pi^e(t)\right)+\phi_2(t)f_2(g(t),m(t),\pi^e(t))$$

$$\max_{g(t)} H(g,m,\pi^e)\ for\ all\ t\in[0,\infty]$$

$$\dot{m}(t)=\frac{\partial H(t)}{\partial\phi_1(t)}=f_1(g(t),m(t),\pi^e(t)) \quad [equation\ of\ motion\ for\ \dot{m}]$$

$$\dot{\pi}^e=\frac{\partial H(t)}{\partial\phi_2(t)}=f_2(g(t),m(t),\pi^e(t)) \quad [equation\ of\ motion\ for\ \dot{\pi}^e]$$

$$\dot{\phi_1}(t)=-\frac{\partial H(t)}{\partial m(t)}+r\phi_1(t) \quad [equation\ of\ motion\ for\ \phi_1]$$

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[4] Similar approach was taken by Matsumoto (2023a) and Matsumoto (2023b), and Taylor (1989).

[5] Explanation on Pontryagin's maximum principle conditions and dynamic optimization are available from Chiang (1992), Chiang and Wainwright (2005).

$$\dot{\phi_2}(t) = -\frac{\partial H(t)}{\partial \pi^e(t)} + r\phi_2(t) \qquad [equation\ of\ motion\ for\ \phi_2]$$

$$\lim_{t\to\infty} \phi_1 e^{-rt} = 0\,,\quad \lim_{t\to\infty} \phi_2 e^{-rt} = 0 \qquad [transversality\ conditions]$$

(30)

A first order condition is required to show that the control of g will be an interior solution.

$$\frac{\partial H}{\partial g(t)} = -2\xi\varepsilon y_g\left[\varepsilon\left(y\left(g,m,\bar{b},\pi^e\right)-\bar{y}\right)+\pi^e-\bar{\pi}\right]-2(1-\xi)y_g\left(y\left(g,m,\bar{b},\pi^e\right)-\bar{y}\right) + \phi_1\left[v\left\{1-\bar{b}\left[\rho_y y_g+\varepsilon y_g+i_{\rho-\pi^e}\rho_y y_g\right]\right\}-m\left\{\varepsilon y_g+i_{\rho-\pi^e}\rho_y y_g\right\}\right]+\phi_2\gamma(1-\theta)\varepsilon y_g = 0$$

(31)

Here, we figure the characteristics of the above equation of the first order condition, which we derived in Eq(31).

$$G(g,m,\pi^e,\phi_1,\phi_2) = 0 \qquad (32)$$

We now solve the total derivation, as shown below. G_g is equivalent to Eq (31).

$$G_m = -\phi_1\varepsilon y_g$$

$$G_{\pi^e} = -2\xi\varepsilon y_g(\varepsilon y_{\pi^e}+1) - 2(1-\xi)y_g y_{\pi^e} < 0$$
$$G_{\phi_1} = v\left(1-\varepsilon y_g\bar{b}\right) - m\varepsilon y_g$$
$$G_{\phi_2} = \gamma(1-\theta)\varepsilon y_g > 0$$

(33)

By combining above equations, we can derive the following, under equilibrium.

$$g_m = \frac{\partial g}{\partial m} = -\frac{G_m}{G_g}$$
$$g_{\pi^e} = \frac{\partial g}{\partial \pi^e} = -\frac{G_{\pi^e}}{G_g} < 0$$
$$g_{\phi_1} = \frac{\partial g}{\partial \phi_1} = -\frac{G_{\phi_1}}{G_g}$$
$$g_{\phi_2} = \frac{\partial g}{\partial \phi_2} = -\frac{G_{\phi_2}}{G_g} > 0$$
$$g = g(m(t),\pi^e(t),\phi_1(t),\phi_2(t))$$

(34)

Further differentiation of Eq(31) with the result of negative shows that the control variable g does maximize the Hamiltonian system.

$$\frac{\partial^2 H}{\partial g^2} = -2{y_g}^2\{\xi\varepsilon^2 - (1-\xi)\} < 0$$

The condition to fulfill above equation is shown below.

$$\varepsilon^2 > \frac{1-\xi}{\xi}\ ; 0 \le \xi \le 1$$

$$\xi > \frac{1}{1+\varepsilon^2}$$

(35)

Solving the maximum-principle conditions give us equations below.

$$\begin{aligned}\dot{m}(t) = \nu \Big\{ & g(m(t),\pi^e(t),\phi_1(t),\phi_2(t)) + \rho\big[y\big(g(m(t),\pi^e(t),\phi_1(t),\phi_2(t)),m,\bar{b},\pi^e\big),m\big]\bar{b} - \tau \\ & - \Big[\big[\varepsilon\big(y\big(g(m(t),\pi^e(t),\phi_1(t),\phi_2(t)),m,\bar{b},\pi^e\big) - \bar{y}\big) + \pi^e \\ & + i\big(\rho(y\big(g(m(t),\pi^e(t),\phi_1(t),\phi_2(t)),m,\bar{b},\pi^e)) - \pi^e\big)\big]b\Big]\Big\} \\ & - m\{\big[\varepsilon\big(y\big(g\big(m(t),\pi^e(t),\phi_1(t),\phi_2(t)\big),m,\bar{b},\pi^e\big) - \bar{y}\big) + \pi^e\big] \\ & + i\big[\rho\big(y\big(g(m(t),\pi^e(t),\phi_1(t),\phi_2(t)),m,\bar{b},\pi^e\big),m\big) - \pi^e\big]\} \\ & = F_1(m(t),\pi^e(t),\phi_1(t),\phi_2(t))\end{aligned}$$

$$\begin{aligned}\dot{\pi}^e(t) = \gamma\{\theta(\bar{\pi} - \pi^e) + (1-\theta)\big[\varepsilon[y(g(m(t),\pi^e(t),\phi_1(t),\phi_2(t)),m,b,\pi^e) - \\ \bar{y}]\big]\} = F_2(m(t),\pi^e(t),\phi_1(t),\phi_2(t))\end{aligned}$$

$$\begin{aligned}\dot{\phi}_1(t) = 2\xi\varepsilon y_m & \big[\varepsilon(y\big(g\big(m(t),\pi^e(t),\phi_1(t),\phi_2(t),m,\bar{b},\pi^e\big) - \bar{y}\big) + \pi^e - \bar{\pi})\big] \\ & + 2(1-\xi)y_m\big[y\big(g\big(m(t),\pi^e(t),\phi_1(t),\phi_2(t)\big),m,\bar{b},\pi^e\big) - \bar{y}\big] \\ & - \phi_1\{v\{\big(\rho_y y_m + \rho_m\big)\bar{b} - \big[\varepsilon y_m + i_{\rho-\pi^e}\big(\rho_y y_m + \rho_m\big)\big]\bar{b}\} \\ & - m\big[\varepsilon y_m + i_{\rho-\pi^e}\big(\rho_y y_m + \rho_m\big)\big] \\ & - \varepsilon(y\big(g\big(m(t),\pi^e(t),\phi_1(t),\phi_2(t),m,\bar{b},\pi^e\big) - \bar{y}\big) + \pi^e \\ & - i\big[\rho\big(y\big(m(t),\pi^e(t),\phi_1(t),\phi_2(t),m,\bar{b},\pi^e\big),m\big) - \pi^e\big]\} - \phi_2\{\gamma[(1-\theta)\varepsilon y_m]\} \\ & + r\phi_1(t) = F_3(m(t),\pi^e(t),\phi_1(t),\phi_2(t);\ r)\end{aligned}$$

$$\dot{\phi}_2(t) = 2\xi(\varepsilon y_{\pi^e} + 1)\big[\varepsilon(y\big(g\big(m(t), \pi^e(t), \phi_1(t), \phi_2(t)\big), m, \bar{b}, \pi^e\big) - \bar{y})\big]$$
$$+ 2(1-\xi)y_{\pi^e}\big[y\big(g\big(m(t), \pi^e(t), \phi_1(t), \phi_2(t)\big), m, \bar{b}, \pi^e\big) - \bar{y}\big]$$
$$- \phi_1\Big\{v\{\rho_y y_{\pi^e}\bar{b} - [\varepsilon y_{\pi^e} + 1 + i_{\rho-\pi^e}(\rho_y y_{\pi^e} - 1)]\bar{b}\}$$
$$- m\{\varepsilon y_{\pi^e} + 1 - i_{\rho-\pi^e}(\rho_y y_{\pi^e} - 1)\}\Big\} - \phi_2\{\gamma[-\theta + (1-\theta)\varepsilon y_{\pi^e}]\} + r\phi_2(t)$$
$$= F_4(m(t), \pi^e(t), \phi_1(t), \phi_2(t); r)$$

(36)

In the state of long-run equilibrium, we expect the output and inflation to equal each of the policy target. The costate variables should also become zero[6]. Applying this assumption to the dynamic equations, we have the following results.

$$y^* = \bar{y}$$
$$\pi^{e*} = \bar{\pi}$$
$$\phi_1{}^* = 0$$
$$\phi_2{}^* = 0$$

(37)

If we put figures below on each dynamic equations, we can analyze the characteristics of the equilibrium.

$$\dot{m} = \dot{\pi}^e = \dot{\phi}_1 = \dot{\phi}_2 = 0\,.$$

[6] This interpretation is explained by Intriligator (1971) and Chiang (1992). According to Intriligator (1971) chapter 14 and Chiang (1992) chapter 8, we have

$$[\phi_1(t)e^{-rt}]_{t=0} = \phi_1(0) = \frac{\partial W^*}{\partial m(0)}, \quad \text{(F1)}$$

$$[\phi_2(t)e^{-rt}]_{t=0} = \phi_2(0) = \frac{\partial W^*}{\partial \pi^e(0)}, \quad \text{(F2)}$$

where W^* is the optimal value of

$$W = \int_0^\infty -\{\xi(\pi(t) - \bar{\pi})^2 + (1-\xi)(y(t) - \bar{y})^2\}e^{-rt}\,dt. \quad \text{(F3)}$$

In this case, we have

$$\frac{\partial W^*}{\partial m(0)} = \int_0^\infty -2\left\{\xi(\pi^*(t) - \bar{\pi})\frac{\partial \pi^*(t)}{\partial m(0)} + (1-\xi)(y^*(t) - \bar{y})\frac{\partial y^*(t)}{\partial m(0)}\right\}e^{-rt}\,dt, \quad \text{(F4)}$$

$$\frac{\partial W^*}{\partial \pi^e(0)} = \int_0^\infty -2\left\{\xi(\pi^*(t) - \bar{\pi})\frac{\partial \pi^*(t)}{\partial \pi^e(0)} + (1-\xi)(y^*(t) - \bar{y})\frac{\partial y^*(t)}{\partial \pi^e(0)}\right\}e^{-rt}\,dt, \quad \text{(F5)}$$

Where $\pi^*(t)$ and $m^*(t)$ are $\pi(t)$ and $m(t)$ at the optimal path. Consider that we are in the equilibrium point such that $m(t) = m^*$, $\pi(t) = \pi^* = \bar{\pi}$, $\pi^e(t) = \pi^{e*} = \bar{\pi}$, $y(t) = y^* = \bar{y}$, $\phi_1(t) = \phi^*$, $\phi_2(t) = \phi^*$ for all $t \geqq 0$. Substituting these conditions to equations (F1), (F2), (F4) and (F5), we obtain

$$\phi_1{}^* = \phi_2{}^* = 0. \quad \text{(F6)}$$

$$\dot{m} = v\{g(m^*, \bar{\pi}, 0, 0) - \tau - b[\rho(\bar{y}, m^*) - \bar{\pi} - i[\rho(\bar{y}, m^*) - \bar{\pi}]]\} - m^*\{\bar{\pi} - i[\rho(\bar{y}, m^*) - \bar{\pi}]\}$$
$$= F(m^*) = 0$$
$$\dot{\pi}^e = 0$$
$$\dot{\phi}_1 = 0$$
$$\dot{\phi}_2 = 0$$

(38)

Below is the partial derivative of $\dot{m}$ with respect to m.

$$F'(m) = v\{g_{m^*} + \bar{b}[\rho_{m^*} + i_{\rho-\pi^e}\rho_{m^*}]\} + i_{\rho-\pi^e}\rho_{m^*m^*} - \bar{\pi} + i[\rho(\bar{y}, m^*) - \bar{\pi}] > 0$$

(39)

This gives the characteristic of optimal level of money capital ratio, m^*, described below.

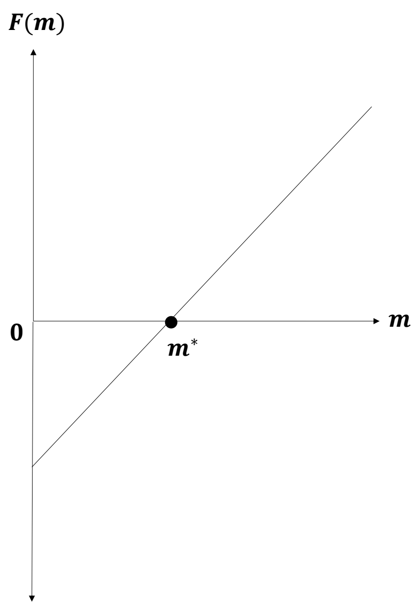

Figure 1

V. Local Stability/Instability of the Long-run Equilibrium Point

In this section, we identify the state of local stability/instability of the long-run equilibrium point of our dynamic system. Jacobian matrix of the four-dimensional dynamic system at the equilibrium point is described below. Here, the liquidity trap is assumed again. In reality, the response of interest rates to increases in money or output is not entirely zero, but for the sake of simplification, it is treated as $\rho_m \approx 0$, $y_m \approx 0$ and $\rho_y \approx 0$.

$$J = \begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{14} \\ F_{21} & F_{22} & F_{23} & F_{24} \\ 0 & 0 & F_{33}(r) & 0 \\ 0 & F_{42} & F_{43} & F_{44}(r) \end{bmatrix}$$

(40)

In Appendix, the detailed expressions of the partial derivatives of Eq (40) are presented. We can write the characteristic equation of the Jacobian matrix as follows.

$$\Gamma(\lambda) \equiv |\lambda I - J| = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0 \ ,$$

where

$$a_1 = -traceJ_4 = -F_{11}(\varepsilon) - F_{22} - F_{33} - F_{44} \ ,$$
$$a_2 = sum\ of\ all\ principal\ second\ order\ minors\ of\ J \ ,$$
$$a_3 = -(sum\ of\ all\ principal\ third\ order\ minors\ of\ J) \ ,$$
$$a_4 = detJ \ .$$

(41)

In the conventional dynamic system in which all initial values of the endogenous variables are given, the equilibrium point of the dynamic system is considered to be locally stable if and only if all of the roots of the characteristic equation have negative real parts, which is defined as "Routh-Hurwitz conditions", shown below. The system is considered as unstable if any one of the roots has a positive real part[7].

$$a_j > 0 \ (j = 1,2,3,4), \ \ a_1 a_2 a_3 - {a_1}^2 a_4 - {a_3}^2 > 0 \ .$$

(42)

Using the results shown in the appendix, we can find that the above condition cannot be attained because of the following inequality.

$$a_1 = -traceJ_4 = -2r < 0 \ .$$

(43)

This tells that at least one root has positive real part, and the dynamic system is unstable in the conventional sense. However, in this 4-dimensional dynamic system derived from the conditions of dynamic optimization, it is not assumed that the initial values of all endogenous variables are given. What is assumed as given in this system are the initial values of two 'state variables,' m and π , and it is anticipated that the initial values of the

[7] We follow steps taken by Asada (2024).

remaining two 'auxiliary variables' can be freely chosen by the consolidated government including the central bank, which is the subject of dynamic optimization. The state variables mentioned here are referred to as pre-determined variables, while the auxiliary variables are called not-pre-determined variables or jump variables. The consolidated government, being the planner, can achieve dynamic optimization only by selecting initial values for the auxiliary variables that satisfy 'transversality conditions' with respect to the given state variables and converge to equilibrium points. The dynamic system described here is considered locally stable and determinate only when the initial values determining such a path are uniquely determined.

Therefore, in the dynamic system described here, it is considered locally “stable” and “determinate” only when the characteristic equation has two roots with negative real parts and two roots with positive real parts. According to Dockner and Feichtinger (1991), if the inequalities represented by $E \equiv a_2 - r^2 < 0$ and $detJ > 0$ simultaneously hold in this dynamic optimization system with two state variables, then the characteristic equation's two roots have negative real parts, and the remaining two roots have positive real parts, leading to the equilibrium point being a locally saddle point. The following verifies whether this definition holds true[8].

Proposition

If the "credibility parameter", θ is sufficiently close to 1, and the inflation target $\bar{\pi}$ satisfies the inequality $\bar{\pi} > i[\rho(\bar{y}, m^*) - \bar{\pi}]$, the dynamic system is locally “stable” and “determinant”.

Proof

We use the solutions which are provided in Appendix to see if the conditions of Dockner and Feichtinger (1991) are met under the condition $\theta = 1$.

$$a_2 = a_2(r) \equiv sum\ of\ all\ principal\ second\ order\ minors\ of\ J$$

$$= \begin{vmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{vmatrix} + \begin{vmatrix} F_{11} & F_{13} \\ F_{31} & F_{33(r)} \end{vmatrix} + \begin{vmatrix} F_{11} & F_{14} \\ F_{41} & F_{44(r)} \end{vmatrix} + \begin{vmatrix} F_{22} & F_{23} \\ F_{32} & F_{33(r)} \end{vmatrix} + \begin{vmatrix} F_{22} & F_{24} \\ F_{42} & F_{44(r)} \end{vmatrix} + \begin{vmatrix} F_{33(r)} & F_{34} \\ F_{43} & F_{44(r)} \end{vmatrix}$$

$$= (F_{11} + F_{22})\{F_{33}(r) + F_{44}(r)\} + F_{33}(r)F_{44}(r) + A$$

$$= \{-\bar{\pi} + i[\rho(\bar{y}, m^*) - \bar{\pi}] - \gamma\}\{\bar{\pi} - i[\rho(\bar{y}, m^*) - \bar{\pi}] + \gamma[\theta - (1-\theta)\varepsilon y_{\pi^e}] + 2r\}$$
$$+ \{[\bar{\pi} - i[\rho(\bar{y}, m^*) - \bar{\pi}] + r][\gamma[\theta - (1-\theta)\varepsilon y_{\pi^e}] + r]\} + A$$

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[8] Follows the method by Asada (2024).

$$= r^2 - r\bar{\pi} + ri[\rho(\bar{y}, m^*) - \bar{\pi}] - r\gamma + B$$

$$\boldsymbol{E(r)} \equiv a_2(r) - r^2 = r\{-\bar{\pi} + i[\rho(\bar{y}, m^*) - \bar{\pi}] - \gamma\} + B \ ,$$

where A and B are independent of r.

$E(r)$ is negative if the condition below is met and r is sufficiently large.

$$i[\rho(\bar{y}, m^*) - \bar{\pi}] < \bar{\pi} + \gamma \ . \tag{44}$$

In terms of $a_4 = detJ$,

$$a_4 = detJ = F_{44}(r)\begin{vmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33}(r) \end{vmatrix} + F_{43}\begin{vmatrix} F_{11} & F_{12} & F_{14} \\ F_{21} & F_{22} & F_{24} \\ F_{31} & F_{32} & F_{34} \end{vmatrix} + F_{42}\begin{vmatrix} F_{11} & F_{13} & F_{14} \\ F_{21} & F_{23} & F_{24} \\ F_{31} & F_{33}(r) & F_{34} \end{vmatrix}$$

$$+ F_{41}\begin{vmatrix} F_{12} & F_{13} & F_{14} \\ F_{22} & F_{23} & F_{24} \\ F_{32} & F_{33(r)} & F_{34} \end{vmatrix}$$

$$= F_{44}(r)F_{33}(r)\begin{vmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{vmatrix} - F_{44}(r)\begin{vmatrix} F_{11} & F_{13} \\ F_{21} & F_{23} \end{vmatrix} + F_{31}\begin{vmatrix} F_{12} & F_{13} \\ F_{22} & F_{23} \end{vmatrix} + C$$

$$= F_{44}(r)F_{33}(r)(F_{11}F_{22} - F_{12}F_{21}) + D$$

$$= \left\{r^2 + r\{\bar{\pi} - i[\rho(\bar{y}, m^*) - \bar{\pi}] + \gamma\}\right\}\left\{\gamma\{\bar{\pi} - i[\rho(\bar{y}, m^*) - \bar{\pi}]\}\right\} + D \ ,$$

where C and D are independent of r.

$a_4 = detJ$ is positive for all sufficiently large values of $r > 0$ if

$$\bar{\pi} > i[\rho(\bar{y}, m^*) - \bar{\pi}]. \tag{45}$$

From the two results above, we can summarize that $E \equiv a_2 - r^2 < 0$ and $detJ > 0$ are satisfied for all sufficiently large $r > 0$ if $\bar{\pi} > i[\rho(\bar{y}, m^*) - \bar{\pi}]$. This result is derived under the assumption that θ =1. However, this result also applies in case of 0< θ <1 as long as θ is close to 1 because of continuity.

□

VI. Economic Interpretation

The proposition we presented, the "credibility parameter" $\theta \approx 1$, signifies a forward-looking expectation formation with a strong influence from the inflation target. This means, the economy can achieve stability of output and inflation at the equilibrium, when the public believes the consolidated government is capable and committed to the inflation target if the discount rate (r) is sufficiently large. Also, the inflation target is required to be sufficiently high, while the nominal interest rate $\rho(\bar{y}, m^*)$ is sufficiently low. The idea that the target needs to be somewhat high is consistent with MMT's argument[9] that setting inflation target too low can come at the expense of employment, and also aligns with the views of so-called mainstream economists, often considered fiscal doves, who argue for a more expansionary fiscal policy[10]. Based on these two assumptions, it can be demonstrated that achieving economic stabilization through a fiscal-policy-led approach, characteristic of MMT, is feasible, which is the main objective of this paper. Conversely, if there is low credibility towards the inflation target, which means the public places more emphasis on past price movements and exhibits backward-looking inflation expectations, and if the inflation target is too low, economic trends are likely to become destabilized.

VII. Concluding Remarks

In this paper, we approached the examination of stability/instability in a dynamic Keynesian model considering MMT through a different perspective by incorporating dynamic optimization, in contrast to the approaches taken by Asada (2020) and Matsumoto (2023c). The dynamic system constructed sets money and inflation expectations as state variables, with an MMT-inspired (fiscally dominant) consolidated government adjusting government expenditure to achieve economic stability. The conclusion suggests a forward-looking environment where economic agents initially trust central bank policy. In the event of a deviation of the inflation rate from the target, the central bank acts to bring it back to the target, ultimately achieving the inflation goal. Additionally, if the consolidated government's inflation target is sufficiently high, the dynamic system converges to an equilibrium point, satisfying stability under sufficiently high discount rate (r). Stability implies that the economy settles at the equilibrium point without experiencing overheating or stagnation, and money supply and inflation expectations do not diverge upward or downward. This argument serves as a counterpoint to frequent criticisms of MMT, asserting that fiscal-dominant economic policies envisioned by MMT cannot stabilize the economy.

One of the common critiques of MMT is that theory lacks mathematical models, which this paper and the aforementioned previous studies aim to address. However, it is important to note that this paper may face criticism from MMT proponents for certain simplifications, keeping central bank's inflation target, and imposing budget constraints, which might be

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[9] Papadimitriou and Wray (1996)

[10] Stiglitz (2008)

considered not entirely MMT-consistent. Consequently, enhancing the model's completeness as a more MMT-centric framework remains a future challenge. Nonetheless, demonstrating the potential for stabilizing the economic system through a dynamic Keynesian model infused with MMT ideas and explaining the existence of optimal paths using dynamic optimization are considered contributions to the discourse.

In addition, there are considerations regarding the dynamic optimization. This model using dynamic optimization assumes that the government is omniscient and has the ability to follow the appropriate path of government expenditure towards the equilibrium point. Therefore, our findings should be considered as a "reference" or "yardstick" to observe how the real-life economic policy is deviating from the optimal path. As such, we may paradoxically say that the real-life policy settings by the policy makers are not optimal.

On the other hand, as mentioned earlier, the analysis of MMT-inspired dynamic models not relying on dynamic optimization are conducted in studies like Asada (2020), Asada, Demetrian, Zimka and Zimková (2023) and Matsumoto (2023). The significance of these analysis lies in the fact that it involves policies of government without complete information of the economic structure.

Appendix

$$F_{11} = -\bar{\pi} + i[\rho(\bar{y}, m^*) - \bar{\pi}]$$

$$F_{12} = v\{g_{\pi^e} - \bar{b}[\varepsilon(y_g g_{\pi^e} + y_{\pi^e}) + 1 - i_{\rho-\pi^e}]\} - m[\varepsilon(y_g g_{\pi^e} + y_{\pi^e}) + 1 + i_{\rho-\pi^e}]$$

$$F_{13} = v\{g_{\phi_1} - \varepsilon y_g g_{\phi_1}\} - m\varepsilon y_g g_{\phi_1}$$

$$F_{14} = v(g_{\phi_2} + \varepsilon y_g g_{\phi_2}\bar{b}) - m\varepsilon y_g g_{\phi_2}$$

$$F_{21} = \gamma[(1-\theta)\varepsilon(y_g g_m + y_m)]$$

$$F_{22} = \gamma[-\theta + (1-\theta)\varepsilon(y_g g_{\pi^e} + y_{\pi^e})]$$

$$F_{23} = \gamma[(1-\theta)\varepsilon y_g g_{\phi_1}]$$

$$F_{24} = \gamma[(1-\theta)\varepsilon y_g g_{\phi_2}]$$

$$F_{31} = 0$$

$$F_{32} = 0$$

$$F_{33}(r) = \bar{\pi} - i[\rho(\bar{y}, m^*) - \bar{\pi}] + r$$

$$F_{34} = 0$$

$$F_{41} = 0$$

$$F_{42} = 2\xi(\varepsilon y_{\pi^e} + 1)\varepsilon(y_g g_{\pi^e} + y_{\pi^e})$$

$$F_{43} = 2\xi(\varepsilon y_{\pi^e} + 1)[\varepsilon(y_g g_{\phi_1})] + 2(1-\xi)y_{\pi^e} y_g g_{\phi_1} - \{v\{(\varepsilon y_{\pi^e} - 1 + i_{\rho-\pi^e})\bar{b}\} - m[\varepsilon y_{\pi^e} + 1 + i_{\rho-\pi^e}]\}$$

$$F_{44}(r) = \gamma[\theta - (1-\theta)\varepsilon y_{\pi^e}] + r$$

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